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VOLUME I

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(Including Liquids and Gases)

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ARITHMETIC

NOTATION AND NUMERATION

1. The art of calculating by numbers is called **arithmetic.**

2. Any single thing is called a **unit.** As *one* crank; *one* lever; *one* interlocking plant, etc.

3. A unit or a collection of units is called a **number.** As *one; two* levers; *two* bolts; *four* signal towers, etc.

4. A number which does not refer to any particular object, or quantity is called an **abstract number.** As *six; eight; ten; eleven,* etc.

5. A number which refers to some particular object or quantity is called a **concrete number.** As *two pounds; ten foundations; fifteen signals, etc.*

6. Numbers which express units of the same kind are called **like numbers.** As *six spikes* and *ten spikes; seven cars* and *ten cars.*

7. Numbers which express units of different kinds are called **unlike numbers.** As *six spikes* and *ten ties; seven cars* and *five bells.*

8. The method of expressing numbers by *figures* or *letters* is called **notation.** The *figure* method is known as **Arabic notation** and the *letter* method is known as **Roman notation.**

9. The art of reading numbers expressed in figures or letters is called **numeration.**

ARABIC NOTATION

10. In Arabic notation ten symbols called **figures** are used, viz:

0	1	2	3	4	5	6	7	8	9
Zero,	*one,*	*two,*	*three,*	*four,*	*five,*	*six,*	*seven,*	*eight,*	*nine.*

The first figure is called **zero, naught** or **cipher,** and has no value when expressed alone. The other figures stand for the words written under them, and have each a given value. All other numbers are written by combining two or more figures. The next higher number than *nine* is *ten,* and is written 10. The figure 1 is in the second place from the right and in this place means not *one unit* but a group of *ten units.* The figure 2 when so placed means *twenty units* and is written 20; the figure 3, *thirty units* and is written 30, etc.

The term **digit** is sometimes applied to these symbols.

11. Any whole number is called an **integer.** As 3, 6, 35, or 251.

12. Figures have **two values,** one of which depends on the position of a figure in relation to other figures.

These values are **simple,** and **local** or **relative.**

13. The value of a figure when standing alone is called its **simple value.** As the value of 3 is three.

14. The *increased* value of a figure by having other figures placed on its right, is called its **local or relative value.**

15. The value of 3 standing alone is 3 units, when a 0 is placed to the right of it, it reads 30 units, its value having been increased *ten* times. By placing another 0 to the right of the 30, it is read 300 units and its value has again been increased *ten* times. By this it will be seen that the value of a figure increases *tenfold* for each figure that is placed to the *right* of it.

16. When a figure is placed in the *third* place from the right it represents *hundreds*. That is, if the figure 1 is placed to the left of two ciphers it means *one hundred units* and is written 100; the figure 2 so placed, *two hundred units* and is written 200, etc.

17. When a figure is placed in the *fourth* place from the right it represents *thousands*. That is, if the figure 1 is placed to the left of three ciphers it means *one thousand units* and is written 1,000; the figure 2 so placed, *two thousand units* and is written 2,000, etc.

18. It will be noticed that we have placed a comma between the figure 1 and the three ciphers. All numbers in figures are spaced into groups of *three* and are pointed off from the *right* in order to aid in the reading of the number.

19. When a figure is placed in the *fifth* place from the right it represents *tens thousands*. That is, if the figure 1 is placed to the left of four ciphers it means *ten thousand units* and is written 10,000; the figure 2 so placed, *twenty thousand units* and is written 20,000, etc.

20. When a figure is placed in the *sixth* place from the right it represents *hundreds thousands*. That is, if the figure 1 is placed to the left of five ciphers it means *one hundred thousand units* and is written 100,000; the figure 2 so placed, *two hundred thousand units* and is written 200,000, etc.

21. When a figure is placed in the *seventh* place from the right it represents *millions*. That is, if the figure 1 is placed to the left of six ciphers it means *one million units* and is written

1,000,000; the figure 2 so placed, *two million units* and is written 2,000,000, etc.

22. The number of ciphers could of course be increased to make the number read to any magnitude. By combining the proper figures any desired number may be written, as in the following:

Twenty-one	21
Five hundred ten	510
Three thousand, forty	3,040
Two hundred one	201
Three hundred twenty-five	325
Eight hundred nine	809
Four hundred seventy-three	473
One thousand, two hundred forty-three	1,243
Nine thousand, one hundred	9,100
One million, two hundred ten thousand	1,210,000

23. The following table shows the **Arabic** system of notation.

Hundreds-billions.	*Tens-billions.*	*Billions.*	*Hundreds-millions.*	*Tens-millions.*	*Millions.*	*Hundreds-thousands.*	*Tens-thousands.*	*Thousands.*	*Hundreds.*	*Tens.*	*Units.*	*Names.*
12th.	11th.	10th.	9th.	8th.	7th.	6th.	5th.	4th.	3rd	2nd.	1st.	*Places.*
6	4	5,	7	0	9,	2	7	8,	0	1	2.	*Figures.*
	4th.			*3rd.*			*2nd.*			*1st.*		*Groups.*
	Billions Group.			*Millions Group.*			*Thousands Group.*			*Units Group.*		

24. Referring to this table, suppose we wish to write in figures, the number *seven hundred five thousand, nine.* We see from the table that *hundreds thousands* come in the *sixth* place, so we place a 7 in the *sixth* place, there being no *tens thousands* a *cipher* is placed in the *fifth* place; the 5 for the *five thousands* comes in the *fourth* place; there being no *hundreds* or *tens* a *cipher* is placed in the *third* and the *second* place; then 9 for the *nine units* is placed in the *first* place. The arrangement of figures will then be 705,009.

25. In writing large numbers it is more convenient to disregard the places and to think of **groups of places.** Thus in writing the number *nine hundred forty million, ninety-one thousand, four hundred forty-four,* the groups of places are 940 *million,* 091 *thousand,* 444 *units,* and the number is 940,091,444.

26. RULE FOR NUMERATION.—*Commence at the right and separate the numbers into groups of three figures each. Then from the left read each group as if it stood alone.*

27. Copy the following numbers, point off into groups and write in words:

(1)	2089	(5)	36845	(9)	207922
(2)	3101	(6)	91002	(10)	318123
(3)	7001	(7)	34627	(11)	418146
(4)	8217	(8)	55555	(12)	983481

28. RULE FOR NOTATION.—*Begin at the left and write the hundreds, tens and units of each group in their proper order; putting ciphers in all vacant places, at the same time separate each group by a comma from the one following.*

29. Write in figures:

Twenty-seven thousand.
Nine hundred forty.

Five thousand, two hundred ten.
Nine million, three hundred five thousand, one.
One hundred one thousand, three hundred fifty.
Seventy-five thousand, ten.
Ninety-seven thousand, five hundred.
Fifty-nine million, two.
Ten hundred fifty-six.
Four hundred sixty-seven thousand, eight hundred forty-one.

ROMAN NOTATION

30. In Roman notation **seven capital letters** are used to express numbers, viz.:

I.	V.	X.	L.	C.	D.	M.
1,	5,	10,	50,	100,	500,	1,000.

The top row gives the capital letters used and the bottom row gives their value in Arabic figures.

31. The following principles of combination are used in Roman notation:

Repeating a letter **repeats** its value. That is, *two* is written II; *three*, III; *twenty*, XX; *thirty*, XXX.

When a letter of *lesser* value is placed in *front* of a letter of a *greater* value, the value of the *lesser* is to be taken from the value of the *greater*. That is, the letter I, placed before the letter V means that *one* is to be taken from *five*. It is written IV and means four; nine is written IX.

When a letter *follows* one of *greater* value it means that the values are to be *added* together. That is, *seven* is written VII; *six*, VI; *thirty-five*, XXXV.

A *bar* placed over any letter *increases* the value thereof *one thousand* fold. That is, $\overline{V}$ means *five thousand ;* $\overline{X}$ means *ten thousand ;* $\overline{L}$, *fifty thousand ;* $\overline{M}$, *one million.*

32. The methods of combination are shown in the following table:

I	=1	X	=10	XXI	=21	C	= 100
II	=2	XI	=11	XXIV	=24	CC	= 200
III	=3	XII	=12	XXIX	=29	CCCC	= 400
IV	=4	XIII	=13	XXX	=30	CD	= 400
V	=5	XIV	=14	XL	=40	D	= 500
VI	=6	XV	=15	L	=50	DCCC	= 800
VII	=7	XVI	=16	LX	=60	M	= 1,000
VIII	=8	XIX	=19	LXX	=70	MMM	= 3,000
IX	=9	XX	=20	XC	=90		

33. Read: XVII, XXV, XIX, VI, III, CXV, CCXLIX, DCCLX, CCCIV, $\overline{\text{X}}$DC.

Write the following numbers in Roman notation: 14, 25, 32, 11, 75, 100.

ADDITION

34. Finding the sum of two or more numbers is called **addition.**

35. Numbers that are to be added together are called **addends.**

36. In addition a sign is used to indicate that the numbers are to be added together. This sign is in the form of a small cross, called **plus** and is written +. When this sign appears between two numbers, as in 4 + 6, it indicates that 6 is to be added to 4. Then instead of writing 4 plus 6, we simply write 4 + 6.

37. 4 + 6 equals 10, but instead of writing the word equals, the sign of **equality** (=) is used, meaning equal to, or is equal to. Therefore as expressed in figures and symbols the problem 4 plus 6 equals 10, is written $4+6=10$.

38. PRINCIPLE.—*Only like numbers can be added.*

TABLE OF ADDITION

0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
1	2	3	4	5	6	7	8	9	10	11	12	13
2	3	4	5	6	7	8	9	10	11	12	13	14
3	4	5	6	7	8	9	10	11	12	13	14	15
4	5	6	7	8	9	10	11	12	13	14	15	16
5	6	7	8	9	10	11	12	13	14	15	16	17
6	7	8	9	10	11	12	13	14	15	16	17	18
7	8	9	10	11	12	13	14	15	16	17	18	19
8	9	10	11	12	13	14	15	16	17	18	19	20
9	10	11	12	13	14	15	16	17	18	19	20	21
10	11	12	13	14	15	16	17	18	19	20	21	22
11	12	13	14	15	16	17	18	19	20	21	22	23
12	13	14	15	16	17	18	19	20	21	22	23	24

This table should be committed to memory.

Explanation.—In the preceding table the figures in heavy type are used as the addends and the sum is found where the columns containing the addends used, intersect. As in $5+7=12$; we begin with the figure 5 in the left hand column and find the sum in the column, under the 7 in the top row of figures. Or in $9+8=17$; we start with the 9 in the left hand column and find the sum in the column, under the 8 in the top row of figures.

39. In addition the figures are generally placed *under one another* as shown in the following problems:

40. Problem.—(1) Add 5, 7 and 9.

Solution.—

$$\begin{array}{r} 5 \\ 7 \\ 9 \\ \hline 21 \end{array}$$ *Ans.*

Explanation.—In problem (1) the figures 5, 7 and 9, are placed as shown with a horizontal line drawn under them. They are then added together and the answer, which in this problem is 21, set under the horizontal line, care being taken to place the 1 in the *units* place and the 2 in the *tens* place.

41. Problem.—(2) Find the sum of 10, 42 and 28.

Solution.—

$$\begin{array}{r} 10 \\ 42 \\ \underline{28} \\ 80 \end{array}, \textit{ Ans.}$$

Explanation.—In problem (2) there are two columns of figures to be added. In this case the *units* are added first. These being 8 and 2, we say, $8+2=10$; the cipher is placed in the *units* place under the horizontal line at the bottom of the *units* column and the 1 is added in with the next column which is *tens;* then we say $1+2+4+1=8$; the 8 is set down in the *tens* place under the horizontal line. The sum or total now reads 80.

42. It will be seen from the above that the solution of problems by addition should begin at the extreme right and that where the sum of any one column is greater than 9, the right hand figure of the result is placed under the column that has just been added and the left hand figure is added to the next column, and so on until all of the columns have been added.

43. Rule.—*Write the addends so that units will be under units, tens under tens, etc.*

Begin at the right, add the columns separately, and write the sum, if it consists of but one figure, under the column added.

If the sum of any one column is greater than 9, *put the right hand figure of the sum under that column and add the remaining figure, or figures, to the next column.*

44. Proof.—*To prove addition, add the columns from the top downward, then if the same result is obtained as by adding from the bottom upward, the answer is probably correct.*

45. Problems.—Add: (1) 2,222+4,362+7,871. (2) 5,726+3,592+7,030+5,020.

(3)	7352	(4)	2742	(5)	29081	(6)	1000
	5812		3681		70345		400
	856		2046		98562		30
	10		6372		81828		5
			8519				40
							1065

(7) A certain railroad has twelve signals at one point, three at another, four at another and five at another. How many signals are there in all?

(8) Along a certain railroad there are three signal towers. In one of these there are sixteen levers, in another eight levers and in the other thirty-two levers. How many levers are there in all the towers?

Answers.—(1) 14,455. (2) 21,368. (3) 14,030. (4) 23,360. (5) 279,816. (6) 2,540. (7) 24 signals. (8) 56 levers.

SUBTRACTION

46. Finding the difference between two numbers is called **subtraction.** The sign used in subtraction is called **minus,** is written —, and when placed between two numbers means that the number *after* the sign is to be subtracted from the number *ahead* of it. As in the problem 4—2=2, which means that 2 is to be taken from the 4, and is read 4 minus 2 equals 2.

47. The number from which another is to be subtracted, is called the **minuend,** and the number to be subtracted, the **subtrahend.**

The result obtained in subtraction is called the **difference** or **remainder.**

48. Principle.—*Only like numbers can be subtracted.*

49. PROBLEM.—(1) Subtract 432 from 784.

SOLUTION.—

$$\begin{array}{rl} 784 & \textit{minuend.} \\ 432 & \textit{subtrahend.} \\ \hline 352 & \textit{remainder or difference, Ans.} \end{array}$$

EXPLANATION.—In subtraction the numbers are so placed that the subtrahend will be under the minuend as shown. The number 432 is the subtrahend and is to be taken, or subtracted from the number 784 which is the minuend. Therefore, the subtrahend is placed under the minuend and a horizontal line drawn under it. We then proceed to subtract by beginning at the right and saying 2 from 4 leaves 2, placing the 2 in the units column under the 2 in the subtrahend; then we say 3 from 8 leaves 5; this we place in the tens column under the 3; then we say 4 from 7 leaves 3, placing this in the hundreds column under the 4. We now have the number 352 under the horizontal line which is the difference, or remainder when 432 is subtracted from 784.

50. PROBLEM.—(2) Subtract 429 from 678.

SOLUTION.—

$$\begin{array}{rl} 678 & \textit{minuend.} \\ 429 & \textit{subtrahend.} \\ \hline 249 & \textit{remainder, Ans.} \end{array}$$

EXPLANATION.—It will be noticed that in problem (2) 678—429, the unit 9 in the subtrahend is greater than the unit 8 in the minuend. We therefore take one of the tens away from the 7 tens in the minuend and add it to the 8 units making 18 units, then we subtract the 9 units from the 18 units and get a remainder of 9 units; the 9 units are placed under the 9 units in the subtrahend, and we proceed with the solution of the problem by subtracting the tens. Instead of saying 2 tens from 7 tens, we say 2 tens from 6 tens leaves 4 tens, because one of the tens was taken away from the 7 tens to make up the 18 units; then we say 4 hundreds from 6 hundreds leaves 2 hundreds. This gives us 678—429 = 249.

51. PROBLEM (3) Subtract 5,876 from 7,004.

SOLUTION.—

$$\begin{array}{rl} 7004 & \textit{minuend.} \\ 5876 & \textit{subtrahend.} \\ \hline 1128 & \textit{remainder, Ans.} \end{array}$$

Explanation.—Since 6 units cannot be subtracted from 4 units, and since there are no tens nor hundreds, one of the 7 thousands must be changed into hundreds, leaving 6 thousands; one of the hundreds must be changed into tens, leaving 9 hundreds; and one of the tens into units, leaving 9 tens and making 14 units. The 6 units may now be subtracted from the 14 units, leaving 8 units; the 7 tens from the 9 tens, leaving 2 tens; the 8 hundreds from the 9 hundreds, leaving 1 hundred; and the 5 thousands from the 6 thousands, leaving 1 thousand. Therefore 7004—5876 = 1128.

52. Rule.—*Write the subtrahend under the minuend so that the units of the subtrahend will be under the units of the minuend; the tens under the tens; the hundreds under the hundreds, etc.*

Begin at the right, subtracting each figure in the subtrahend from the corresponding figure in the minuend, and writing the result beneath them.

If a figure in the subtrahend has a greater value than the corresponding figure in the minuend, increase the figure in the minuend by ten by prefixing a unit of the next higher order, subtracting as before. Now diminish the next figure of the minuend by one and subtract.

53. Proof.—*The sum of the subtrahend and the remainder is equal to the minuend.*

54. Problems.—Subtract: (1) 24–10. (2) 364–274. (3) 5,284–3,372.

(4)	2075	(5)	37521	(6)	3214	(7)	49281
	100		24632		1472		20956

(8) In excavating for a trench for a certain purpose, I find that it must be 7 feet deep and 6 feet wide to conform with a standard. If it is 4 feet deep and 2 feet wide when measured, how much deeper and wider must it be excavated?

(9) How much must be added to 740 to make it equal 920?

Answers.—(1) 14. (2) 90. (3) 1,912. (4) 1,975. (5) 12,889. (6) 1,742. (7) 28,325. (8) 3 feet deeper and 4 feet wider. (9) 180.

MULTIPLICATION

55. The process producing a result equivalent to adding a number to itself a number of times, is called **multiplication.**

56. The number to be multiplied is called the **multiplicand.**

57. The number by which the multiplicand is multiplied is called the **multiplier.**

58. The result obtained by multiplication is called the **product.**

59. The multiplicand and the multiplier are called the **factors** of the product.

60. The sign of multiplication is an oblique cross ×, it means **multiplied by** or **times.** As 5×2 means 5 *multiplied by* 2, or 5 *times* 2.

61. PRINCIPLE.—*The multiplier is regarded as an abstract number. The multiplicand and the product must be like numbers.*

62. For convenience the multiplier is written under the multiplicand. As 9 × 5 is written

$$\begin{array}{rl} 9 & \textit{multiplicand.} \\ \underline{5} & \textit{multiplier.} \\ 45 & \textit{product, Ans.} \end{array}$$

63. The *greater* number is usually treated as the multiplicand, and the *smaller* one as the multiplier.

MULTIPLICATION TABLE

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

This table should be committed to memory.

Explanation.—In the preceding table the figures in heavy type are used as *multipliers* and *multiplicands* and the *result* is read in the space where the horizontal and vertical columns containing these numbers intersect. As 9×7 are 63. The answer will be found in the ninth column from the top and seven columns to the right. Or 7×9 are 63, the answer is found in the seventh column from the top and nine columns to the right.

64. Problem.—(1) Multiply 32 by 25.

Solution.—

32 *multiplicand.*
25 *multiplier.*

160 *partial product No. 1.*
64 *partial product No. 2.*

800 *total product, Ans.*

Explanation.—Numbers *greater* than 9 are multiplied by setting down the problem as shown in the solution of problem (1), with the multiplier written under the multiplicand. We begin at the right to multiply, saying 5 times 2 units equal 10 units; we set the cipher down in the units column under the horizontal line, reserving the 1 which is to be added to the product of the next operation; we next multiply the 3 tens in the multiplicand by the 5 units in the multiplier, saying 5 times 3 tens equals 15 tens, to which we add the 1 ten left over from the preceding operation, which gives us 16 tens; we set the 16 down with the 6 tens under the 2 tens in the multiplier. This gives us a partial product of 160 units. We next multiply the figures in the multiplicand by the figure in the tens place of the multiplier. This being a 2, or 2 tens, we say 2 tens times 2 units equals 4 tens; we place the 4 tens in the tens column under the 6 tens in the first partial product; then we say 2 tens times 3 hundreds equals 6 hundreds; we write the 6 hundreds in the hundreds column under the 1 hundred in the first partial product; we now have a second partial product of 6 hundreds, 4 tens, and no units. The next part of the operation is to add together the partial products as per Art. 43, which gives us a total product of 800.

65. Problem.—(2) Multiply 46 by 20.

Solution.—
46 *multiplicand.*
20 *multiplier.*
920 *product, Ans.*

Explanation.—When the right hand figure, or figures, in the multiplier are ciphers, as many ciphers must be placed to the right of the product, as there are ciphers to the right of the multiplier, and the figures in the multiplicand are then multiplied by the remaining figures in the multiplier.

66. Problem.—(3) Multiply 175 by 103.

Solution.—
175 *multiplicand.*
103 *multiplier.*
525 *partial product No. 1.*
1750 *partial product No. 2.*
18,025 *total product, Ans.*

Explanation.—In problem (3) the multiplier is set down under the multiplicand in the usual manner; then proceed as follows: Taking first the 3 units in the multiplier we multiply each figure in the multiplicand by it and place the partial product so obtained under the horizontal line in the usual manner; there being no tens in the multiplier a cipher is placed in the tens place in the next partial product; we then begin multiplying the figures in the multiplicand by the last figure in the multiplier and place the result to the left of the cipher in the second partial product. There being no more figures in the multiplier we now find the sum of the two partial products and get the total product which is 18,025.

67. Rule.—*Write the multiplier under the multiplicand, with units under units, tens under tens, etc.*

Begin with the units and successively multiply each figure in the multiplicand by each figure in the multiplier.

Place the right hand figure of each partial product under the figure of the multiplier used to obtain it. Then add the partial products.

68. Proof.—*To prove, multiply the multiplier by the multiplicand. If the same answer is obtained the work is probably correct.*

69. **Problems.**—Multiply: (1) 275 × 20. (2) 9,134 × 100. (3) 246 × 48. (4) 41,645 × 99. (5) 895 × 507. (6) 9,675 × 314. (7) 87,241 × 237. (8) 58,325 × 901.

(9) I need 27 times as many pounds of sulphate of copper as I have on hand. If I have on hand 356 pounds, how many more pounds must I obtain?

(10) If, in laying one mile of track, 6,202 ties are needed, how many ties will be needed for laying thirteen miles of track?

Answers.—(1) 5,500. (2) 913,400. (3) 11,808. (4) 4,122,855. (5) 453,765. (6) 3,037,950. (7) 20,676,117. (8) 52,550,825. (9) 9,612 pounds. (10) 80,626 ties.

DIVISION

70. The process of finding how many times one number is contained in another is called **division.**

71. The number to be divided is called the **dividend.**

72. The number by which we divide is called the **divisor.**

73. The result is called the **quotient.**

74. The part of the dividend remaining when the division is not exact is called the **remainder.**

75. The sign of division is ÷ and is read **divided by.** As $4 \div 2$ reads, 4 divided by 2.

76. Division is also indicated by writing the *dividend* above the *divisor* separated by a horizontal line. As $\frac{4}{2}$ indicates that 4 is to be divided by 2.

77. There are *two* methods of dividing numbers, one called *long* division and the other called *short* division.

78. **Problem.**—(1) Divide 486 by 2 using short division.

Solution.— *divisor.* 2)486 *dividend.*

243 *quotient, Ans.*

Explanation.—After writing the numbers in the relation shown in the solution of example (1), we begin at the left and divide the hundreds, tens and units successively by the divisor 2. Commencing with the hundreds we find that 2 is contained in 4 hundreds, 2 hundreds times; the 2 is placed directly under the 4 in the hundreds place; we then divide the 8 tens by 2, getting 4 tens; the 4 tens are placed in the tens place under the 8 tens; we then divide the 6 units by 2 which gives us 3 units; the 3 is placed in the units place under the 6 units. We now have $486 \div 2 = 243$.

79. PROBLEM.—(2) Divide 48 by 3.

SOLUTION.— *divisor.* 3)48 *dividend.*
16 *quotient, Ans.*

EXPLANATION.—We begin by dividing the 4 tens by 3; we find that 3 is contained in 4 tens, 1 tens times, leaving a remainder of 1 ten; the 1 ten obtained from this division is placed in the tens place under the 4 tens; the 1 remaining ten is added in with the 8 units in the dividend making 18 units; the 18 units are now divided by the divisor 3 and the result, which is 6 units is placed in the units place in the quotient under the 8 units in the dividend. Therefore $48 \div 3 = 16$.

80. PROBLEM.—(3) Divide 8,572 by 21 using long division.

SOLUTION.— *dividend.*
divisor. 21)8572($408\frac{4}{21}$ *quotient, Ans.*
84
172
168
4 *remainder.*

EXPLANATION.—A problem in long division is solved as follows: after writing the numbers in the relation shown, we begin at the left of the dividend to divide. We find that 21 is not contained in 8 thousands any thousands times; therefore the thousands are united with the 5 hundreds, making a total of 85 hundreds; 21 is contained in 85 hundreds 4 hundreds times with a remainder of 1 hundred; the 4 hundreds are written as the first figure of the quotient, and the divisor is multiplied by this figure giving a product of 84 hundreds; we then subtract 84 hundreds from 85 hundreds and get a remainder of 1 hundred, which is placed in the hundreds place as shown, and the 7 tens in the dividend brought down and united with it, making 17 tens; we find that 21 tens in not contained in 17 tens, any tens times; therefore we write a cipher in the tens place in the quotient and bring down the 2 units in the dividend and unite them with the 17 tens, making a total of 172 units; we find that

21 is contained in 172 units, 8 units times and that 8 units times 21 units are 168 units; we place the 8 in the units place in the quotient and subtract the 168 units from the 172 units and obtain a remainder of 4 units; the remainder of 4 units is written over the divisor and is a part of the quotient. Therefore $8{,}572 \div 21 = 408\frac{4}{21}$.

81. RULE.—*Write the divisor at the left of the dividend with a curved line between them.*

Find how many times the divisor is contained in the smallest number of figures (to the left) of the dividend that will contain it and write the quotient as the first figure in the answer.

Multiply the divisor by this quotient and subtract the product obtained from the partial dividend used, and to the remainder annex the next figure of the dividend for the new partial dividend.

Divide as before, until all the figures contained in the dividend have been used.

If any partial dividend will not contain the divisor, write a cipher in the quotient, then annex the next figure in the dividend and proceed as before.

If there is a remainder after the last division write it after the quotient, with the divisor under it.

82. PROOF.—*Multiply the divisor by the quotient, and to the product add the remainder, if there is one. If the problem has been properly solved the product will be equal to the dividend.*

83. PROBLEMS.—Divide: (1) 378 ÷ 2. (2) 3,102 ÷ 13. (3) 605 ÷ 71. (4) 210 ÷ 28. (5) 5,270 ÷ 10. (6) 13,189 ÷ 7. (7) 2,708 ÷ 9. (8) 37,827 ÷ 33.

(9) I wish to divide five thousand spikes among twenty-five sections. How many spikes will each section receive?

(10) (*a*) If a carload of pipe contains seven thousand, eight hundred ninety feet, how many piles of pipe, each consisting of two hundred fifty-three feet, can be made? (*b*) If there be any left, state how much.

Answers.—(1) 189. (2) $238\frac{8}{13}$. (3) $8\frac{67}{71}$. (4) $7\frac{14}{25}$. (5) 527. (6) $1,884\frac{4}{7}$. (7) $300\frac{5}{8}$. (8) $1,146\frac{2}{33}$. (9) 200 spikes. (10) (*a*) 31 piles. (*b*) 47 feet.

EXAMINATION QUESTIONS AND PROBLEMS

(1) The velocity of an express train is sixty-three miles per hour. How many miles can it run in seventeen hours?

(2) How far will one hundred lengths of pipe reach if laid end to end, assuming that each length is sixteen feet long?

(3) A car contains the following: Five boxes weighing seven pounds each; seven boxes weighing one hundred twenty pounds each; three boxes weighing forty-nine pounds each and ten boxes whose combined weight is ninety pounds. (*a*) How many boxes are there in all? (*b*) How much do they weigh?

(4) How many stakes will be required for a stake line, if they are to be placed 14 feet apart, providing the two end stakes are to be 3,850 feet apart?

(5) What is the difference between a concrete number and an abstract number?

(6) In nine weeks there are 63 days. How many days are there in one week?

(7) What is the answer to 5,181 divided by 33?

(8) What is the difference between addition and subtraction?

(9) Why can not 256 be subtracted from 198?

(10) Multiply 53 by 17, add 21, subtract 56, then divide by 7. What is the answer?

(11) Express: 1908, 1780, 59, 37, in Roman notation.

(12) How many are 25 times 53, plus 70, minus 320, divided by 5?

(13) How many holes will have to be drilled through the rails in bonding 650 joints, if four holes are required for each joint?

(14) Where wire lines turn at right angles, grooved chain wheels are used. If it takes 42 inches of chain for each wheel, how many inches will it take for 37 wheels?

(15) If a pipe compensator weighs 90 pounds, how many will it take to weigh 8,100 pounds?

(16) (*a*) How many cells of gravity battery can be renewed with 316 pounds of sulphate of copper, provided it takes 3 pounds per cell? (*b*) If there be any left state how much.

(17) Add: 243,890 + 109,190 + 243,021 + 461,087 + 109,908.

(18) From: 45,567 + 90,786 + 57,765 + 32,080 subtract 205 + 36,081 + 30,000.

(19) What is Arabic notation?

(20) In multiplying two abstract numbers, which would you use as the multiplier?

(21) What is the difference between multiplication and division?

(22) Why must numbers that are to be added together, be like numbers?

(23) Can an abstract number be divided by an abstract number?

(24) A train leaves A and proceeds west at the rate of sixty miles per hour; another train leaves A at the same time and proceeds east at the rate of thirty miles per hour. How far apart will they be in 5 hours?

(25) After having multiplied 37 by 15, I find that I should have added them. What is the difference between the answer I obtained and the one I should have obtained?

ANSWERS TO EXAMINATION PROBLEMS

(1) 1,071 miles.
(2) 1,600 feet.
(3) (*a*) 25 boxes. (*b*) 1,112 pounds.
(4) 276 stakes.
(5) ?
(6) 7 days.
(7) 157.
(8) ?
(9) ?
(10) $123\frac{5}{7}$.
(11) ?
(12) 215.
(13) 2,600 holes.
(14) 1,554 inches.
(15) 90 compensators.
(16) (*a*) 105 cells. (*b*) 1 pound.
(17) 1,167,096.
(18) 159,912.
(19) ?
(20) ?
(21) ?
(22) ?
(23) ?
(24) 450 miles.
(25) 503.

FACTORING

84. The **factors** of a number are those numbers which multiplied together produce the number.

85. The **factors** of 40 are 4 and 10; or 5 and 8; or 2, 5 and 4; or 10, 2 and 2.

86. The process of separating a number into its factors is called **factoring.**

87. Any number which cannot be divided by any other number except *itself* and *1* is a **prime** number.

88. Any factor that is a prime number is termed a **prime factor.**

89. Any number which can be divided by other numbers besides itself and 1 is a **composite** number.

90. Any number that is exactly divisible by 2 is called an **even** number, as 2, 4, 6, 8. Any number that is not exactly divisible by 2 is called an **odd** number, as 3, 5, 7, 9.

91. Numbers are said to be **prime to one another** when any two of them cannot be divided by any number except 1. 3, 5, 7 and 13, are prime to one another, as also are 22, 23 and 25.

92. To find the prime factors of a number divide the given number by any prime number that will exactly divide it. Divide the quotient that is obtained again by a prime number and so continue until the final quotient secured is one. The several divisors will be the prime factors.

93. **Problem.**—Find the prime factors of 2,520.

Solution.—

$$\begin{array}{r|l} 2 & \underline{2520} \\ 2 & \underline{1260} \\ 2 & \underline{630} \\ 3 & \underline{315} \\ 3 & \underline{105} \\ 5 & \underline{35} \\ 7 & \underline{7} \\ & 1 \end{array}$$

Explanation.—Since the number is an even one, the prime number, 2, is taken for the first divisor. Since the quotient thus obtained is even, 2 can again be taken as a divisor. This procedure should be followed until the last quotient is 1, then as indicated, the prime factors of 2,520 are 2, 2, 2, 3, 3, 5 and 7.

94. Rule.—*Divide the given number by any prime number that will exactly divide it. Divide the quotient again by a prime number and so continue until the quotient is one. The several divisors will be the prime factors.*

95. Proof.—*To prove this multiply the prime factors thus obtained together; this will give the original number.*

96. **Problems.**—Find the prime factors of the following:

(1) 260. (2) 375. (3) 345. (4) 462. (5) 420.

Answers.—(1) 2, 2, 5, 13. (2) 3, 5, 5, 5. (3) 3, 5, 23. (4) 2, 3, 7, 11. (5) 2, 2, 3, 5, 7.

CANCELLATION

97. Rejecting equal factors from both dividend and divisor is called cancellation. This process does not alter the quotient.

98. Problem.—Divide $6 \times 36 \times 30 \times 18$ by $3 \times 8 \times 10 \times 24$.

Solution.—

$$\frac{\overset{3}{\cancel{6}} \times \overset{3}{\cancel{36}} \times \overset{\cancel{3}}{\cancel{30}} \times \overset{9}{\cancel{18}}}{\cancel{3} \times \underset{\underset{2}{\cancel{4}}}{\cancel{8}} \times \cancel{10} \times \underset{2}{\cancel{24}}} = \frac{81}{4} = 20\tfrac{1}{4}. \quad \textit{Ans.}$$

Explanation.—As indicated the *dividend* is written *above* the *divisor* with a line between them. Since 12 is a *factor* of both 24 and 36 it may be divided into these numbers, the quotients, 2 and 3, respectively, being placed *below* and *above* them, and 24 and 36 being *cancelled* as shown; 10, being a factor of 30, is divided into 30, and the quotient 3 is written *above* 30, 10 and 30 being *cancelled.* As 3 is a *factor* of 3, these two numbers should be *cancelled;* the quotient being 1, it is not necessary to indicate it. Since 2 is a *factor* of 8 and 18 it may be divided into these numbers and the quotients, 4 and 9, respectively, placed *below* and *above* them. Finally we find that 2 is a *factor* of both 4 and 6, therefore it may be divided into these numbers, the quotients, 2 and 3, respectively, being placed *below* and *above* them.

As there are no more *factors common* to *both* the dividend and the divisor the cancellation is now complete, and the next step is to find the *product* of the *factors not cancelled* in the dividend, this being $3 \times 3 \times 9$, or 81, and to find the *product* of the *factors not cancelled* in the divisor, this being 2×2, or 4. Then placing 81 over 4 we get the quotient $\frac{81}{4}$, or $20\frac{1}{4}$.

99. Rule.—*Reject from the dividend and divisor all factors common to both and then divide the product of the remaining factors of the dividend by the product of the remaining factors of the divisor.*

When all the factors of both dividend and divisor are cancelled the quotient is 1, for the dividend contains the divisor once.

100. **Problems.**—Divide the following using cancellation:

(1) $\dfrac{30\times42\times56\times20}{28\times36\times56\times10}$ (2) $\dfrac{10\times20\times8\times30}{5\times40\times6\times8}$

(3) $\dfrac{50\times48\times60\times24}{20\times36\times40\times12}$ (4) $\dfrac{16\times60\times45\times40}{9\times20\times14\times32}$

(5) $\dfrac{63\times80\times45\times50}{5\times70\times100\times75}$ (6) $\dfrac{16\times35\times48\times90}{12\times48\times30\times10}$

(7) $\dfrac{85\times14\times20\times36}{60\times5\times42\times3}$ (8) $\dfrac{95\times24\times36\times84}{8\times10\times18\times6}$

Answers.—(1) $2\frac{1}{2}$. (2) 5. (3) 10. (4) $21\frac{3}{7}$. (5) $4\frac{8}{25}$. (6) 14. (7) $22\frac{2}{3}$. (8) 798.

FRACTIONS

101. When we divide anything into a number of equal parts, these parts are called **fractions.**

102. Fractions are expressed by writing *two numbers,* one above the other with a line between them. The number which is written *below* the line and which shows into how many parts the thing has been divided is called the **denominator.** The number which is written *above* the line and which shows how many parts form the fraction is called the **numerator.** As in the fraction $\frac{3}{7}$, 7 is the **denominator** and shows that something has been divided into 7 equal parts, and 3 is the **numerator** and shows that the fraction contains 3 of the 7 equal parts.

103. The unit which is divided into equal parts is called the **unit of the fraction.**

104. A fraction whose unit has been divided into *any number* of equal parts is called a **common fraction**, as $\frac{3}{4}$, $\frac{5}{7}$, $\frac{7}{10}$, $\frac{81}{1000}$. A fraction whose unit has been divided into *tenths, hundredths, thousandths,* etc., is called a **decimal fraction**, as $\frac{3}{10}$, $\frac{19}{100}$, $\frac{85}{1000}$.

105. The *numerator* and the *denominator* are called the **terms** of the fraction.

106. A fraction whose numerator is *less* than its denominator is called a **proper fraction,** as $\frac{3}{4}$, $\frac{6}{7}$, $\frac{5}{8}$. From this it is seen that a proper fraction is less than 1.

107. A fraction whose numerator *equals* or *exceeds* its denominator is called an **improper fraction,** as $\frac{6}{6}$, $\frac{18}{8}$, $\frac{19}{16}$. From this it is seen that an improper fraction is equal to 1 or more than 1.

108. When a whole number and a fraction are expressed together they are called a **mixed number,** as $3\frac{3}{4}$, $9\frac{5}{7}$, $16\frac{5}{8}$.

109. A fraction also expresses unexecuted division. The denominator is the **divisor** and the numerator is the **dividend,** as $\frac{15}{3} = 15 \div 3$.

NOTE.—*In reading fractions the numerator is read first, then the denominator: thus,* $\frac{5}{8}$ *is read five-eighths;* $\frac{3}{12}$, *three-twelfths;* $\frac{14}{44}$, *fourteen-forty-fourths; etc.*

REDUCTION OF FRACTIONS

110. The process of changing the forms of fractions without changing their value is called **reduction of fractions.**

111. A fraction is expressed in **higher terms,** when its denominator and numerator are expressed in *greater numbers,* and in **lower terms,** when its denominator and numerator are expressed in *smaller numbers.* Thus $\frac{3}{8}$ expressed in higher terms equals $\frac{6}{16}$, or $\frac{9}{24}$, while $\frac{9}{24}$, or $\frac{6}{16}$, expressed in lower terms equals $\frac{3}{8}$.

112. A fraction is expressed in its **lowest terms,** when its numerator and denominator have *no common divisor,* as $\frac{3}{4}$, $\frac{5}{8}$, $\frac{7}{9}$.

TO REDUCE A FRACTION TO HIGHER TERMS

113. PRINCIPLE.—*Multiplying or dividing both terms of a fraction by the same number does not change the value of the fraction.*

114. **PROBLEM.—Reduce $\frac{3}{5}$ to 45ths.**

SOLUTION.—

$$\frac{3}{5} = \frac{3 \times 9}{5 \times 9} = \frac{27}{45}, \quad \textit{Ans.}$$

EXPLANATION.—Whereas 45 divided by 5 equals 9, we must multiply both terms of the fraction by 9 which gives $\frac{27}{45}$.

115. RULE.—*Divide the denominator of the required fraction by the denominator of the given fraction and multiply the numerator and the denominator of the given fraction by the quotient thus obtained.*

116. Problems.—Reduce: (1) $\frac{3}{8}$ to 16ths. (2) $\frac{7}{10}$ to 20ths. (3) $\frac{5}{8}$ to 32nds. (4) $\frac{1}{9}$ to 18ths. (5) $\frac{3}{7}$ to 14ths. (6) $\frac{5}{6}$ to 18ths. (7) $\frac{3}{7}$ to 21sts. (8) $\frac{1}{4}$ to 8ths. (9) $\frac{1}{11}$ to 33rds. (10) $\frac{12}{32}$ to 64ths.

Answers.— (1) $\frac{6}{16}$. (2) $\frac{14}{20}$. (3) $\frac{20}{32}$. (4) $\frac{2}{18}$. (5) $\frac{6}{14}$. (6) $\frac{15}{18}$. (7) $\frac{9}{21}$. (8) $\frac{2}{8}$. (9) $\frac{3}{33}$. (10) $\frac{24}{64}$.

TO REDUCE A FRACTION TO ITS LOWEST TERMS

117. Problem.—Reduce $\frac{60}{75}$ to its lowest terms.

Solution.—
$$\frac{60 \div 5 = 12}{75 \div 5 = 15}\ \text{then}\ \frac{12 \div 3 = 4}{15 \div 3 = 5},\quad \textit{Ans.}$$

Explanation.—Whereas the fraction is to be reduced to its lowest terms we must continue to divide *both* terms of the fraction by common divisors until such a division is no longer possible. We therefore divide the *numerator*, 60, by 5 and get 12 for a new numerator; then we divide the *denominator*, 75, by 5 and get 15 for a new denominator. We now have the fraction $\frac{12}{15}$ *both* terms of which are divisible by 3, we therefore divide the *numerator*, 12, by 3 and get 4 for the new numerator, then we divide the *denominator*, 15, by 3 and get 5 for the new denominator. We now have $\frac{4}{5}$. Therefore $\frac{60}{75}$ reduced to its lowest terms is equal to $\frac{4}{5}$.

118. Rule.—*Divide both terms of the fraction by any common divisor, continue to do so until the terms have no common divisor.*

119. Problems.—Reduce to lowest terms: (1) $\frac{5}{10}$. (2) $\frac{7}{21}$. (3) $\frac{8}{32}$. (4) $\frac{9}{15}$. (5) $\frac{15}{45}$. (6) $\frac{32}{64}$. (7) $\frac{108}{244}$. (8) $\frac{92}{100}$. (9) $\frac{125}{250}$. (10) $\frac{240}{280}$.

Answers.— (1) $\frac{1}{2}$. (2) $\frac{1}{3}$. (3) $\frac{1}{4}$. (4) $\frac{3}{5}$. (5) $\frac{1}{3}$. (6) $\frac{1}{2}$. (7) $\frac{27}{61}$. (8) $\frac{23}{25}$. (9) $\frac{1}{2}$. (10) $\frac{6}{7}$.

TO REDUCE A WHOLE OR MIXED NUMBER TO AN IMPROPER FRACTION

120. Problem.—Reduce $346\frac{1}{3}$ to an improper fraction.

Solution.—

$$\begin{array}{r} 346 \\ 3 \\ \hline 1038 \\ 1 \\ \hline 1039 \end{array} = \frac{1039}{3}, \textit{ Ans.}$$

Explanation.—Whereas, in *one* whole number there are 3 *thirds*, in 346 whole numbers, there are 346 times 3 thirds, or 1038 *thirds*. Then to the 1038 thirds we must add the 1 third in the fraction of the mixed number making in all 1039 *thirds*. Therefore $346\frac{1}{3}$ reduced to an improper fraction equals $\frac{1039}{3}$.

121. Rule.—*Multiply the whole number by the denominator of the fraction, then add the numerator.*

122. Proof.—*Divide the numerator by the denominator and the result will be the whole or mixed number.*

TO REDUCE AN IMPROPER FRACTION TO A WHOLE OR MIXED NUMBER

123. Problem.—Reduce $\frac{376}{5}$ to a whole or mixed number.

Solution.— $\frac{376}{5} = 376 + 5 = 75\frac{1}{5}$, *Ans.*

Explanation.—Whereas 5 *fifths* are equal to *one* whole number, 376 *fifths* are equal to as many whole numbers as 5 is contained times in 376, or 75 and $\frac{1}{5}$. Therefore $\frac{376}{5} = 75\frac{1}{5}$.

124. Rule.—*Divide the numerator of the improper fraction by the denominator.*

125. Proof.—*Multiply the whole number by the denominator and add the numerator of the fraction; write the sum over the denominator.*

126. Problems.—Reduce the following to whole or mixed numbers:

(1) $\frac{36}{2}$. (2) $\frac{52}{3}$. (3) $\frac{37}{8}$. (4) $\frac{269}{14}$. (5) $\frac{310}{6}$.

Reduce the following to improper fractions:

(6) $52\frac{1}{8}$. (7) $185\frac{1}{3}$. (8) $9\frac{1}{8}$. (9) $71\frac{2}{13}$. (10) $28\frac{2}{5}$.

Answers.— (1) 18. (2) $17\frac{1}{3}$. (3) $4\frac{5}{8}$. (4) $19\frac{3}{14}$. (5) $51\frac{2}{3}$. (6) $\frac{417}{8}$. (7) $\frac{556}{3}$. (8) $\frac{73}{8}$. (9) $\frac{925}{13}$. (10) $\frac{142}{5}$.

127. **Similar** fractions are fractions that have the *same* denominator, as $\frac{3}{8}$, $\frac{5}{8}$, $\frac{7}{8}$.

128. **Dissimilar** fractions are fractions that have *not* the same denominator, as $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{8}$.

129. A **common denominator** is the denominator of similar fractions; thus 8 is the **common denominator** of $\frac{3}{8}$, $\frac{5}{8}$, $\frac{7}{8}$.

130. The **least common denominator** is the smallest number into which all the denominators of a given number of fractions can be divided without a remainder.

TO REDUCE DISSIMILAR FRACTIONS TO SIMILAR FRACTIONS HAVING A LEAST COMMON DENOMINATOR

131. Problem.—Reduce the fractions $\frac{3}{16}$, $\frac{3}{4}$, $\frac{5}{8}$ and $\frac{1}{3}$, to equivalent fractions having the least common denominator.

Solution.—

1st.

$$\begin{array}{r|rrrr} 4 & 16 & 4 & 8 & 3 \\ \hline 2 & 4 & 1 & 2 & 3 \\ \hline 2 & 2 & 1 & 1 & 3 \\ \hline 3 & 1 & 1 & 1 & 3 \\ \hline & 1 & 1 & 1 & 1 \end{array}$$

$4 \times 2 \times 2 \times 3 = 48.$

2nd.

$$\left.\begin{array}{l} \dfrac{3 \times 3}{16 \times 3} = \dfrac{9}{48} \\[2ex] \dfrac{3 \times 12}{4 \times 12} = \dfrac{36}{48} \\[2ex] \dfrac{5 \times 6}{8 \times 6} = \dfrac{30}{48} \\[2ex] \dfrac{1 \times 16}{3 \times 16} = \dfrac{16}{48} \end{array}\right\} \textit{Ans.}$$

Explanation.—1st. To find the *least common denominator* place the denominators of all the fractions in a horizontal row as shown, and proceed to find the *largest factor* which will divide the *greatest number* of denominators and use it as a first divisor. In this case we find that 4 will divide 16, 4 and 8; we therefore use 4 as the first divisor and divide 16, 4 and 8, by it; 4 is contained into 16 *four* times, we place the 4 *under* the 16; 4 is contained in 4 *once*, we place 1 *under* the 4; 4 is contained in 8 *twice*, we place the 2 *under* the 8; 4 is not contained in 3 so we bring the 3 down. We now have 4, 1, 2 and 3 in the second row. We find that 2 will divide 4 and 2, therefore we use 2 as the next divisor. 2 is contained in 4 twice; we place the 2 under the 4. 2 is contained in 2, once, we place the 1 under the 2; 2 is not contained in 1, nor in 3 without a remainder, so we bring down the 1 and the 3. We now have 2, 1, 1, 3, in the third row. As 2 and 3 are the only numbers greater than 1 in this row, we again divide by 2, and then by 3 as shown. The last row is now 1, 1, 1, 1. Therefore the factors of the least common denominator of $\frac{3}{16}$, $\frac{3}{4}$, $\frac{5}{8}$, and $\frac{1}{3}$, are 4, 2, 2, 3. These multiplied together equal 48, the least common denominator.

2nd. The next step in the solution is to reduce the given fractions to fractions whose denominators are 48. To do this we must multiply both terms of each fraction by a number which will make the denominator 48. The denominator of the first fraction is contained in 48 *three* times, so we must multiply both terms, 3 and 16, by 3, which gives $\frac{9}{48}$. The denominator of the second fraction is contained in 48 *twelve* times, so both terms, 3 and 4, must be multiplied by 12; each term of the third fraction by 6; and each term of the fourth fraction by 16. The four given fractions $\frac{3}{16}$, $\frac{3}{4}$, $\frac{5}{8}$, and $\frac{1}{3}$, are equal respectively to $\frac{9}{48}$, $\frac{36}{48}$, $\frac{30}{48}$, and $\frac{16}{48}$.

132. Rule.—*Find the least common denominator, multiply both terms of the fraction by the quotient derived from dividing the least common denominator by the denominator of the fraction.*

133. **Problems.**—Reduce the following fractions to fractions having a least common denominator:

(1) $\frac{1}{8}$, $\frac{3}{4}$, $\frac{5}{16}$. (2) $\frac{9}{15}$, $\frac{17}{8}$, $\frac{12}{32}$. (3) $\frac{5}{16}$, $\frac{7}{24}$, $\frac{21}{30}$. (4) $\frac{1}{7}$, $\frac{5}{8}$, $\frac{3}{16}$, $\frac{9}{32}$, $\frac{18}{64}$.

Answers.— (1) $\frac{2}{16}$, $\frac{12}{16}$, $\frac{5}{16}$. (2). $\frac{288}{480}$, $\frac{1020}{480}$, $\frac{180}{480}$. (3) $\frac{75}{240}$, $\frac{70}{240}$, $\frac{168}{240}$. (4) $\frac{64}{448}$, $\frac{280}{448}$, $\frac{84}{448}$, $\frac{126}{448}$, $\frac{126}{448}$.

ADDITION

134. Principle.—*Only similar fractions can be added.*

135. Problem.—$\frac{3}{8}+\frac{5}{8}+\frac{7}{8}$.
Solution.—$\frac{3}{8}+\frac{5}{8}+\frac{7}{8}=\frac{15}{8}=1\frac{7}{8}$, *Ans.*

Explanation.—In this problem the fractions are similar fractions, so to add them the *numerators* 3, 5 and 7 only, are added and their sum 15 is placed over the common denominator 8. This gives $\frac{15}{8}$ as the sum which reduced to a mixed number equals $1\frac{7}{8}$.

136. Dissimilar fractions must be reduced to similar fractions before they can be added.

137. Problem.—$\frac{1}{8}+\frac{2}{5}+\frac{3}{10}$.
Solution.—$\frac{1}{8}+\frac{2}{5}+\frac{3}{10}=\frac{5}{40}+\frac{16}{40}+\frac{12}{40}=\frac{33}{40}$, *Ans.*

Explanation.—In this problem the fractions are *dissimilar* so the first step in the solution is to reduce them to *similar* fractions (Art. 132). The common denominator is found to be 40, and the given fractions to be equal to $\frac{5}{40}$, $\frac{16}{40}$, and $\frac{12}{40}$, respectively. These fractions are similar and can be added by *adding* the *numerators* 5, 16 and 12, and placing the sum over the common denominator 40; this gives $\frac{33}{40}$ as the sum of the fractions.

138. Rule.—*First reduce to similar fractions having a least common denominator, then add the numerators. Place the sum so obtained over the denominator and reduce the fraction to its lowest terms.*

TO ADD WHOLE AND MIXED NUMBERS

139. When whole and mixed numbers are to be added, add the fractions and whole numbers separately, combine the sums thus obtained to obtain the total sum.

140. PROBLEM.—$12\frac{3}{8}+6\frac{7}{8}+7$.

SOLUTION.—

$$\text{(1st.)}\quad \begin{array}{r} 12 \\ 6 \\ 7 \\ \hline 25 \end{array} \qquad \text{(2nd.)}\quad \tfrac{3}{8}+\tfrac{7}{8}=\tfrac{10}{8}=1\tfrac{2}{8}=1\tfrac{1}{4}. \qquad \text{(3rd.)}\quad \begin{array}{r} 25 \\ 1\frac{1}{4} \\ \hline 26\frac{1}{4}, \end{array}\ \textit{Ans.}$$

EXPLANATION.—(1st). Add the *whole* numbers 12, 6 and 7, which gives 25. (2nd). Add the *fractions* $\frac{3}{8}$ and $\frac{7}{8}$ getting $\frac{10}{8}$ or $1\frac{1}{4}$. (3rd.) *Add* the *sums* of the *first* and *second* operations, 25 and $1\frac{1}{4}$, which gives $26\frac{1}{4}$ as the sum of the mixed numbers.

141. RULE.—*Add the whole numbers. Then reduce the fractions to similar fractions, adding their sum to the whole number, first reducing an improper fraction to a whole or mixed number.*

142. PROBLEMS.—(1) $2\frac{1}{8}+4\frac{7}{10}$. (2) $12\frac{3}{4}+7\frac{1}{2}$. (3) $14\frac{3}{7}+7\frac{5}{8}$.
(4) $2\frac{1}{2}+7\frac{1}{4}+3\frac{5}{8}+1\frac{3}{16}$. (5) $9\frac{3}{64}+10\frac{1}{8}+7\frac{5}{32}+4\frac{1}{2}$.
(6) $10\frac{15}{16}+8\frac{7}{8}+3\frac{1}{4}+5\frac{1}{2}$. (7) $5\frac{1}{16}+2\frac{3}{4}+7\frac{1}{8}+1\frac{19}{32}$.
(8) $7\frac{2}{3}+8\frac{1}{5}+19\frac{9}{15}$. (9) $45\frac{1}{2}+37\frac{1}{8}+54\frac{3}{4}$.
ANSWERS.— (1) $6\frac{33}{40}$. (2) $20\frac{1}{4}$. (3) $22\frac{3}{56}$. (4) $14\frac{9}{16}$.
(5) $30\frac{53}{64}$. (6) $28\frac{9}{16}$. (7) $16\frac{7}{16}$. (8) $35\frac{7}{15}$. (9) $137\frac{3}{8}$.

SUBTRACTION

143. PRINCIPLE.—*Only similar fractions can be subtracted.*

144. PROBLEM.—From $\frac{11}{12}$ subtract $\frac{3}{4}$.
SOLUTION.— $\frac{11}{12}-\frac{3}{4}=\frac{11}{12}-\frac{9}{12}=\frac{2}{12}=\frac{1}{6}$, *Ans.*

EXPLANATION.—As the fractions are *dissimilar* they must be reduced to *similar* fractions before subtracting. The *numerator* of the *subtrahend* is then subtracted from that of the *minuend.*

145. Problems.— (1) $\frac{19}{21}-\frac{2}{3}$. (2) $\frac{34}{36}-\frac{4}{9}$. (3) $\frac{39}{45}-\frac{7}{15}$.
(4) $\frac{23}{27}-\frac{3}{4}$. (5) $\frac{49}{60}-\frac{7}{15}$. (6) $\frac{3}{8}-\frac{2}{7}$. (7) $\frac{67}{75}-\frac{5}{9}$.
(8) $\frac{71}{72}-\frac{9}{24}$. (9) $\frac{43}{51}-\frac{8}{17}$. (10) $\frac{12}{13}-\frac{47}{65}$.

Answers.— (1) $\frac{5}{21}$. (2) $\frac{1}{2}$. (3) $\frac{2}{5}$. (4) $\frac{11}{108}$. (5) $\frac{7}{20}$.
(6) $\frac{5}{56}$. (7) $\frac{76}{225}$. (8) $\frac{11}{18}$. (9) $\frac{19}{51}$. (10) $\frac{1}{5}$.

TO SUBTRACT WHOLE OR MIXED NUMBERS

146. When whole or mixed numbers are to be subtracted, subtract the fractions and whole numbers separately.

147. Problem.—Subtract $7\frac{5}{8}$ from $9\frac{7}{8}$.

Solution.— *minuend* $9\frac{7}{8}$
subtrahend $7\frac{5}{8}$
remainder $2\frac{2}{8}=2\frac{1}{4}$, *Ans.*

Explanation.—In the problem from $9\frac{7}{8}$ subtract $7\frac{5}{8}$ we subtract 7 from 9 which leaves 2, then subtracting $\frac{5}{8}$ from $\frac{7}{8}$ we get $\frac{2}{8}$, combining this with the *whole* number in the quotient we get $2\frac{2}{8}$, reducing which we get $2\frac{1}{4}$.

148. Rule.—*Subtract the whole numbers and the fractions separately. Combine the results.*

149. Problems.— (1) $147\frac{4}{5}-98\frac{2}{3}$. (2) $986\frac{19}{20}-798\frac{4}{7}$.
(3) $291\frac{8}{9}-199\frac{5}{8}$. (4) $87\frac{4}{9}-69$. (5) $1982\frac{12}{13}-797\frac{5}{6}$.
(6) $1231\frac{5}{12}-695\frac{2}{5}$. (7) $429\frac{6}{17}-371\frac{7}{51}$. (8) $919\frac{2}{3}-199\frac{7}{16}$.
(9) $16425\frac{12}{49}-9876\frac{4}{56}$. (10) $489\frac{77}{120}-298\frac{17}{32}$.

Answers.— (1) $49\frac{2}{15}$. (2) $188\frac{53}{140}$. (3) $92\frac{19}{72}$. (4) $18\frac{4}{9}$.
(5) $1185\frac{7}{78}$. (6) $536\frac{1}{60}$. (7) $58\frac{11}{51}$. (8) $720\frac{11}{48}$. (9) $6549\frac{17}{98}$.
(10) $191\frac{53}{480}$.

150. When the minuend is a whole number, or the fraction in the minuend is *smaller* than the fraction in the subtrahend, one of the *whole numbers* in the minuend must be reduced to a fraction of the same denomination as the fraction in the subtrahend, in order to have a fraction in the minuend from which to subtract the fraction in the subtrahend.

151. PROBLEM.— $10\frac{3}{4}-5\frac{7}{8}$.

SOLUTION.— $10\frac{3}{4}=10\frac{6}{8}=9\frac{14}{8}$

$$\begin{array}{r} 9\frac{14}{8} \\ 5\frac{7}{8} \\ \hline 4\frac{7}{8} \end{array}$$ *Ans.*

EXPLANATION.—Whereas the fraction in the *subtrahend* is *greater* than the fraction in the *minuend,* one of the *whole* numbers in the minuend must be reduced to *8ths;* so reducing gives us $\frac{8}{8}+\frac{6}{8}$, or $\frac{14}{8}$, as the fraction in the minuend and leaves 9 as the whole number. Then the new minuend is $9\frac{14}{8}$ instead of $10\frac{6}{8}$ and we can now subtract $\frac{7}{8}$ from $\frac{14}{8}$ which leaves $\frac{7}{8}$, we can also subtract 5 *whole* numbers from 9 *whole* numbers which leaves 4 whole numbers. Combining the 4 *whole* numbers with the fraction $\frac{7}{8}$ we get $4\frac{7}{8}$.

152. RULE.—*Reduce one of the whole numbers in the minuend to a fraction and subtract in the usual way.*

153. PROBLEMS.—(1) $8\frac{1}{6}-6\frac{1}{4}$. (2) $15\frac{1}{8}-7\frac{1}{4}$. (3) $21\frac{3}{14}-18\frac{5}{7}$. (4) $8\frac{1}{8}-5\frac{2}{3}$. (5) $24\frac{5}{14}-23\frac{1}{2}$.

ANSWERS.— (1) $1\frac{11}{12}$. (2) $7\frac{7}{8}$. (3) $2\frac{1}{2}$. (4) $2\frac{11}{24}$. (5) $\frac{6}{7}$.

MULTIPLICATION

TO MULTIPLY A FRACTION BY A FRACTION

154. To multiply a fraction by a fraction, multiply the numerators for the new numerator and the denominators for the new denominator.

155. PROBLEM.—$\frac{9}{16}\times\frac{8}{15}$.

SOLUTION.— $\frac{9}{16}\times\frac{8}{15}=\frac{9\times 8}{16\times 15}=\frac{72}{240}=\frac{3}{10}$, *Ans.*

EXPLANATION.—In this example the *numerator*, 9, of the *first* fraction is *multiplied* by the *numerator*, 8, of the *second* fraction to get the *numerator* of the result, which is 72; the *denominator*, 16, of the *first* fraction is *multiplied* by the *denominator*, 15, of the *second* fraction to get the *denominator* of the *result*, which is 240. Then the result $\frac{72}{240}$, is reduced to its *lowest* terms giving $\frac{3}{10}$ as the final product.

156. When possible cancellation should be used before multiplying.

157. PROBLEM.—$\frac{9}{16}\times\frac{8}{15}$.
SOLUTION.—

$$\frac{9}{16}\times\frac{8}{15}=\frac{\overset{3}{\cancel{9}}}{\underset{2}{\cancel{16}}}\times\frac{\overset{1}{\cancel{8}}}{\underset{5}{\cancel{15}}}=\frac{3}{10},\quad \textit{Ans.}$$

EXPLANATION.—This problem is the same as in Art. 155. The solution given in Art. 155 can be simplified if the factor 8 which is *common* to the *first* denominator 16, and the *second* numerator 8, and the factor 3 which is common to the *first* numerator 9 and the *second* denominator 15, are cancelled as shown above, before the *numerators* and *denominators* are multiplied.

158. In the multiplication of fractions the word "*of*" means the same as ×, or times. Thus $\frac{9}{16}$ of $\frac{8}{15}=\frac{9}{16}\times\frac{8}{15}$.

159. RULE.—*Multiply the numerator by the numerator and the denominator by the denominator.*

160. PROBLEMS.—(1) $\frac{3}{8}\times\frac{7}{10}$. (2) $\frac{9}{16}\times\frac{5}{16}$. (3) $\frac{5}{7}\times\frac{3}{5}$.
(4) $\frac{15}{32}\times\frac{1}{2}$. (5) $\frac{3}{4}\times\frac{17}{64}$. (6) $\frac{8}{9}\times\frac{5}{13}$. (7) $\frac{13}{16}\times\frac{7}{8}$.
(8) $\frac{24}{25}\times\frac{7}{10}$. (9) $\frac{2}{3}\times\frac{1}{4}$. (10) $\frac{3}{8}\times\frac{7}{9}$.

ANSWERS.— (1) $\frac{21}{80}$. (2) $\frac{45}{256}$. (3) $\frac{3}{7}$. (4) $\frac{15}{64}$. (5) $\frac{51}{256}$.
(6) $\frac{40}{117}$. (7) $\frac{91}{128}$. (8) $\frac{84}{125}$. (9) $\frac{1}{6}$. (10) $\frac{7}{24}$.

TO MULTIPLY A FRACTION BY A WHOLE NUMBER

161. To multiply a fraction by a whole number multiply the numerator or divide the denominator by the whole number.

162. PROBLEM.—Multiply $\frac{7}{8}$ by 4.

1st method. Multiplying the numerator by the whole number.

SOLUTION.—$\frac{7}{8}\times4=\frac{28}{8}=3\frac{1}{2}$, *Ans.*

2nd method. Dividing the denominator by the whole number.

SOLUTION.—$\frac{7}{8}\times4=\frac{7}{2}=3\frac{1}{2}$, *Ans.*

EXPLANATION.—Using the 1st method we *multiply* the *numerator* 7, by the whole number 4, and get the product $\frac{28}{8}$ which equals $3\frac{1}{2}$. Using the 2nd method we *divide* the *denominator* 8, by the whole number 4, and get the quotient $\frac{7}{2}$, which also equals $3\frac{1}{2}$.

163. RULE.—*Multiply the numerator or divide the denominator by the whole number.*

164. PROBLEMS.— (1) $\frac{3}{9}\times5$. (2) $\frac{7}{8}\times9$. (3) $\frac{15}{16}\times13$.
(4) $\frac{11}{10}\times12$. (5) $\frac{17}{32}\times4$. (6) $\frac{32}{64}\times18$. (7) $\frac{19}{64}\times7$.
(8) $\frac{1}{2}\times42$. (9) $\frac{16}{32}\times17$. (10) $\frac{21}{24}\times8$.

ANSWERS.— (1) $1\frac{2}{3}$. (2) $7\frac{7}{8}$. (3) $12\frac{3}{16}$. (4) $13\frac{1}{5}$. (5) $2\frac{1}{8}$.
(6) 9. (7) $2\frac{5}{64}$. (8) 21. (9) $8\frac{1}{2}$. (10) 7.

TO MULTIPLY A MIXED NUMBER BY A WHOLE NUMBER

165. To multiply a mixed number by a whole number, multiply the whole number and the fraction of the mixed number separately, then add the products.

166. PROBLEM.—$4\frac{5}{8}\times10$.

SOLUTION.—

$$\begin{array}{l} 4\frac{5}{8} \\ \underline{10} \\ 40\frac{50}{8} \quad \text{then } \frac{50}{8}=6\frac{1}{4} \\ \qquad\quad \text{then } 40+6\frac{1}{4}=46\frac{1}{4}, \ \textit{Ans.} \end{array}$$

EXPLANATION.—Whereas the product of a *mixed* number by a *whole* number is equal to the product of the *whole* number in the *multiplicand* by the *whole* number in the *multiplier, plus* the product of the *fraction* by the *multiplier;* we may multiply the *whole* numbers 4 and 10 together; this gives us 40. Then we multiply the *numerator* 5 of the fraction $\frac{5}{8}$ by the *multiplier* 10. This gives us $\frac{50}{8}$ which reduced to a mixed number gives us

6 whole numbers and $\frac{2}{8}$, or $\frac{1}{4}$. Now adding the *sums* of the operations we have $46\frac{1}{4}$.

167. RULE.—*Multiply the whole numbers and the fraction separately, reduce the improper fraction, if there is one, to a whole or mixed number and find the sum of the two products.*

168. PROBLEMS.— (1) $2\frac{2}{3} \times 7$. (2) $13\frac{15}{16} \times 9$. (3) $27\frac{3}{4} \times 15$. (4) $12\frac{1}{15} \times 37$. (5) $14\frac{1}{8} \times 21$. (6) $32\frac{5}{8} \times 19$. (7) $63\frac{2}{3} \times 5$. (8) $8\frac{9}{10} \times 13$. (9) $12\frac{7}{8} \times 3$. (10) $14\frac{5}{8} \times 17$.

ANSWERS.—(1) $18\frac{2}{3}$. (2) $125\frac{7}{16}$. (3) $416\frac{1}{4}$. (4) $446\frac{7}{15}$. (5) $296\frac{5}{8}$. (6) $619\frac{7}{8}$. (7) $318\frac{1}{3}$. (8) $115\frac{7}{10}$. (9) $38\frac{5}{8}$. (10) $248\frac{5}{8}$.

TO MULTIPLY A MIXED NUMBER BY A MIXED NUMBER

169. To multiply a mixed number by a mixed number, reduce them to improper fractions and then multiply the numerators for the new numerator and the denominators for the new denominator and reduce to a whole or mixed number.

170. PROBLEM.—$2\frac{1}{2} \times 1\frac{1}{8}$.
SOLUTION.—$2\frac{1}{2} \times 1\frac{1}{8} = \frac{5}{2} \times \frac{9}{8} = \frac{45}{16} = 2\frac{13}{16}$, *Ans.*

EXPLANATION.—The mixed numbers $2\frac{1}{2}$ and $1\frac{1}{8}$ must first be reduced to *improper* fractions. (Art. 121.) These *improper* fractions, $\frac{5}{2}$ and $\frac{9}{8}$, are then *multiplied* and the result is reduced to a mixed number giving $2\frac{13}{16}$ as the product.

171. RULE.—*Reduce both the multiplicand and multiplier to improper fractions, cancel all the factors common to any numerator and any denominator, then multiply the numerators for the numerator of the product, and the denominators for the denominator of the product. Reduce the product to a whole or mixed number.*

172. Problems.—(1) $2\frac{2}{3}\times3\frac{1}{5}$. (2) $8\frac{1}{2}\times7\frac{3}{4}$. (3) $9\frac{7}{8}\times5\frac{3}{5}$. (4) $15\frac{1}{2}\times14\frac{1}{2}$. (5) $12\frac{3}{4}\times15\frac{1}{32}$. (6) $18\frac{7}{10}\times19\frac{3}{5}$.

Answers.— (1) $8\frac{8}{15}$. (2) $65\frac{7}{8}$. (3) $55\frac{3}{10}$. (4) $224\frac{1}{4}$. (5) $191\frac{83}{128}$. (6) $366\frac{13}{25}$.

DIVISION

TO DIVIDE A FRACTION BY A FRACTION

173. To divide a fraction by a fraction, multiply the dividend by the divisor inverted.

174. Use cancellation when possible.

175. The fraction $\frac{3}{8}$ inverted reads $\frac{8}{3}$.

176. Problem.—$\frac{7}{8}\div\frac{3}{8}$.

Solution.— $\frac{7}{8}\div\frac{3}{8}=\frac{7}{8}\times\frac{8}{3}=\frac{56}{24}=2\frac{1}{3}$. *Ans.*

Solution.— $\frac{7}{8}\div\frac{3}{8}=\frac{7}{\cancel{8}}\times\frac{\cancel{8}}{3}=\frac{7}{3}=2\frac{1}{3}$. *Ans.*

Explanation.—Whereas the fraction $\frac{3}{8}$ when *inverted* reads $\frac{8}{3}$, and whereas the *dividend* is *multiplied* by the *divisor inverted*, we simply multiply 7×8, which gives 56 for the *new* numerator and then multiply 8×3, which gives us 24 for the *new* denominator; then reducing $\frac{56}{24}$ to a whole or mixed number we get $2\frac{1}{3}$; or using cancellation we cancel the 8 in the denominator and the 8 in the numerator and get $\frac{7}{3}$ which reduced to a whole or mixed number gives us $2\frac{1}{3}$.

177. Rule.—*Multiply the terms of the dividend by the terms of the divisor inverted and reduce to a whole or mixed number.*

178. Problems.—(1) $\frac{7}{8}\div\frac{1}{3}$. (2) $\frac{5}{8}\div\frac{3}{8}$. (3) $\frac{13}{16}\div\frac{3}{8}$. (4) $\frac{8}{25}\div\frac{9}{16}$. (5) $\frac{3}{10}\div\frac{2}{7}$. (6) $\frac{8}{9}\div\frac{7}{8}$. (7) $\frac{3}{16}\div\frac{2}{5}$. (8) $\frac{19}{32}\div\frac{3}{8}$. (9) $\frac{4}{7}\div\frac{5}{8}$. (10) $\frac{3}{7}\div\frac{5}{8}$.

Answers.—(1) $2\frac{5}{8}$. (2) $1\frac{2}{3}$. (3) $2\frac{1}{6}$. (4) $\frac{128}{225}$. (5) $1\frac{1}{20}$. (6) $1\frac{1}{63}$. (7) $\frac{15}{32}$. (8) $1\frac{7}{12}$. (9) $\frac{32}{35}$. (10) $\frac{24}{35}$.

TO DIVIDE A FRACTION BY A WHOLE NUMBER

179. To divide a fraction by a whole number, either divide the numerator or multiply the denominator by the whole number.

180. PROBLEM.—$\frac{9}{12} \div 3$.
1st method. By dividing the numerator.
SOLUTION.—$\frac{9}{12} \div 3 = \frac{3}{12} = \frac{1}{4}$, *Ans.*

2nd method. By multiplying the denominator.
SOLUTION.—$\frac{9}{12} \div 3 = \frac{9}{36} = \frac{1}{4}$, *Ans.*

EXPLANATION.—Using the 1st method we *divide* the *numerator* by the whole number 3, getting $\frac{3}{12}$, which reduced to its lowest terms equals $\frac{1}{4}$. Using the 2nd method we *multiply* the *denominator* by the whole number 3, getting $\frac{9}{36}$, which reduced to its lowest terms, also equals $\frac{1}{4}$.

181. RULE.—*Divide the numerator or multiply the denominator of the fraction by the whole number and reduce.*

182. PROBLEMS.—(1) $\frac{12}{32} \div 4$. (2) $\frac{21}{16} \div 10$. (3) $\frac{9}{13} \div 7$.
(4) $\frac{21}{64} \div 8$. (5) $\frac{12}{15} \div 13$. (6) $\frac{18}{30} \div 15$. (7) $\frac{9}{13} \div 2$.
(8) $\frac{12}{16} \div 11$. (9) $\frac{16}{21} \div 2$. (10) $\frac{14}{17} \div 12$.

ANSWERS.— (1) $\frac{3}{32}$. (2) $\frac{21}{160}$. (3) $\frac{9}{91}$. (4) $\frac{21}{512}$. (5) $\frac{4}{65}$.
(6) $\frac{1}{25}$. (7) $\frac{9}{26}$. (8) $\frac{12}{176}$. (9) $\frac{8}{21}$. (10) $\frac{7}{102}$.

TO DIVIDE A MIXED NUMBER BY A WHOLE NUMBER

183. To divide a mixed number by a whole number, divide the whole number as in ordinary division; then reduce the remainder to an improper fraction and divide the numerator or multiply the denominator by the divisor.

184. Problem.—Divide $18\frac{1}{8}$ by 5.

Solution.— $18\frac{1}{8} \div 5 = 5)18\frac{1}{8}(3$

$$\begin{array}{l} 15 \\ \hline 3\frac{1}{8} = \frac{25}{8} \text{ and } \frac{25}{8} \div 5 = \frac{5}{8}. \end{array}$$

Then $3 + \frac{5}{8} = 3\frac{5}{8}$, *Ans.*

Explanation.—First we *divide* the *whole* number 18 by 5 and obtain 3 and a remainder of 3; then we *reduce* the remainder 3 to $\frac{24}{8}$ and add the $\frac{1}{8}$ in the dividend making $\frac{25}{8}$: we now *divide* the *numerator* 25, by the *divisor* 5 and obtain $\frac{5}{8}$. Adding the fraction $\frac{5}{8}$ to the 3 in the quotient, we get $3\frac{5}{8}$.

185. Rule.—*Divide the whole number by the divisor in the usual way. Reduce the remainder, if any, to an improper fraction having the same denominator as the fraction in the mixed number, add to this the fraction of the mixed number and divide the numerator or multiply the denominator by the divisor.*

186. Problems.—(1) $15\frac{1}{2} \div 7$. (2) $24\frac{1}{3} \div 9$. (3) $31\frac{7}{10} \div 5$.
(4) $4\frac{1}{8} \div 3$. (5) $56\frac{17}{32} \div 5$. (6) $27\frac{1}{2} \div 4$. (7) $32\frac{1}{8} \div 5$.
(8) $365\frac{13}{64} \div 8$. (9) $97\frac{3}{16} \div 7$.

Answers.— (1) $2\frac{3}{14}$. (2) $2\frac{19}{27}$. (3) $6\frac{17}{50}$. (4) $1\frac{3}{8}$.
(5) $11\frac{49}{160}$. (6) $6\frac{7}{8}$. (7) $6\frac{17}{40}$. (8) $45\frac{333}{512}$. (9) $13\frac{99}{112}$.

TO DIVIDE A MIXED NUMBER BY A MIXED NUMBER

187. It is necessary to reduce mixed numbers to improper fractions before dividing.

188. Problem.—$3\frac{1}{8} \div 2\frac{1}{4}$.

Solution.—$3\frac{1}{8} = \frac{25}{8}$.
$2\frac{1}{4} = \frac{9}{4}$.

then $\frac{25}{8} \div \frac{9}{4} = \frac{25}{\cancel{8}_{2}} \times \frac{\cancel{4}}{9} = \frac{25}{18} = 1\frac{7}{18}$ *Ans.*

Explanation.—Reducing $3\frac{1}{8}$ to an improper fraction we get

$\frac{25}{8}$, and reducing $2\frac{1}{4}$ to an improper fraction we get $\frac{9}{4}$; then inverting the divisor $\frac{9}{4}$ we get $\frac{4}{9}$, and multiplying the dividend by it, gives us, using cancellation, $\frac{25}{18}$. This reduced to a whole or mixed number gives us $1\frac{7}{18}$.

189. Rule.—*Reduce to improper fractions and divide in the usual manner, using cancellation when possible.*

190. Problems.—(1) $2\frac{1}{2} \div 1\frac{1}{4}$. (2) $3\frac{8}{10} \div 7\frac{1}{9}$. (3) $4\frac{1}{8} \div 9\frac{1}{4}$. (4) $20\frac{1}{32} \div 3\frac{1}{2}$. (5) $70\frac{3}{16} \div 65\frac{1}{8}$. (6) $15\frac{5}{8} \div 7\frac{1}{16}$. (7) $12\frac{1}{2} \div 3\frac{3}{5}$.
Answers.— (1) 2. (2) $\frac{171}{320}$. (3) $\frac{33}{74}$. (4) $5\frac{81}{112}$. (5) $1\frac{81}{1042}$. (6) $2\frac{24}{113}$. (7) $3\frac{17}{36}$.

COMPLEX FRACTIONS

191. Expressions in unexecuted division of fractions are frequently written in the form of a fraction. When thus expressed they are called **complex fractions.** As $\frac{3}{4} \div \frac{1}{2}$ when expressed $\frac{\frac{3}{4}}{\frac{1}{2}}$, and $\frac{1}{8} \div 9$ when expressed $\frac{\frac{1}{8}}{9}$, and $9 \div \frac{1}{8}$ when expressed $\frac{9}{\frac{1}{8}}$, are complex fractions.

192. When a *heavy line* separates the parts of a complex fraction, it indicates that all *below* the line is to be divided into all *above* the line.

193. More intricate complex fractions are as follows:

$$\frac{\frac{3}{4} \div \frac{1}{4}}{\frac{4}{5} + \frac{5}{8}} \qquad \frac{2\frac{1}{4} + \frac{3}{8}}{1\frac{1}{2} + 2\frac{3}{4}}$$

These are solved like other problems in division of fractions.

194. Problems.—(1) $\frac{3\frac{1}{8}}{5}$ (2) $\frac{\frac{4}{\frac{1}{2}}}{3}$ (3) $\frac{4\frac{7}{8}}{6\frac{1}{2}}$

(4) $\frac{\frac{5}{32} \text{ of } 10\frac{5}{8}}{\frac{13}{80} \text{ of } 9\frac{1}{10}}$ (5) $\frac{\frac{11}{16} \text{ of } 9\frac{1}{3}}{\frac{5}{8} \text{ of } 2\frac{1}{2}}$

Answers.—(1) $\frac{5}{8}$. (2) $2\frac{2}{3}$. (3) $\frac{3}{4}$. (4) $1\frac{1161}{9464}$. (5) $5\frac{7}{150}$.

EXAMINATION PROBLEMS

(1) Give ten different factors of 90.

(2) Find the sum of $\frac{1}{8}$, $\frac{3}{4}$, $\frac{5}{8}$, $\frac{13}{16}$.

(3) Find the product of $\frac{3}{4}$, $\frac{5}{8}$, $\frac{1}{9}$.

(4) Find the quotient of $\frac{3}{16} \div \frac{5}{8}$.

(5) Find the product of $3\frac{1}{8}$, $4\frac{1}{2}$, $5\frac{1}{9}$.

(6) Reduce to a simple fraction, $\dfrac{3\frac{24}{35} \text{ of } 1\frac{1}{6}}{4\frac{1}{8} \text{ of } 110\frac{1}{6}}$.

(7) F nd the prime factors of 6,480.

(8) The signal tower of an interlocking plant cost 945 dollars, which was $\frac{3}{16}$ of the total cost of the entire installation. What was the total cost?

(9) At signal No. 21, 30 dollars were expended for repairs, which is $\frac{3}{32}$ of the first cost of the signal. At signal No. 32, 35 dollars were expended for repairs which is $\frac{7}{72}$ of the first cost of the signal. On which signal was the first cost the greater, and how much?

(10) Two men, A and B, contract to paint a signal tower for 38 dollars. If the rate of pay per day for both men is the same, how much will each receive if A works $2\frac{1}{2}$ days and B works $4\frac{2}{5}$ the length of time that A works?

(11) The difference between $\frac{7}{8}$ and $\frac{11}{12}$ of a certain number is 250. What is the number?

(12) A train travelled from noon to a time equal to $\frac{4}{5}$ of 15 hours. How far did it travel if it maintained a speed of $38\frac{1}{2}$ miles per hour?

(13) A railroad was leased for 99 years for the sum of 52,864 dollars per year. If $\frac{4}{7}$ of the total sum is still due, how much has been paid?

(14) A number is equal to two times the difference between $\dfrac{65\frac{1}{2}}{2}$ and $\dfrac{14\frac{1}{3}}{5}$. What is the number?

(15) The repairs on the signal plant for 1895 cost $3\frac{1}{4}$ times as much as the repairs in 1893, and the repairs in 1893 were equal to $\frac{1}{20}$ the difference between $\frac{3}{5}$ and $\frac{1}{5}$ of the total cost of the installation. The cost of the installation was 19,500 dollars. What was the cost of the repairs in 1895?

(16) What is the difference in the cost of two automatic signals if $\frac{1}{4}$ of the total cost is 125 dollars, assuming the cost of one of the signals to be $\frac{2}{5}$ of the total cost?

(17) Train A travels at the rate of 88 ft. per second for a period of 2,130 seconds. Train B travels at the rate of 2,169 ft. per minute for a period of $45\frac{1}{2}$ minutes. (*a*) Which train travels the greater distance? (*b*) How much greater?

(18) How much will have to be added to $\frac{13\frac{5}{16}}{3}$ in order that we can subtract $\frac{25\frac{7}{8}}{4}$ and have a remainder of 5?

(19) A railroad company owes 253,060 dollars. By saving $\frac{3}{8}$ of its earnings for a period of 30 years it can pay the debt and have 3,000 dollars left. What are its annual earnings?

(20) A railroad company paid 235,000 dollars in 1902 for signal installations. $\frac{2}{3}$ of $\frac{1}{4}$ of this sum was for repairs. How much did the repairs cost?

ANSWERS TO EXAMINATION PROBLEMS

(1) ?

(2) $2\frac{5}{16}$.

(3) $\frac{5}{96}$.

(4) $\frac{8}{10}$.

(5) $71\frac{7}{8}$.

(6) $\frac{8440}{881848}$.

(7) 2, 2, 2, 2, 3, 3, 3, 3, 5.

(8) 5,040 dollars.

(9) Signal No. 32 cost 40 dollars more.

(10) A, $7\frac{1}{27}$ dollars; B, $30\frac{26}{27}$ dollars.

(11) 6,000.

(12) 462 miles.

(13) 2,242,944 dollars.

(14) $59\frac{28}{30}$.

(15) $1{,}267\frac{1}{2}$ dollars.

(16) 100 dollars.

(17) (*a*) Train A. (*b*) $88{,}750\frac{1}{2}$ feet.

(18) $6\frac{78}{96}$.

(19) $22{,}760\frac{8}{9}$ dollars.

(20) $39{,}166\frac{2}{3}$ dollars.

DECIMALS

195. The divisions of any thing into *tenths, hundredths, thousandths, etc.*, are called **decimal divisions.**

196. One or more of these *decimal divisions*, is called a *decimal fraction*, commonly known as a **decimal.**

197. A *decimal* is a fraction whose *denominator* is *ten*, or some *power* of *ten*. As $\frac{1}{10}$, $\frac{1}{100}$, $\frac{1}{1000}$, etc.

198. In expressing *decimals* we do not write out the denominator as in *common* fractions, but instead, a period, called a **decimal point,** is placed to the left of the numerator. That is $\frac{1}{10}$ is expressed .1, and $\frac{5}{10}$, .5.

199. If the denominator is *higher* than ten, as one hundred, one thousand, etc., as many ciphers as are necessary are placed between the *decimal point* and the *figure*, or *figures*, in the numerator. As $\frac{1}{100}$ is expressed .01, $\frac{1}{1000}$ is expressed .001, etc.

200. The number of figures in the decimal is always *equal* to the number of *ciphers* in the denominator of the fraction. As $.003 = \frac{3}{1000}$ and as $.034 = \frac{34}{1000}$.

201. Only figures placed to the *right* of a decimal point are affected by it. As shown in the following table:

5 . 5 = $5\frac{5}{10}$ = *Five and five tenths.*
5 . 0 5 = $5\frac{5}{100}$ = *Five and five hundredths.*
5 . 0 0 5 = $5\frac{5}{1000}$ = *Five and five thousandths.*
5 . 0 0 0 5 = $5\frac{5}{10000}$ = *Five and five ten thousandths.*
5 . 0 0 0 0 5 = $5\frac{5}{100000}$ = *Five and five hundred thousandths.*

Units Decimal 1st 2nd 3rd 4th 5th Places

202. Adding ciphers to the *right* of a decimal does not alter the **value** of the decimal. As $.7=\frac{7}{10}$, $.70=\frac{70}{100}$, but as $\frac{7}{10}=\frac{70}{100}$ .7 must equal .70.

DECIMAL NOTATION TABLE

203. The following numeration table shows the relation of decimals to whole numbers.

Billions.	*Hundreds-millions.*	*Tens-millions.*	*Millions.*	*Hundreds-thousands.*	*Tens-thousands.*	*Thousands.*	*Hundreds.*	*Tens.*	*Units.*	*Decimal Point.*	*Tenths.*	*Hundredths.*	*Thousandths.*	*Ten-thousandths.*	*Hundred-thousandths.*	*Millionths.*	*Ten-millionths.*	*Hundred-millionths.*	*Billionths.*
2	2	2	2	2	2	2	2	2	2	.	2	2	2	2	2	2	2	2	2

204. It will be noted that the decimals *decrease* by the scale of ten to the right of the decimal point, and the whole numbers *increase* by the scale of ten to the left of the decimal point.

TO REDUCE DECIMALS TO COMMON FRACTIONS

205. Problem.—Reduce .25 to a common fraction.

Solution.— $.25=\frac{25}{100}$

$$\frac{25}{100}=\frac{25\div25}{100\div25}=\frac{1}{4},\ \textit{Ans.}$$

Explanation.—The *decimal* fraction .25 (twenty-five hundredths), can be written as a *common* fraction if we use 25 as the numerator and 1, with as many ciphers placed to the right as there are decimal places in .25, as the denominator; there are 2 decimal places in .25, so the denominator of the fraction will be 100. The first step in the solution of the problem then is to write the decimal fraction .25 as a common fraction, $\frac{25}{100}$. The next step is to reduce the common fraction to its lowest terms. The highest common divisor of both numerator and denominator is 25; so we divide the numerator 25 by 25 and get 1 as a numerator of the result, and then divide the denominator 100, by 25, which gives us 4 as the denominator of the result. Therefore the decimal .25 expressed in a common fraction equals $\frac{1}{4}$.

206. Rule.—*Substitute the proper denominator for the decimal point and reduce the common fraction so obtained to its lowest terms.*

207. Problems.—Reduce to common fractions:

(1) .150. (2) .85. (3) .675. (4) .251. (5) .815. (6) .623.
(7) .7624. (8) .818. (9) .314. (10) .728.

Answers.— (1) $\frac{3}{20}$. (2) $\frac{17}{20}$. (3) $\frac{27}{40}$. (4) $\frac{251}{1000}$. (5) $\frac{163}{200}$.
(6) $\frac{623}{1000}$. (7) $\frac{953}{1250}$. (8) $\frac{409}{500}$. (9) $\frac{157}{500}$. (10) $\frac{91}{125}$.

TO REDUCE COMMON FRACTIONS TO DECIMALS

208. Problem.—Reduce $\frac{5}{8}$ to a decimal.

Solution.— $\frac{5}{8}=\frac{5.000}{8}=625$, *Ans.*

Explanation.—To reduce the common fraction $\frac{5}{8}$ to a decimal we place the decimal point to the *right* of the numerator, 5, and then annex *ciphers*. Next the numerator with its annexed ciphers is divided by the denominator 8; this division is carried on until there is no remainder. If the last of the ciphers is reached before the division comes out even, more ciphers may be added. In this example the division comes out even after three ciphers have been used and the quotient is 625. The last step is to point off in the quotient, 625, three decimal places, beginning at the right, because there were three ciphers annexed.

209. Rule.—*Annex ciphers to the numerator and divide by the denominator, point off as many decimal places in the quotient as there are annexed ciphers.*

210. Problems.—Reduce to decimal fractions: (1) $\frac{3}{5}$. (2) $\frac{5}{8}$. (3) $\frac{7}{8}$. (4) $\frac{1}{2}$. (5) $\frac{7}{10}$. (6) $\frac{144}{1200}$. (7) $\frac{3}{8}$. (8) $\frac{37}{40}$. (9) $\frac{32}{64}$.

Answers.—(1) .6. (2) .625. (3) .875. (4) .5. (5) .7. (6) .12. (7) .375. (8) .925. (9) .5.

211. It frequently happens in the reduction of a common fraction to a decimal that the division is not exact. In such cases the remainder may be expressed as a fraction, or by the incomplete division sign plus, as follows:

$$\frac{1}{3}=3)\underline{1.000}$$
$$.333\tfrac{1}{3} \text{ or } .333+, \ \textit{Ans.}$$

212. Rule.—*Affix ciphers to the numerator and continue to divide until 3 or 4 decimal places are reached. Then affix the plus sign.*

213. Problems.—Reduce to decimal fractions:

(1) $\frac{2}{3}$. (2) $\frac{1}{7}$. (3) $\frac{4}{11}$. (4) $\frac{2}{9}$. (5) $\frac{1}{17}$.

Answers.— (1) .6666+. (2) .1428+. (3) .363+.
(4) .2222+. (5) .0588+.

ADDITION

214. In adding decimals they are placed, and added, as in the addition of whole numbers, care being taken to arrange the *decimal points* beneath *one another*.

215. Problem.—Add: .3, .9, .7.

Solution.—

$$\begin{array}{r} .3 \\ .9 \\ .7 \\ \hline 1.9 \end{array}, \ \textit{Ans.}$$

Explanation.—In the above problem the decimals are set down as shown, with the *decimal points under one another*. The numbers are then added in the usual way, the sum being 1 unit and 9 tenths units. The 9 tenths are placed to the right of the decimal point and the 1 unit to the left of the decimal point.

216. RULE.—*Place the decimals in columns the same as for ordinary addition with the decimal points under one another; then add in the usual way.*

217. PROBLEMS.— (1) .37+.42+.67+.1.
(2) .137+.035+.61+.918+.001.
(3) .451+.682+.312+.102+.02.
(4) .505+.681+.984+.602+.009.
(5) 10.5+8.01+510.17+36.87.
(6) 794.01+851.97+381.11+425.037.
(7) 952.45+365.91+16.451+32.65.
(8) 95+101.02+.81+3+636.003.
(9) 100.02+305.2+684.82+900.
(10) 375.75+258.5+687.25+25.366.

ANSWERS.—(1) 1.56. (2) 1.701. (3) 1.567. (4) 2.781. (5) 565.55. (6) 2452.127. (7) 1367.461. (8) 835.833. (9) 1990.04. (10) 1346.866.

SUBTRACTION

218. In subtracting decimals the *minuend* and *subtrahend* are placed and subtracted as in the subtraction of whole numbers, care being taken to arrange the *decimal points beneath one another.*

219. PROBLEM.—.326—.125.

SOLUTION.—

.326 *minuend.*
.125 *subtrahend.*
.201 *remainder, Ans.*

EXPLANATION.—In this problem we have to subtract one decimal from another. This subtraction is carried on like subtraction of ordinary numbers. The *decimal point* of the *subtrahend* is always placed *directly under* the *decimal point* of the *minuend.* So the numbers are placed as shown, the figures 1, 2, and 5, of the subtrahend coming under the figures 3, 2, and 6, of the minuend. The subtraction is then carried on in the usual way and a decimal point is placed in the remainder directly under that in the subtrahend. Therefore the result is .201.

220. RULE.—*Place the subtrahend under the minuend, having the decimal point of the subtrahend directly beneath that of the minuend. Subtract as in the subtraction of whole numbers, and place a decimal point in the remainder directly under that in the subtrahend.*

221. PROBLEMS.—(1) .427—.225. (2) .126—.085. (3) .964—.709. (4) .1239—.0743. (5) .49765—.39827. (6) .4707—.2499. (7) .694—.375. (8) .9263—.0905. (9) .7326—.2378. (10) .7285—.5287.

ANSWERS.—(1) .202. (2) .041. (3) .255. (4) .0496. (5) .09938. (6) .2208. (7) .319. (8) .8358. (9) .4948. (10) .1998.

TO SUBTRACT A DECIMAL FROM A WHOLE NUMBER

222. Annex as many ciphers to the whole *number* as there are *decimal places* in the decimal, or subtrahend.

223. PROBLEM.—Subtract .305 from 27.

SOLUTION.—

27.000	*minuend.*
.305	*subtrahend.*
26.695	*remainder, Ans.*

EXPLANATION.—In this problem a decimal is to be subtracted from a whole number. Every whole number may have a decimal point placed at its right and have ciphers annexed without affecting its value. To subtract the decimal .305 from the whole number 27, we first write the 27 with a decimal point to its right and three ciphers annexed (27.000) because there are three decimal places in the subtrahend. We then place the subtrahend under the minuend so that the decimal point of the subtrahend comes directly beneath that of the minuend; then subtract as if the numbers 27.000 and .305 were whole numbers. The last step in the solution is to place a decimal point in the remainder directly under that in the subtrahend. This gives 26.695 as the remainder.

224. Rule.—*Place a decimal point to the right of the whole number and annex as many ciphers as there are decimal places in the subtrahend. Place the subtrahend under the minuend so that the decimal point of the subtrahend comes directly beneath that of the minuend. Subtract as in whole numbers, and point off the remainder by putting a decimal point directly under that of the subtrahend.*

225. Problems.—(1) 375—.375. (2) 49—.484. (3) 728—.42. (4) 4—.785. (5) 7—.975. (6) 38—.4295. (7) 425—.9875. (8) 38—.2698. (9) 59—.98765. (10) 46—.23787.

Answers.—(1) 374.625. (2) 48.516. (3) 727.58. (4) 3.215. (5) 6.025. (6) 37.5705. (7) 424.0125. (8) 37.7302. (9) 58.01235. (10) 45.76213.

TO SUBTRACT A DECIMAL FROM A DECIMAL WHEN THERE ARE A GREATER NUMBER OF DECIMAL PLACES IN THE SUBTRAHEND THAN IN THE MINUEND

226. Add as many ciphers to the minuend as are necessary to make the number of decimal places in the minuend and the subtrahend equal.

227. Problem.—Subtract .065 from .3.

Solution.—

.300	*minuend.*
.065	*subtrahend.*
.235	*remainder,* *Ans.*

Explanation.—In this problem we have to subtract the decimal, .065, from the decimal .3 which has fewer decimal places. As ciphers may be annexed to the right of a decimal without changing its value we annex enough ciphers to the .3 to make the number of decimal places in the minuend equal to the number in the subtrahend. There are three decimal places in the subtrahend and one in the minuend so we must annex two ciphers to the minuend .3, which gives us .300. We may then finish the subtraction as in Art. 221. This gives a remainder of .235.

228. Rule.—*To subtract a decimal from another decimal having a smaller number of decimal places, annex sufficient ciphers to the minuend to make the number of decimal places in the minuend equal to the number in the subtrahend. Place the subtrahend under the minuend having the decimal point of the subtrahend beneath that of the minuend. Subtract as in whole numbers and point off by placing a decimal point in the remainder directly under that in the subtrahend.*

229. Problems.—(1) .7—.527. (2) .21—.1856. (3) 43.42—7.3479. (4) 249.8—34.729. (5) 64.29—2.7963. (6) 7.6—.928. (7) 1.02—.7247. (8) .64—.00266. (9) .023—.01695. (10) .1—.09975.

Answers.—(1) .173. (2) .0244. (3) 36.0721. (4) 215.071. (5) 61.4937. (6) 6.672. (7) .2953. (8) .63734. (9) .00605. (10) .00025.

MULTIPLICATION

230. In the multiplication of decimals the decimal point is disregarded, until the process of multiplication is completed, the procedure being the same as in whole numbers. As many decimal places are pointed off in the product *commencing at the right,* as there are decimal places in the multiplicand and multiplier *combined,* prefixing ciphers if necessary.

231. Problem.—Multiply .625 by 11.

Solution.—

$$\begin{array}{rl} .625 & \textit{multiplicand.} \\ 11 & \textit{multiplier.} \\ \hline 625 & \\ 625 & \\ \hline 6.875 & \textit{product, Ans.} \end{array}$$

Explanation.—In this problem we multiply as in whole numbers, then point off three decimal places in the product for there are three in the multiplicand and none in the multiplier.

232. NOTE.—*The ciphers directly to the right of the decimal point need not be taken into consideration in multiplying, as they merely determine the number of decimal places.*

233. PROBLEM.—Multiply 346 by .004.

SOLUTION.—

$$\begin{array}{r} 346 \\ .004 \\ \hline 1.384, \end{array} \quad \textit{Ans.}$$

234. PROBLEM.—Multiply .346 by .004.

SOLUTION.—

$$\begin{array}{r} .346 \\ .004 \\ \hline .001384, \end{array} \quad \textit{Ans.}$$

235. RULE.—*Multiply as in whole numbers. Then point off as many decimal places in the product, as there are decimal places in the multiplicand and multiplier combined.*

236. PROBLEMS.—(1) 429×.75. (2) 876×2.95. (3) .0405×125. (4) .375×96. (5) 4.96×8.05. (6) .987×6.009. (7) 70.693×.827. (8) .0096×.0475. (9) 90.09×7.625. (10) 7.625×98.7.

ANSWERS.—(1) 321.75. (2) 2584.2. (3) 5.0625. (4) 36. (5) 39.928. (6) 5.930883. (7) 58.463111. (8) .000456. (9) 686.93625. (10) 752.5875.

DIVISION

237. In the division of decimals the decimal point is disregarded until the process of division is completed, the procedure being the same as in the division of whole numbers. As many decimal places are pointed off in the quotient, beginning at the right, as the decimal places in the dividend exceed those in the divisor, prefixing ciphers if necessary.

238. PROBLEMS.—(1) Divide .425 by 5.

SOLUTION.—

$$\begin{array}{rl} \textit{divisor}\ \ 5)\underline{.425} & \textit{dividend.} \\ .085 & \textit{quotient,}\ \ \textit{Ans.} \end{array}$$

EXPLANATION.—In this problem there are no decimal places in the divisor and three in the dividend; the number of decimal places in the dividend exceeds the number of decimal places in the divisor by three, therefore three places must be pointed off in the quotient.

(2) Divide 16.33 by 2.3.

SOLUTION.—

$$\begin{array}{l} 2.3)16.33(7.1,\ \ \textit{Ans.} \\ \underline{161} \\ 23 \\ \underline{23} \end{array}$$

EXPLANATION.—In this case the number of places in the dividend exceeds the number in the divisor by one and one place must be pointed off in the quotient.

239. RULE.—*Divide the same as whole numbers and point off as many decimal places in the quotient as those in the dividend exceed those in the divisor.*

240. NOTE.—*The number of decimal places in the dividend must at least equal the number of decimal places in the divisor.*

TO DIVIDE WHEN THE NUMBER OF DECIMALS IN THE DIVISOR EXCEED THOSE IN THE DIVIDEND

241. PROBLEM.—Divide 13 by .25.

SOLUTION.—

$$\begin{array}{l} .25)13.00(52,\ \ \textit{Ans.} \\ \underline{125} \\ 50 \\ \underline{50} \end{array}$$

EXPLANATION.—In this problem there are no places in the dividend and two in the divisor, so two ciphers must be annexed to the dividend; then the number of places in the dividend equals the number in the divisor and none need be pointed off in the quotient.

242. Problem.—Divide .045 by 1.45.

Solution.— 1.45).0450000(.03103+, *Ans.*
435
150
145
500
435
65

Explanation.—In this problem we find that the dividend .045 will not contain the divisor 1.45 therefore ciphers must be annexed to the dividend .045, before we can divide.

243. Rule.—*Annex the required number of ciphers to the dividend and proceed as in ordinary decimal division.*

244. Problems.—(1) 86.75 ÷ 12.5. (2) 70.3 ÷ 9.25. (3) 1.8696 ÷ 3.28. (4) 86.875 ÷ 69.5. (5) 24.8 ÷ .3875. (6) 4875 ÷ 6.25. (7) .22125 ÷ 59. (8) .91 ÷ 175. (9) 64.8 ÷ .18. (10) 8.75 ÷ .4375.

Answers.—(1) 6.94. (2) 7.6. (3) .57. (4) 1.25. (5) 64. (6) 780. (7) .00375. (8) .0052. (9) 360. (10) 20.

SIGNS OF AGGREGATION

245. The following series of figures show the application of the Signs of Aggregation:

(*a*) $26 \times 13 \div 8 + 4 - 1 = 45\frac{1}{4}$, *Ans.*
(*b*) $8 + 4 \times 26 \div 13 - 1 = 15$, *Ans.*
(*c*) $26 \times 13 \div \overline{8 + 4} - 1 = 27\frac{1}{6}$, *Ans.*
(*d*) $26 \times 13 \div (8 + 4) - 1 = 27\frac{1}{6}$, *Ans.*
(*e*) $26 \times [(13 \div 8) + 4] - 1 = 145\frac{1}{4}$, *Ans.*
(*f*) $26 \times \{[13 \div (8 + 4)] - 1\} = 2\frac{1}{6}$, *Ans.*

246. The various operations in connection with the series of figures as shown in line (*a*) are performed in order from left to right. In *all* cases the operations of Multiplication and Division must precede those of Addition and Subtraction. The signs of Addition and Subtraction are of equal importance and the operations denoted by them are performed in the order given from left to right.

247. The lines (*a*) and (*b*) explain the relative values of the different signs; for instance, in row (*a*) 26 is multiplied by 13, then the product divided by 8, to this is added 4, and 1 is subtracted from the sum thus obtained giving an answer of $45\frac{1}{4}$. It will be noted that the figures in line (*b*) are the same as those indicated in line (*a*) but the arrangement is different. In this case the first step is to multiply 4 by 26, dividing the product by 13, adding 8 to the quotient thus obtained, and subtracting 1 from this sum, giving an answer of 15.

248. The series of figures in rows (*c*), (*d*), (*e*) and (*f*) indicate the use of the Signs of Aggregation which are as follows:

Vinculum	———.
Parenthesis	( ).
Brackets	[ ].
Braces	{ }.

249. The numbers within the Signs of Aggregation are to be regarded as *parts* or *factors* of the *same quantity*.

250. In performing a series of operations, which operations are indicated by Signs of Aggregation and by the signs of Multiplication, Division, Addition and Subtraction, the numbers within the Signs of Aggregation should be operated upon first. As in problem (*c*) the first step is to multiply 26 by 13, dividing the product thus obtained by the sum of 8 and 4 which are included in the vinculum; the last operation is to subtract 1 from the quotient obtained giving an answer of $27\frac{1}{6}$. The Parenthesis in problem (*d*), takes the place of the vinculum in problem (*c*); the operations are the same, and consequently the answer is the same. Problem (*e*) shows the use of the Brackets. The operations of this problem are as follows; 13 is divided by 8, and 4 is added to the quotient thus obtained; the result of these operations being multiplied by 26 and finally 1 is subtracted from the product obtained, which gives an answer of $145\frac{1}{4}$. Problem (*f*) indicates the use of the Braces. The first operation in this problem is to add 4 and 8 together; 13 is divided by the sum thus obtained, after which 1 is subtracted from the result of these operations; the final step being to multiply the remainder by 26 which gives an answer of $2\frac{1}{6}$.

251. PROBLEMS.—Find the value of:

(1) $12\times10-6\div2+1$.
(2) $12\times(10-6)\div\overline{2+1}$.
(3) $12\times[(10-6)\div2]+1$.
(4) $12\times\left\{10-[6\div(2+1)]\right\}$.

ANSWERS.—(1) 118. (2) 16. (3) 25. (4) 96.

PERCENTAGE

252. The term **Percentage** is based on the Latin words, *per centum*, and means by the *hundred.* If a man has 500 sheep, and out of each 100, 5 die, his loss would be 5 *per hundred*, or in other words, 5 *per cent.*

253. The sign used in percentage is % and means **per cent.** Thus, 6% is read 6 per cent.

254. Percentage is an arithmetical operation by which some result is found by the application of per cent.

255. Any per cent may be written as a decimal or fraction whose denominator is 100. Thus, 5 per cent is equal to the decimal .05, or to the fraction $\frac{5}{100}$. The decimal expression is more commonly used.

256. The **Base**, the **Rate**, the **Percentage**, the **Amount**, and the **Difference** are component parts of a problem in percentage.

257. The quantity of which the per cent is to be found is called the **Base.**

258. The number of *hundreds*, or per cent, of the base to be taken is called the **Rate.**

259. The number obtained by finding a number of hundredths of the base is called the **Percentage.**

260. The base plus the percentage is called the **Amount.**

261. The base minus the percentage is called the **Difference.**

TO FIND THE PERCENTAGE WHEN THE BASE AND RATE ARE GIVEN

262. Problem.—Find 5% of 300.

Solution.—5% of $300=\overset{\textit{rate.}}{.05}\times\overset{\textit{base.}}{300}=15$ *percentage*, *Ans.*

Explanation.—To find 5% of 300 we first express the rate 5% as a decimal .05. The percentage equals the base times the rate; therefore we multiply the base 300 by the rate expressed as a decimal .05 and get 15 as the percentage.

263. Rule.—*The base times the rate equals the percentage.*

264. Problems.—Find the following:

(1) 8% of 1,500. (2) 3% of 1,424. (3) 16% of 120.

Answers.—(1) 120. (2) 42.72. (3) 19.2.

TO FIND THE AMOUNT WHEN THE BASE AND RATE ARE GIVEN

265. Problem.—What is the amount if 500 is increased by 12% of itself?

Solution.— 12% of 500=60

percentage $60+\overset{\textit{base.}}{500}=560$ *amount*, *Ans.*

Explanation.—Whereas 12% of 500 is 60, the base 500 is increased by 60. The amount is therefore 500+60, or 560.

266. Rule.—*Find the percentage and add to the base.*

267. Problems.—(1) Find the amount if 1,260 is increased by 10% of itself.

(2) 750 is increased by 12½% of itself. What is the amount?

(3) 1,785 is increased by 8% of itself. What is the amount?

Answers.—(1) 1,386. (2) 843.75. (3) 1,927.8.

TO FIND THE DIFFERENCE WHEN THE BASE AND RATE ARE GIVEN

268. PROBLEM.—600 is decreased by 8% of itself. How much is left?

SOLUTION.—	600	*base.*	then	600
	.08	*rate.*		48
	48.00	*percentage.*	*difference*	552, *Ans.*

EXPLANATION.—In order to find the difference we must first find the percentage. This is 48. Subtracting 48 from 600 we get 552, the difference.

269. RULE.—*Find the percentage and subtract it from the base.*

270. PROBLEMS.—(1) A man bought a watch for 75 dollars and sold it at a loss of 40 per cent. How much did he sell it for?

(2) A railroad was sold for 15% less than it cost. If the cost was 25,000,000 dollars, how much was it sold for?

(3) A storehouse which cost 25,000 dollars was destroyed by fire. If the insurance was 95 per cent of the cost, how much was lost?

ANSWERS.—(1) 45 dollars. (2) 21,250,000 dollars. (3) 1,250 dollars.

TO FIND THE RATE WHEN THE BASE AND PERCENTAGE ARE GIVEN

271. PROBLEM.—If from the given number 360, 240 is taken away, what per cent has been taken?

SOLUTION.— $\frac{\textit{percentage}=240}{\textit{base}\ =360}=\frac{2}{3}=66\frac{2}{3}$ *per cent, Ans.*

EXPLANATION.—Whereas the percentage divided by the base equals the rate, therefore 240 divided by 360 equals $66\frac{2}{3}\%$.

272. RULE.—*The percentage divided by the base equals the rate.*

273. Problems.—(1) What per cent of 700 men are 300 men?

(2) What per cent of 1,280 is 240?

(3) Out of 75 compensators 31 were defective. What per cent were in good condition?

(4) I have on hand 890 pipe carriers, and need 958 more. What per cent of the whole number required must I order?

(5) What per cent of 657 is 345?

Answers.—(1) $42\frac{6}{7}\%$. (2) $18\frac{3}{4}\%$. (3) $58\frac{2}{3}\%$. (4) $51\frac{194}{231}\%$. (5) $52\frac{112}{219}\%$.

TO FIND THE BASE WHEN THE PERCENTAGE AND RATE ARE GIVEN

274. Problem.—I have on hand 36 pounds of putty, which is 40% of an entire order. How many pounds were there in the whole amount?

Solution.—

percentage.
rate .40)36.00(90 *base, Ans.*
36 0
0

Explanation.—Whereas the percentage divided by the rate equals the base, therefore 36 divided by .40 equals 90.

275. Rule.—*Divide the percentage by the rate.*

276. Problems.—(1) 75% of the number of miles of a certain railroad is 390 miles. How many miles are there in all?

(2) I find that 3,720 bond wires are 15% of the total number on hand. How many are there in all?

(3) 9,416 is 20% of what number?

(4) 38 is 70% of what number?

(5) 782 is 10% of what number?

Answers.—(1) 520. (2) 24,800. (3) 47,080. (4) $54\frac{2}{7}$. (5) 7,820.

TO FIND THE BASE WHEN THE AMOUNT AND RATE ARE GIVEN

277. Problem.—After adding 50% to a certain sum I had 600. How much had I at first?

Solution.—

$$\overset{\textit{rate.}}{1+.50=1.50}$$

$$\begin{array}{l} \qquad\ \ \textit{amount.} \\ 1.50)600.00(400 \quad \textit{base, Ans.} \\ \qquad\ \underline{600} \\ \qquad\quad 00 \end{array}$$

Explanation,—Whereas the amount divided by 1 + the rate equals the base, therefore 600 divided by 1.50 equals 400.

278. Rule.—*Divide the amount by 1 plus the rate.*

279. Problems.—(1) To what number must I add 75% of itself to get 82$\frac{1}{2}$?

(2) What number increased by 75% of itself will make 609?

(3) A railroad company increased its captial stock 36%. After the increase was made the capital stock of the company was 2,679,064 dollars. What was the original capital stock?

(4) An inventory of stock shows on hand a supply of 1,105$\frac{11}{25}$ pounds of nails which is an increase of 12% over a previous inventory. What number was on hand at the time of the previous inventory?

(5) To what number must I add 61% of itself to make 59,338$\frac{4}{25}$?

Answers.—(1) 47$\frac{1}{7}$. (2) 348. (3) 1,969,900 dollars. (4) 987. (5) 36,856.

TO FIND THE BASE WHEN THE DIFFERENCE AND RATE ARE GIVEN

280. Problem.—From what number can we take 30% of itself and have 419 left over?

SOLUTION.—

```
               rate.
           1—.30=.70

           difference.
          .70)419.00(598 4/7  base, Ans.
              350
              ---
              690
              630
              ---
               600
               560
               ---
                40   4
                -- = -
                70   7
```

EXPLANATION.—Whereas the difference divided by 1 — the rate equals the base, therefore 419 divided by .70 equals $598\frac{4}{7}$.

281. RULE.—*Divide the difference by 1 minus the rate.*

282. PROBLEMS.—(1) What number may be diminished by 25% of itself and still have 450 left?

(2) A railroad company purchased 5,070 pounds of rails, which was 35% less than a prior purchase. How many pounds of rails were there in the prior purchase?

(3) From what number can we take 24% of itself and have 446.12 left.

(4) From what number can we take 31% of itself and have 6,900 left.

(5) From what number can we take $12\frac{1}{2}$% of itself and have 79,625 left?

ANSWERS.—(1) 600. (2) 7,800. (3) 587. (4) 10,000. (5) 91,000.

283. FORMULAS.—

Where, $B = \text{Base}$, Then, $P = R \times B$.

$R = \text{Rate}$, $R = P \div B$.

$P = \text{Percentage}$, $B = P \div R$.

$A = \text{Amount}$, $A = B + B \times R$.

$D = \text{Difference}$. $D = B - B \times R$.

$B = A \div (1 + R)$.

$B = D \div (1 - R)$.

PROFIT AND LOSS

284. Gain or loss is reckoned at a certain per cent.

285. Problem. A bought land for 500 dollars and sold it for 425 dollars. What was his loss per cent?

Solution.—

$$(1)\quad \begin{array}{r} 500 \\ 425 \\ \hline 75 \end{array}\ \textit{loss.} \qquad (2)\quad \begin{array}{l} 500)75.00(15\%,\ \textit{Ans.} \\ \quad\ \ \underline{500} \\ \quad\ \ 2500 \\ \quad\ \ \underline{2500} \end{array}$$

Explanation.— (1) We find the loss in dollars by subtracting the selling price from the cost. (2) We divide the loss in dollars, 75, by the cost, 500, to find the loss per cent.

286. Rule.—*Divide the gain or loss by the base. The quotient expressed in hundredths will be the gain or loss per cent.*

287. Problems.—(1) A man bought property for 1,000 dollars, spent 200 dollars for improvements and then sold for 1,320 dollars. What was his gain per cent ?

(2) I pay 45 dollars for a machine and sell it for 49.05 dollars. What is my gain per cent ?

(3) A railroad company has capital stock amounting to 2,065,725 dollars. During a certain year its profits were 247,887 dollars. What was the per cent gain for the stockholders?

(4) What per cent is lost if goods costing 125 dollars are sold for $115.62\frac{1}{2}$ dollars?

(5) A man bought a house for 7,000 dollars; he spent 125 dollars for painting and 650 dollars for other repairs. He then sold it for 7,000 dollars. What was his loss per cent ?

Answers.—(1) 10%. (2) 9%. (3) 12%. (4) $7\frac{1}{2}\%$. (5) $9\frac{301}{311}\%$.

EXAMINATION QUESTIONS AND PROBLEMS

(1) Add 29.0146, 3146.5, 2109, .62417, 14.16.

(2) Subtract 2.138 from 91.73.

(3) Multiply 79.347 by 23.15.

(4) Divide 12 by .7854.

(5) (*a*) Reduce $\frac{8}{25}$ to a decimal.

(*b*) Reduce .625 to a common fraction.

(6) The circumference of a circle is 3.1416 times the diameter. What is the circumference of a circle whose diameter is 28 inches.

(7) A rod is equal to 16.5 feet. How many rods in 396 feet?

(8) A locomotive wheel is eight feet in diameter. How many times will it turn in going 5,280 feet if the circumference is 3.1416 times the diameter?

(9) Find 16% of 52.87.

(10) A company which had stock amounting to 2,500,000 dollars increased its stock 17%. What was the total amount of stock after the increase?

(11) A man had 125 pounds of nails and used 37½% of them. How many pounds has he left?

(12) A earned 55 dollars per month and saved 10 dollars. What per cent did he save?

(13) 8,789 is 17% of what number?

(14) What number increased by 13% of itself is equal to 3,248¾?

(15) After a man had lost 15% of his property he had 5,525 dollars. What was his original property?

(16) Explain the difference between common fractions and decimals.

(17) (*a*) What is the effect if ciphers are annexed to the right of a decimal? (*b*) What is the effect if they are placed directly to the right of the decimal point, before the figures of the decimal?

(18) What is percentage?

(19) What are the component parts of an example in percentage?

(20) A man had 2,500 dollars and lost 375 dollars. What per cent did he lose?

ANSWERS TO EXAMINATION PROBLEMS

(1) 5,299.29877.
(2) 89.592.
(3) 1,836.88305.
(4) 15.278 +.
(5) (*a*) .12.
(*b*) $\frac{5}{8}$.
(6) 87.9648 inches.
(7) 24 rods.
(8) 210 +.
(9) 8.4592.
(10) 2,925,000 dollars.
(11) $78\frac{1}{8}$ pounds.
(12) $18\frac{2}{11}\%$.
(13) 51,700.
(14) 2,875.
(15) 6,500 dollars.
(16) ?
(17) ?
(18) ?
(19) ?
(20) 15%.

DENOMINATE NUMBERS

288. A **denominate number** is a concrete number in which the unit of measure is established by law or custom. As 10 quarts, 3 pints, 5 yards, 2 feet, or 10 inches.

289. There are two kinds of denominate numbers, simple and compound.

290. A **simple denominate number** is a number which is composed of units of *one denomination* only. As 10 quarts, 3 pints, 7 dollars, 10 hours, or 4 feet.

291. A **compound denominate number** is a number composed of units of *two or more denominations* that are related to each other. As 5 yards 2 feet 10 inches; 10 quarts 3 pints; 1 hour 20 minutes 10 seconds.

292. Only denominate numbers of the *same denomination* can be combined into one term; that is, we cannot add 6 quarts and 5 pints, but we can add 6 quarts and 5 quarts.

293. A **standard unit** is a unit of measure from which other units of the same kind can be derived. The standard unit of length is the yard; of dry measure, the Winchester bushel.

294. The ratio by which numbers increase and decrease is called a **scale.**

295. The scale is either uniform or varying. In United States currency the scale is uniform. In square measure it is varying. 144 sq. in. equal 1 sq. ft., 9 sq. ft. equal 1 sq. yd., etc.

296. There are six kinds of measures:

1st. Measures of **Extension.**
2nd. Measures of **Capacity.**
3rd. Measures of **Weight.**
4th. Measure of **Money, or value.**
5th. Measure of **Time.**
6th. Measure of **Angles.**

MEASURES OF EXTENSION

297. There are three measures of extension; Linear Measure, Square Measure, and Cubic Measure.

LINEAR MEASURE

298. **Linear measure** is used in measuring *length only*.

Table

12	Inches	(in.)	make 1 Foot	(ft.)
3	Feet		make 1 Yard	(yd.)
5½	Yards or 16½ Feet		make 1 Rod	(rd.)
320	Rods		make 1 Mile	(mi.)

SQUARE MEASURE

299. **Square measure** is used for measuring surfaces. Anything having *length* and *breadth* is a **surface.**

Table

144	square inches	(sq. in.)	make 1 square foot	(sq. ft.)
9	square feet		make 1 square yard	(sq. yd.)
30¼	square yards		make 1 square rod	(sq. rd.)
160	square rods		make 1 Acre	(A.)
640	Acres		make 1 square mile	(sq. mi.)

CUBIC MEASURE

300. Anything that has *length, breadth,* and *thickness* is a **solid.**

301. **Cubic measure** is used in measuring the *contents* of solids.

Table

1,728	cubic inches	(cu. in.)	make 1 cubic foot	(cu. ft.)
27	cubic feet		make 1 cubic yard	(cu. yd.)
128	cubic feet		make 1 cord	(C.)

SURVEYOR'S LINEAR MEASURE

302. **Gunters chain** is the unit employed in surveying. It is 66 feet long.

Table

7.92 Inches	make 1 Link	(l.)
25 Links	make 1 Rod	(rd.)
4 Rods, or 100 Links	make 1 Chain	(ch.)
80 Chains	make 1 Mile	(mi.)

SURVEYOR'S SQUARE MEASURE

Table

625 square links	(sq. l.)	make 1 square rod	(sq. rd.)
16 square rods		make 1 square chain	(sq. ch.)
10 square chains		make 1 Acre	(A.)
640 Acres		make 1 square mile	(sq. mi.)

MEASURES OF CAPACITY

303. There are two measures of capacity; Liquid Measure and Dry Measure.

LIQUID MEASURE

304. **Liquid measure** is used in measuring liquids.

Table

4 Gills	(gi.)	make 1 Pint	(pt.)
2 Pints		make 1 Quart	(qt.)
4 Quarts		make 1 Gallon	(gal.)

DRY MEASURE

305. **Dry measure** is used in measuring grain, roots, fruit, etc.

Table

2 pints	(pt.)	make 1 Quart	(qt.)
8 Quarts		make 1 Peck	(pk.)
4 Pecks		make 1 bushel	(bu.)

MEASURES OF WEIGHT

306. There are three measures of weight in common use. Avoirdupois, Troy, and Apothecaries' weight.

AVOIRDUPOIS WEIGHT

307. **Avoirdupois weight** is used for determining the quantity. of all ordinary objects, as iron, steel, hay, grain, etc.

Table

16 Ounces	(oz.)	make 1 Pound	(lb.)
100 Pounds		make 1 Hundred-weight	(cwt.)
20 Hundred-weight		make 1 Ton	(T.)

TROY WEIGHT

308. **Troy weight** is used in weighing gold, silver, and jewels.

Table

24 Grains	(gr.)	make 1 Pennyweight	(pwt.)
20 Pennyweights		make 1 Ounce	(oz.)
12 Ounces		make 1 Pound	(lb.)

APOTHECARIES' WEIGHT

309. **Apothecaries' weight** is used by apothecaries in weighing medicines, and has been purposely omitted.

MEASURE OF VALUE

310. The ordinary measure of value is **money.**

UNITED STATES MONEY

311. The *unit* of United States Money is the **Dollar.**

Table

10 Mills	(m.)	make 1 Cent	(ct.)
10 Cents		make 1 Dime	(d.)
10 Dimes		make 1 Dollar	($)
10 Dollars		make 1 Eagle	(E.)

MEASURE OF TIME

312. The ordinary divisions of time are given in the following table.

Table

60 Seconds	(sec.)	make 1 Minute	(min.)
60 Minutes		make 1 Hour	(hr.)
24 Hours		make 1 Day	(da.)
7 Days		make 1 Week	(wk.)
365 Days		make 1 Year	(yr.)
366 Days		make 1 Leap Year	(yr.)
100 Years		make 1 Century	(cen.)

MEASURE OF ANGLES

313. **Circular,** or **Angular Measure** is used to measure arcs of circles, and angles.

Table

60 Seconds	(″)	make 1 Minute	(′)
60 Minutes		make 1 Degree	(°)
360 Degrees		make 1 Circle	(Cir.)

REDUCTION

314. **Reduction** of denominate numbers is the process of changing their *denomination without* changing their *value.* They may be changed from a lower to a higher, or from a higher to a lower denomination. As pounds may be changed to ounces, or ounces changed to pounds.

315. Denominate numbers are changed to a lower denomination by multiplying, and to a higher denomination by dividing.

316. PROBLEM.—Change 2 hours to minutes.

SOLUTION.—

$$\begin{array}{r} 60 \ \textit{min.} \\ 2 \\ \hline 120 \ \textit{min.}, \quad \textit{Ans.} \end{array}$$

EXPLANATION.—Whereas there are 60 minutes in 1 hour, in 2 hours there are 2 times 60 minutes, or 120 minutes. Therefore 2 hr. equal 120 min.

317. **Problem.**—Change 7,200 seconds to hours.

Solution.—

60)7200(120 *min.*
60
120
120
0

60)120(2 *hr.*, *Ans.*
120

Explanation.—Whereas there are 60 seconds in one minute and 60 minutes in one hour, therefore 7,200 seconds divided by 60 and the quotient thus obtained again divided by 60 will give the number of hours.

318. **Problem.**—Reduce 9 yd. 7 ft. 2 in. to inches.

Solution.—

9 *yd.*
3
27 *ft.*
7 *ft.*
34 *ft.*
12
68
34
408 *in.*
2 *in.*
410 *in.*, *Ans.*

Explanation.—Whereas there are 3 feet in one yard, in 9 yards there are 9 times 3 feet, or 27 feet, to which are added the 7 feet given in the problem, making 34 feet. Then, whereas in 1 foot there are 12 inches, in 34 feet there are 34 times 12 inches, or 408 inches; to this 408 inches are added the 2 inches given in the problem, making 410 inches. Therefore 9 yards 7 feet 2 inches equal 410 inches.

319. **Problem.**—Reduce 4,690 sq. in. to higher denominations.

Solution.—

1st 144)4690 *sq. in.*(32 *sq. ft.*
432
370
288
82 *sq. in.*

2nd 9)32 *sq. ft.*(3 *sq. yd.*
27
5 *sq. ft.*

Then 4,690 sq. in.=3 sq. yd. 5 sq. ft. 82 sq. in., *Ans.*

Explanation.—We first find how many square feet there are in 4,690 square inches. Whereas in 1 square foot there are 144 square inches, in 4,690 square inches there are as many square feet as 144 is contained in 4,690. 144 is contained in 4,690, 32 times with a remainder of 82. Therefore in 4,690 square inches there are 32 square feet and 82 square inches. The next thing to find out is how many square yards there are in 32 square feet. Whereas in 1 square yard there are 9 square feet, in 32 square feet there are as many square yards as 9 is contained in 32. 9 is contained in 32, 3 times with a remainder of 5. Therefore in 32 square feet there are 3 square yards and 5 square feet to which we add the remainder of 82 square inches, getting a result of 3 square yards 5 square feet 82 square inches.

320. Rule.—*To reduce, or change, from a higher to a lower denomination:*

(1) *Multiply the number in the highest denomination by as many of the next lower denomination as are equal to 1 in the higher denomination.*

(2) *To the product add the number, if any, which was in the lower denomination, and set down the amount.*

(3) *Reduce this amount in the same way multiplying by as many units of the next lower denomination as are equal to 1 unit of the amount.*

(4) *Proceed in this manner until all the denominations have been reduced to the lowest denomination.*

321. Rule.—*To reduce, or change, from a lower to a higher denomination:*

(1) *Divide the lowest denomination by as many units as are contained in the next higher denomination, and reserve the remainder.*

(2) *Divide the quotient so obtained by as many units as are contained in the next higher denomination, and reserve the remainder.*

(3) *Continue in this manner until the quotient represents the highest denomination and write it as such; then successively write the several remainders as the succeeding lower denominations.*

322. PROBLEMS.—(1) How many square inches are there in a board 14 inches wide and 12½ feet long?

(2) How many bushels are there in 10,080 pints?

(3) How many cubic inches are there in 1 cubic yard 4 cubic feet 6 cubic inches?

(4) How many ¼ mile lengths are there in 11,120 feet of wire?

(5) Reduce 9 tons 3 hundred-weight 7 pounds 9 ounces to ounces.

ANSWERS.—(1) 2,100 sq. in. (2) 157½ bu. (3) 53,574 cu. in. (4) $8\frac{14}{33}$. (5) 292,921 oz.

323. In the processes of adding, subtracting, multiplying, and dividing compound denominate numbers, the principles of operation involved are the same as in ordinary processes, the only difference being that caused by the varying scale of the denominations.

ADDITION

324. In adding compound numbers the several denominations are written in columns, like denominations under one another, as in ordinary addition. They are then added separately and the sum of the lower denominations reduced to terms of the next higher denominations.

325. PROBLEM.—Add 30 rd. 7 yd. 2 ft. 9 in. to 18 rd. 0 yd. 1 ft. 5 in.

SOLUTION.—

rd.	yd.	ft.	in.	
30	7	2	9	
18	0	1	5	
48	7	3	14	
49	2	2	8,	*Ans.*

(*a*) 9 in.+5 in.=14 in. and 14 in.= 1 ft. 2 in.

(*b*) 2 ft. + 1 ft. = 3 ft. and 3 ft. + 1 ft. 2 in. = 4 ft. 2 in. = 1 yd. 1 ft. 2 in.

(*c*) 7 yd. + 0 yd. = 7 yd. and 7 yd. + 1 yd. 1 ft. 2 in. = 8 yd. 1 ft. 2 in. = 1 rd. 2 yd. 2 ft. 8 in.

(*d*) 30 rd. + 18 rd. = 48 rd. and 48 rd. + 1 rd. 2 yd. 2 ft. 8 in. = 49 rd. 2 yd. 2 ft. 8 in.

EXPLANATION.—In solving a problem in the addition of denominate numbers the quantities are first written in columns, like denominations under like denominations, as shown. We then add, beginning for convenience with the column of lowest denomination, in this case inches.

(*a*) The sum of the inches, 14, is greater than a foot, so we reduce the 14 inches to 1 foot 2 inches.

(*b*) We then add the feet, getting 3 feet, and to this add the 1 foot 2 inches obtained from the inches, making in all 4 feet 2 inches. 4 feet is more than a yard so we change 4 feet to yards, which makes 1 yard 1 foot; to this is added the inches, giving in all 1 yard 1 foot 2 inches.

(*c*) The sum of the yards is 7 and to this must be added the 1 yard 1 foot 2 inches obtained from the previous additions making in all 8 yards 1 foot 2 inches. 8 yards equal 1 rod 2½ yards; and ½ yard equals 1 foot 6 inches, that is 8 yards equal 1 rod 2 yards 1 foot 6 inches. To this must be added the 1 foot 2 inches we had previously, making 1 rod 2 yards 2 feet 8 inches.

(*d*) The sum of the rods is 48 and to this must be added the 1 rod 2 yards 2 feet 8 inches obtained from the previous operations, making in all 49 rods 2 yards 2 feet 8 inches. 49 rods cannot be reduced to any higher denomination so the solution is complete and the result is 49 rods 2 yards 2 feet 8 inches.

326. RULE.—*Write the numbers under one another as in ordinary addition, with like denominations under like denominations. Add each column of denominations separately and reduce the several results to their proper denominations.*

327. Problems.—Find the sum of:

(1)

cu. yd.	cu. ft.	cu. in.
36	20	1320
15	14	780
10	0	607

(2)

Da.	hr.	min.	sec.
27	23	45	30
30	10	8	17
45	21	31	49
10	5	14	58

(3)

T.	cwt.	lb.	oz.
15	8	96	12
28	0	72	5
14	18	81	15
71	1	35	10

Answers.—(1) 62 cu. yd. 8 cu. ft. 979 cu. in. (2) 114 da. 12 hr. 40 min. 34 sec. (3) 129 T. 9 cwt. 86 lb. 10 oz.

SUBTRACTION

328. In subtracting denominate numbers they are set down as in the ordinary subtraction of whole numbers, care being taken to have the denominations in the subtrahend under the denominations of a similar kind in the minuend.

329. Problem.—From 30 yards 1 foot 6 inches take 25 yards 2 feet 5 inches.

Solution.—

yd.	ft.	in.
30	3+1	6
25	2	5
4	2	1, *Ans.*

Explanation.—We begin at the lowest denomination to subtract. In this case it is inches therefore we subtract the 5 inches in the subtrahend from the 6 inches in the minuend and write the remainder as a part of the answer. We now find that we cannot subtract 2 feet from 1 foot, therefore we reduce one of the yards, of the 30 yards, to feet; 1 yard is equal to 3 feet, which we add to the 1 foot in the minuend. This gives us 4 feet; we may now subtract the 2 feet in the subtrahend from the 4 feet in the minuend and get a remainder of 2 feet which we write as a part of the answer. Having previously taken 1 yard from the 30 yards we now have 29 yards in the minuend; subtracting 25 yards from 29 yards leaves 4 yards, this we write as the last figure in the answer and now have 4 yd. 2 ft. 1 in.

330. Rule.—*Write the numbers under one another as in ordinary subtraction, with like denominations under like denominations. Subtract each denomination separately and reduce the several results to their proper denominations.*

331. Problems.—

(1) From 49° 52′ 51″ take 38° 53′ 40″.
(2) From 5 cwt. 38 lb. 5 oz. take 96 lb. 15 oz.
(3) From 9 mi. 1 rd. 11 ft. 6 in. take 57 rd.
(4) From 4 yr. 19 wk. 8 da. take 3 yr. 41 wk. 9 da.

Answers.—(1) 10° 59′ 11″. (2) 4 cwt. 41 lb. 6 oz. (3) 8 mi. 264 rd. 11 ft. 6 in. (4) 29 wk. 6 da.

MULTIPLICATION

332. In multiplying denominate numbers each denomination is multiplied separately and then the lower denominations are raised to higher denominations if possible. *The multiplier must be an abstract number.*

333. Problem.—Multiply 5 bu. 3 pk. 4 qt. 1 pt. by 5.

Solution.—

	bu.	pk.	qt.	pt.	
	5	3	4	1	
				5	
	25	15	20	5	
reduced gives	29	1	6	1,	*Ans.*

Explanation.—In this problem the denominate numbers are set down in the usual way; then each number is multiplied by the multiplier. Beginning with the lowest denomination, which is pints, we multiply the 1 pint by 5, this gives us 5 pints; then we multiply the 4 quarts by 5 and get 20 quarts; then we multiply the 3 pecks by 5 and get 15 pecks; lastly we multiply the 5 bushels by 5 and get 25 bushels. We now have 25 bushels 15 pecks 20 quarts 5 pints which reduced to proper denominations gives us 29 bushels 1 peck 6 quarts 1 pint.

334. Rule.—*Multiply each denomination separately. Then reduce the products to proper denominations.*

335. Problems.—(1) Multiply 4 days 9 hours 40 minutes 20 seconds by 16.

(2) Multiply 40 acres 90 square rods 56 square feet by 13.

(3) How much is 5 times 9 gallons 3 quarts 1 pint?

(4) Multiply 93 cubic yards 26 cubic feet 735 cubic inches by 9.

Answers.—(1) 70 da. 10 hr. 45 min. 20 sec. (2) 527 A. 52 sq. rd. 183 sq. ft. 72 sq. in. (3) 49 gal. 1 qt. 1 pt. (4) 845 cu. yd. 21 cu. ft. 1431 cu. in.

DIVISION

336. In dividing denominate numbers each denomination is divided separately, beginning with the highest. Where there is a remainder resulting from any such division it is reduced to terms of the next lower denomination. This operation is repeated until the last denomination is reached. If there is a remainder after the last denomination has been divided, this remainder forms a fractional part of the last denomination.

337. Problem.—Divide 28 bushels 3 pecks 7 quarts 1 pint by 6.

```
Solution.—6)28 bu.
      ⎧4 bu.......4+4 bu.
      ⎪              4 bu.=16 pk.
      ⎪                     3 pk.
      ⎪                  6)19 pk.
      ⎪3 pk..................3+1 pk.
      ⎪                        1 pk.=8 qt.
Ans.  ⎨                             7 qt.
      ⎪                         6)15 qt.
      ⎪2 qt...........................2+3 qt.
      ⎪                              3 qt.=6 pt.
      ⎪                                    1 pt.
      ⎪                                 6)7 pt.
      ⎩1⅙ pt..............................1⅙ pt.
```

Explanation.—In this problem the 28 bushels 3 pecks 7 quarts 1 pint are set down as shown. We then divide 28 bushels by 6 and find it gives us 4 and a remainder of 4 bushels; the quotient 4 is set down as the first denomination of the answer, the remainder of 4 bushels is reduced to pecks and added to the 3 pecks in the problem making 19 pecks. The 19 is now divided by 6 getting a quotient of 3 and a remainder of 1 peck. The quotient 3 is written as the second denomination in the answer, and the 1 peck is reduced to quarts and added to the 7 quarts in the problem, making in all 15 quarts; we now divide the 15 quarts by 6 and get a quotient 2 and a remainder of 3 quarts. The quotient 2 is written as the third denomination of the answer and the remaining 3 quarts are reduced to pints and added to the 1 pint in the problem, making in all 7 pints. We now divide 7 by 6 and get a quotient 1 and a remainder of 1. We write the quotient 1 as the last denomination of the answer, and, whereas there are no lower denominations in the problem, the remainder 1 together with the divisor 6 is written as a fractional part of the last denomination.

338. Rule.—*Divide each denomination separately. Begin at the highest denomination. If there is a remainder reduce it to terms of the next lower denomination and add it to the number in the next lower denomination. Then divide as before. Repeat the process until all the denominations have been divided. If any denomination be less than the divisor reduce it to terms of the next lower denomination before dividing.*

339. Problems.—(1) Divide 360° 40 ′ 32″ by 9.

(2) Divide 90 acres 87 square rods 42 square feet by 16.

(3) Divide into 5 parts, a wire 9 miles 87 rods 14 feet 10 inches long.

(4) Divide 9 days 20 hours 41 minutes 56 seconds by 4.

Answers.—(1) 40° 4′ $30\frac{2}{9}''$. (2) 5 A. 105 sq. rd. 121 sq. ft. $105\frac{3}{4}$ sq. in. (3) 1 mi. 273 rd. 3 yd. $6\frac{4}{5}$ in. (4) 2 da. 11 hr. 10 min. 29 sec.

THE METRIC SYSTEM

340. The **metric system** is based on the meter as a unit, all the other units of measure employed in the system being derived from it. It is used by nearly all the civilized countries of the world with the notable exception of the United States and Great Britain. The system has a decimal scale.

341. The *unit* of measure, the **Meter,** is very nearly one ten millionth part of the distance from the equator to the poles.

342. By using *Greek* and *Latin prefixes* the several other denominations are formed. **Deci** means $\frac{1}{10}$; **Centi,** $\frac{1}{100}$; **Milli,** $\frac{1}{1000}$. While **Deka** means 10; **Hekto,** 100; **Kilo,** 1,000; **Myria,** 10,000.

343. Therefore, **Decimeter** means *one-tenth* of a meter; **Centiliter,** *one-hundredth* of a liter, etc. While **Hektometer** means *one hundred* meters; **Kilometer,** *one thousand* meters; **Dekaliter,** *ten* liters, etc.

344. There are five kinds of metric measures:

1st. Measures of **Length.**
2nd. Measures of **Surface.**
3rd. Measures of **Volume.**
4th. Measures of **Capacity.**
5th. Measures of **Weight.**

MEASURES OF LENGTH

345. The **meter** is the unit of length and equals 39.37 inches.

Table

10 Millimeters	(mm.)	make 1 Centimeter	(cm.)
10 Centimeters		make 1 Decimeter	(dm.)
10 Decimeters		make 1 Meter	(m.)
10 Meters		make 1 Dekameter	(Dm.)
10 Dekameters		make 1 Hektometer	(Hm.)
10 Hektometers		make 1 Kilometer	(Km.)
10 Kilometers		make 1 Myriameter	(Mm.)

MEASURES OF SURFACE

346. The unit of a surface is a square whose sides equal the linear unit. Therefore in the metric system of surface measures the area of a square whose side is 10 millimeters, is 100 sq. millimeters, or 1 sq. centimeter.

One square meter equals 1.196+ sq. yd.

Table

100 Sq. Millimeters	make 1 Sq. Centimeter
100 Sq. Centimeters	make 1 Sq. Decimeter
100 Sq. Decimeters	make 1 Sq. Meter
100 Sq. Meters	make 1 Sq. Dekameter
100 Sq. Dekameters	make 1 Sq. Hektometer
100 Sq. Hektometers	make 1 Sq. Kilometer

MEASURES OF VOLUME

347. The unit of volume is a cube whose height, breadth, and thickness, is equal to the linear unit. Therefore in the metric system of volumes, the volume of a cube whose edge is 10 millimeters, is 1,000 cu. millimeters, or 1 cu. centimeter.

One cubic meter equals 1.308 cu. yd.

Table

1,000 Cu. Millimeters	make 1 Cu. Centimeter
1,000 Cu. Centimeters	make 1 Cu. Decimeter
1,000 Cu. Decimeters	make 1 Cu. Meter

MEASURES OF CAPACITY

348. The unit of capacity is the **liter** and contains a volume equal to a cubic decimeter.

One liter is equal to 1.0567 qt.

Table

10 Milliliters	(ml.)	make 1 Centiliter	(cl.)
10 Centiliters		make 1 Deciliter	(dl.)
10 Deciliters		make 1 Liter	(l.)
10 Liters		make 1 Dekaliter	(Dl.)
10 Dekaliters		make 1 Hektoliter	(Hl.)
10 Hektoliters		make 1 Kiloliter	(Kl.)

MEASURES OF WEIGHT

349. The **gram** is the unit of weight and is equal to a cubic centimeter of distilled water at its greatest density.

One gram equals 15.43248 grains. One kilogram equals 2.20462 lb.

Table

10 Milligrams	(mg.)	make 1 Centigram	(cg.)
10 Centigrams		make 1 Decigram	(dg.)
10 Decigrams		make 1 Gram	(g.)
10 Grams		make 1 Dekagram	(Dg.)
10 Dekagrams		make 1 Hektogram	(Hg.)
10 Hektograms		make 1 Kilogram	(Kg.)
10 Kilograms		make 1 Myriagram	(Mg.)
10 Myriagrams		make 1 Quintal	(Q.)
10 Quintals		make 1 Tonneau	(T.)

REDUCTION

350. In the metric system reduction to higher or lower denominations is accomplished by changing the position of the decimal point.

351. In measures of volume it requires 1,000 units of a lower denomination to make 1 unit of a higher denomination. Therefore the decimal point must be removed three places. As 1,000. cu. mm. = 1.000 cu. cm.

352. In measures of surfaces it requires 100 units of a lower denomination to make 1 unit of a higher denomination. Therefore the decimal point must be removed two places. As 100. sq. dm. = 1.00 sq. m.

353. In other metric measures the decimal point is removed one place. As 10. Hm. = 1.0 Km.

354. Problem.—How many gallons of oil will a tank 8 meters long, 5 meters deep, and 4 meters wide, hold?

Solution.—Whereas $8 \times 5 \times 4 = 160$ (cu. m.),
and 1 (cu. m.) = 1,000 (cu. dm.).
Therefore, 160 (cu. m.) = 160,000 (cu. dm.),
and whereas 1 (l.) = 1 (cu. dm.).
Therefore, 160,000 (cu. dm.) = 160,000 (l.),
and $160{,}000 \times 1.0567 \div 4 = 42{,}268$ gallons, *Ans.*

Explanation.—Whereas a tank 8 meters long, 5 meters wide, and 4 meters deep contains 160 cubic meters, and whereas 1 cubic meter is equal to 1,000 cubic decimeters; therefore, 160 cubic meters equals 160,000 cubic decimeters, and whereas 1 liter is equal to 1 cubic decimeter; therefore, 160,000 cubic decimeters equals 160,000 liters, and whereas 1 liter equals 1.0567 quarts, 160,000 liters equals 169,072 quarts, and whereas 1 gallon contains 4 quarts, therefore a tank 8 meters long, 5 meters deep and 4 meters wide contains 42,268 gallons.

355. Problems.—(1) What is the area in square feet of a surface 8 decimeters long and 9 centimeters wide?

(2) A cubic inch of cast iron weights ¼ lb. What will be the weight in metric measures of a piece of cast iron 10 in. long, 4 in. wide, and 2½ in. thick?

(3) If a pipe discharge into a tank 5.1 Dl. per minute how long will it take to fill a tank ⅔ full providing it is 5 m. long, 3 m. wide and 4 m. deep?

(4) How many gallons of water will the above tank contain when it is half full?

(5) One cubic centimeter of water weighs 1 gram; the weight of mercury is 13.5 times that of water. What is the weight of 9 deciliters of mercury?

Answers.—(1) .775+ sq. ft. (2) 11.339+ (Kg.). (3) 13 hr. 4 min. 18$\frac{14}{17}$ sec. (4) 7,925¼ gal. (5) 12.15 (Kg.).

EXAMINATION QUESTIONS AND PROBLEMS

(1) What is a denominate number?

(2) In what way does the metric system of weights and measures differ from the ordinary, or English system?

(3) How many cubic feet are there in a box 2 meters long. 12 decimeters wide, and 98 centimeters deep?

(4) Reduce ¾ mi. to inches.

(5) Reduce 1.9 bu. to pints.

(6) Reduce 14,849,500 sq. in. to higher denominations.

(7) Reduce to cu. yd. 575,424 cu. in.

(8) Add:

rd.	yd.	ft.	in.
27	4	2	8
19	5	2	7
8	2	1	11
17	3	0	8

(9) Add:

gal.	qt.	pt.
14	3	1
9	2	1
27	1	0
25	2	1

(10) Subtract 7 bu. 3 pk. 3 qt. 1 pt. from 10 bu.

(11) Find the difference between 10 T. 2 cwt. and 124,760 oz.

(12) Multiply 7 da. 15 hr. 25 min. 34 sec. by 15.

(13) If a box contains 4 bu. 2 pk. 3 qt. 1 pt. how much will 75 such boxes contain?

(14) A circle is to be divided into 17 equal parts. What will be the number of degrees, minutes and seconds in each part?

(15) A train running 90 mi. makes the distance in 1 hr. 25 min. What is its rate per hour?

(16) What will it cost to build 1.75 mi. of fence at 55 cents per yard?

(17) How many steel rails 30 ft. long will be needed to build 4 mi. of double track railway?

(18) A locomotive burns 15 T. 2 cwt. 21 lb. of coal in going 80 mi. How much does it burn in going 1 mi?

ANSWERS TO EXAMINATION PROBLEMS

(1) ?

(2) ?

(3) 83.063 + cu. ft.

(4) 47,520 in.

(5) 121.6 pt.

(6) 2 A. 58 sq. rd. 23 sq. yd. 4 sq. ft. 4 sq. in.

(7) $12\frac{1}{3}$ cu. yd.

(8) 73 rd. 5 yd. 1 ft. 10 in.

(9) 77 gal. 1 qt. 1 pt.

(10) 2 bu. 4 qt. 1 pt.

(11) 6 T. 4 cwt. 2 lb. 8 oz.

(12) 16 wk. 2 da. 15 hr. 23 min. 30 sec.

(13) 345 bu. 2 pk. 6 qt. 1 pt.

(14) $21° \; 10' \; 35\frac{5}{17}''$.

(15) $63\frac{9}{17}$ mi. per hr.

(16) $1,694.

(17) 2,816 rails.

(18) 3 cwt. 77 lb. $12\frac{1}{5}$ oz.

INVOLUTION

356. The process of multiplying a number *by itself* a number of times is called **involution.** As 2×2×2, etc., and 3×3×3 ×3, etc.

357. The result obtained by this process is called a **power** of the number. As in 2×2×2=8, 8 is a power of 2.

358. The *power* of a number *varies* according to the *number of times* the number has been multiplied by itself. When a number is multiplied by itself *once*, the result is called the *second power* of the number. As 2×2=4, 4 is the second power of 2. When a number is multiplied by itself 3 times the product is called the *fourth power* of the number. As 3×3×3×3=81, 81 is the *fourth power* of 3. Or 5×5×5×5=625, 625 is the *fourth power* of 5.

359. Any power of any number can be found by multiplying the number by itself the required number ot times. As 2×2×2 ×2×2×2×2×2=256, 256 is the 8th power of 2. Therefore, a power is a quantity produced by multiplying a given number by itself a number of times.

360. The first power of a number is called the **root.** As in 2×2=4, 2 is the *root,* or *first power.*

361. The following table shows the number 4 raised to the *first, second, third,* and *fourth* powers.

4=4; 4 *is the root or first power of* 4.
4×4=16; 16 *is the second power or square of* 4.
4×4×4=64; 64 *is the third power or cube of* 4.
4×4×4×4=256; 256 *is the fourth power of* 4.

362. In writing problems in involution they are not usually expressed by writing them as in 3×3×3=27, but a small *index figure* called the **exponent** is used. It is written to the right and above the root and *indicates the power required.* As 3^3 indicates that the *third* power (or cube) of 3 is required; 3^5 indicates that the *fifth* power of 3 is required; etc.

363. The powers of a whole number, except the first power, are always greater than the number; while the powers of a fraction, except the first power, are always less than the fraction. As $2 \times 2 = 4$, while $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$.

364. The principle of decrease is the same in decimals. As in $.6 \times .6 \times .6 \times .6 = .1296$.

365. PROBLEM.—$.5^4$.

SOLUTION.—

$$\begin{array}{rl} .5 & \textit{first power.} \\ \underline{.5} & \\ .25 & \textit{second power.} \\ \underline{.5} & \\ .125 & \textit{third power.} \\ \underline{.5} & \\ .0625 & \textit{fourth power,} \quad \textit{Ans.} \end{array}$$

EXPLANATION.—Multiplying the decimal root by itself we obtain the second power, which is .25; then multiplying the second power by the decimal root .5 we obtain the third power; then multiplying the third power by the decimal root .5 we obtain the fourth power which is .0625, a cipher having been prefixed to the 625 to obtain the right number of decimal places.

366. RULE.—*Multiply the root by itself until it has been used as a factor the number of times indicated by the exponent.*

367. PROBLEMS.—Find the following: (1) 12^3. (2) 10^5. (3) 5^7. (4) 3^9. (5) $(\frac{1}{5})^3$. (6) $(\frac{7}{8})^4$. (7) $(\frac{3}{5})^4$. (8) $(\frac{1}{9})^5$.

ANSWERS.—(1) 1,728. (2) 100,000. (3) 78,125. (4) 19,683. (5) $\frac{1}{125}$. (6) $\frac{2401}{4096}$. (7) $\frac{81}{625}$. (8) $\frac{1}{59049}$.

EVOLUTION

368. **Evolution** is the process of finding the *roots* of numbers.

369. It was demonstrated in involution that the root of a number, is that number which multiplied by itself a certain number of times produces the number. The square root of 4 is 2, because $2\times2=4$; the cube root of 27 is 3, because $3\times3\times3=27$; the fifth root of 32 is 2, because $2\times2\times2\times2\times2=32$, and so on.

370. From this it is seen that **involution** is the process of *raising* numbers to required *powers* by the multiplication of equal factors, while **evolution** is the process of *finding roots* by the analysis of numbers.

371. When the root of a number is to be found a sign called the **radical sign** is used. It is written thus $\sqrt{\ }$ and is placed before the number and the vinculum is used in connection with it. Thus, $\sqrt{49}$.

372. To indicate what root of a number is to be found a small figure called the **index** is placed *above* and to the *left* of the radical sign. Thus $\sqrt[2]{4}$; $\sqrt[3]{9}$; $\sqrt[4]{243}$, etc.

$\sqrt[2]{4}$ indicates that the *second*, or *square root* of 4 is to be found.

$\sqrt[3]{9}$ indicates that the *third*, or *cube root* of 9 is to be found.

$\sqrt[4]{243}$ indicates the *fourth root* of 243 is to be found.

373. When the *second*, or square root of a number is to be found the index figure is *omitted* and the sign only is used. Thus $\sqrt{16}$; $\sqrt{8}$, etc. $\sqrt{16}$ indicates that the square root of 16 is to be found. $\sqrt{8}$ indicates that the square root of 8 is to be found, etc.

374. When we wish to extract the root of a number we must first separate the number into certain groups, or periods, beginning at the right. The number of figures contained in each group will depend on the nature of the root to be found. If the *square root* is to be found the number must be separated into groups of *2 figures* each.

375. In the problem $\sqrt{275}$, we separate the number 275 into groups of 2 figures beginning at the *right*. This gives us 2'75. We find that the first group we pointed off contains two figures, 7 and 5. The last contains but one figure, 2. In the problem $\sqrt{1,375}$ we point off into groups of two figures each, and then have two figures, 7 and 5, in the first group and 1 and 3 in the last group, as 13'75.

376. PROBLEM.—Find the square root of 729.

SOLUTION.—

Space 1.	*Space 2.*	*Space 3.*
2 *com. divisor.*	7'29	27, *Ans.*
2	4	
40 *trial divisor.*	329	
7	329	
47 *com. divisor.*	000	

EXPLANATION.—For convenience in explaining, a diagram has been constructed, divided into three spaces numbered 1, 2 and 3. The number 729 whose square root is to be found, is written in *space 2* and separated into periods, or groups, of two figures each beginning at the right. This operation gives us one group of two figures, 2 and 9, and the 7 is then the only figure in the next group. We now find that there is no whole number which when once multiplied by itself will produce 7; therefore, we find the highest number whose square is less than 7; this number we find to be 2, as $2\times2=4$; we therefore use 2 as the first divisor. This we call a complete divisor. We write 2 in *space 1*, and in *space 3* as a part of the answer. Then squaring 2, we write the product 4, under the 7 in *space 2*. We next subtract the 4 from the 7 in *space 2* and bring down the next group of figures. We now have 329 as a new dividend in *space 2*. We next add another 2 to the 2 in *space 1* and get 4, to this 4 we affix a cipher and thus obtain 40; we use the 40 as a trial divisor and divide it into 329. We find that it is contained in 329, 8 times; we add the 8 to the forty in *space 1* and multiply the 48 thus obtained by 8. This gives us 384. Whereas 384 is larger than 329, the quotient 8 is larger than the required figure in the root. We then take the next number smaller than 8 and add it to the 40 in the place of 8, this gives 47 as the complete divisor, we now multiply the 47 in *space 1* by 7 and place the product 329 under the dividend in *space 2* and subtract it. The last product 329 not being greater than the dividend 329, it follows that 7 is the next figure of the root sought. We therefore place it in *space 3* as the second figure in the root. There being no more groups of figures to bring down, and no remainder after subtracting the product of 7 and 47, we find that the extraction of the square root of 729 is completed and that the said root is 27.

377. Problem.—$\sqrt{17{,}424}$.

Solution.—

Space 1.	*Space 2.*	*Space 3.*
1	1'74'24	132, *Ans.*
1	1	
20	74	
3	69	
23	524	
3	524	
260	000	
2		
262		

Explanation.—Proceeding as in the previous problem we write the number whose root is to be found in *space 2* and point off into periods of 2 figures each, beginning at the right. We find that we have two periods of two figures each, 2 and 4, and 7 and 4, and one period of one figure. The only figure in the first group to the left is 1. The root of 1, is 1, because 1×1=1. We place a 1 in *space 1*, and as the first figure in our answer in *space 3*; we also write it under the 1 in *space 2*. We write another 1 under the 1 in *space 1*, add them and get 2, to this we annex a cipher and get 20. We next subtract 1 from 1 in *space 2* and get no remainder. We then bring down the next group 74; using 20 as a trial divisor we divide it into 74 and get 3; we add the 3 to the trial divisor, 20, which gives us 23 as a complete divisor. We write a 3 as the second figure in our answer. Then multiplying 23 by 3 we obtain 69, which we subtract from 74, getting a remainder of 5; we bring down the next period of figures which is 24 and annex it to the 5 getting 524. We next add a 3 to the complete divisor, 23, getting 26. We now affix a cipher to the 26, making 260. We use 260 as a trial divisor and divide the new dividend 524 by it. We find that 260 is contained in 524 twice, we therefore place a 2 in the answer in *space 3* and also add a 2 to the trial divisor 260; this gives us a complete divisor of 262; we now multiply 262 by 2 and find that it is equal to 524. We write the 524 under the new dividend in *space 2* and subtract it, getting no remainder. Whereas there are no more groups of figures to bring down the extraction of the square root of 17,424 is now complete and we find it to be 132.

378. RULE.—(1) *Separate the number into periods of two figures each, beginning at the right.*

(2) *Find the greatest square in the left hand period, and write the root of this period for the first figure in the answer.*

(3) *Square this root and subtract the result from the left hand period.*

(4) *Bring down the next period and annex it to the remainder obtained, and use it as a new dividend.*

(5) *Add the root already found to itself, and affix a cipher.*

(6) *Use the number thus formed as a trial divisor and divide the new dividend by it, add the quotient thus obtained to the trial divisor and also place it in the root. Multiply the complete divisor thus obtained by the figure just placed in the root and subtract the product from the dividend.*

(7) *Bring down the next period of figures and annex it to the remainder.*

(8) *Proceed as before until all the periods have been operated upon.*

(9) *If the number of which the square root is sought is not a perfect square, annex periods of decimal ciphers and continue the process as before.*

379. The square root of a decimal is found by first separating the decimal into groups of two figures each, beginning at the decimal point and passing to the right, and then proceeding with the operation the same as in whole numbers. If necessary a cipher is added to complete the last group.

380. Problem.—Find the square root of .000625.

Solution.—

Space 1.	*Space 2.*	*Space 3.*
2	.00'06'25	.025, *Ans.*
2	00	
40	06	
5	4	
45	225	
	225	
	000	

Explanation.—After setting down the decimal .000625 as shown in the solution we find the root of the first group. This group is all ciphers, therefore the first figure of the root is a cipher. We place a cipher in the answer, and bring down the next group which consists of a cipher and a 6; we find the next figure in the root to be 2 because 2×2=4 and 4 is less than 6. We write a 2 in the root and also in *space 1;* we subtract the square of 2 which is 4 from the 6 in *space 2* and bring down the next group of figures and affix it to the remainder 2, getting 225. We then add a 2 to the 2 in *space 1* getting 4; we now affix a cipher to the 4 and get 40, which we divide into 225 and get 5 as the third figure of the root; we now add the 5 to the 40 in *space 1* and get 45 for a complete divisor. Multiplying 45 by 5 we get 225; subtracting this from the 225 in *space 2* we have no remainder and as there are no more periods to bring down, the extraction of the square root of .000625 is completed and the root is .025.

381. Rule.—(1) *Separate into periods of two figures each beginning at the decimal point.*

(2) *Proceed as in extracting the square root of whole numbers.*

(3) *Point off as many decimal places in the root as there are groups of figures in the decimal part of the power.*

382. Problems.—(1) $\sqrt{2,025}$. (2) $\sqrt{1,024}$. (3) $\sqrt{.0676}$. (4) $\sqrt{17.3056}$. (5) $\sqrt{\frac{5}{8}}$.

Answers.—(1) 45. (2) 32. (3) .26. (4) 4.16. (5) .790+.

RATIO

383. **Ratio** is the *numerical measure* of the *relation* of numbers.

384. Certain relations exist between numbers. When we compare 2 to 4 we find that 2 is ½ of 4. When we compare 8 to 16 we find that 8 is ½ of 16. Comparing 8 and 24 we find that 8 is ⅓ of 24.

385. The process of comparing one number with another is called **finding the ratio** of the two numbers compared. In making such a comparison the numbers must always be of the *same* kind. That is, both numbers must be either *abstract* numbers, or *concrete* numbers of the same denomination.

386. The two numbers compared are called the **terms** of the ratio.

387. When the terms of a ratio are considered together they are called a **couplet.**

388. The first term is called the **antecedent,** and the second term the **consequent.**

389. In the ratio 10 to 2, 10 and 2 form the *couplet* of which 10 is the *antecedent* and 2 the *consequent.*

390. In the ratio 2 to 10, 2 and 10 form the *couplet* of which 2 is the *antecedent* and 10 is the *consequent.*

391. In a ratio the *antecedent* is the *dividend,* and the *consequent* is the *divisor.*

392. There are *three* ways of expressing the ratio between two numbers, as 2 : 4, or $2 \div 4$, or $\frac{2}{4}$, all of which mean the same thing and may be read "2 is to 4."

393. The **reciprocal** of a number is 1 divided by the number. The *reciprocal* of 1 is 1, because $1 \div 1 = 1$, but the *reciprocal* of 2 is $\frac{1}{2}$ because $1 \div 2 = \frac{1}{2}$; the *reciprocal* of 10 is $\frac{1}{10}$; the *reciprocal* of $\frac{1}{2}$ is $\frac{2}{1}$, or 2, because $1 \div \frac{1}{2} = 2$; the *reciprocal* of $\frac{3}{4}$ is $\frac{4}{3}$ or $1\frac{1}{3}$, because $1 \div \frac{3}{4} = 1\frac{1}{3}$.

394. QUESTION.—What is the ratio of 10 to 2?

ANSWER.—10 is 5 times 2. As will be seen by this the measure of relation between 10 and 2 is 5; therefore, the ratio of 10 to 2 is *five*.

395. QUESTION.—What is the relation of 2 to 10?

ANSWER.—2 is $\frac{1}{5}$ of 10; here the ratio is *one fifth*.

396. In both of these cases we divided the *antecedent* by the *consequent* and obtained the *ratio*. In the first case we sought the ratio of 10 to 2 and found it to be five; here 10 is the antecedent and 2 is the consequent. In the second case we sought the ratio of 2 to 10 and found it to be one fifth. Here 2 is the antecedent and 10 the consequent.

PRINCIPLE.—*The antecedent divided by the consequent equals the ratio.*

397. In comparing, or finding the ratio of, two quantities, one of which weighs 75 lbs. and the other 35 lbs., the heavier is to the lighter in the ratio of 75 : 35.

398. Changing this expression to the *fractional* form as shown in Art. 392 we have $\frac{75}{35}$, then dividing both terms by 35, the resulting consequent is $\frac{2\frac{1}{7}}{1}$, or $2\frac{1}{7}$: 1. Therefore, the larger is $2\frac{1}{7}$ times as large as the smaller.

399. If the comparison is to be made in the reverse sequence; that is, if we compare the lighter to the heavier quantity then the expression becomes 35 : 75.

400. Changing the expression to the fractional form we have $\frac{35}{75}$ then reducing this to its lowest terms we have $\frac{\frac{7}{15}}{1}$ or $\frac{7}{15}$: 1.

Therefore, the smaller quantity is $\frac{7}{15}$ times as large as the larger quantity.

401. A ratio may be *squared, cubed,* or *raised* to any *power.* If we take the ratio 3 : 7 and cube it we get $3^3 : 7^3$ from the fact that a fraction may be cubed; as $(\frac{3}{7})^3 = \frac{3^3}{7^3} = 3^3 : 7^3$.

402. Rule.—*Divide the antecedent by the consequent.*

403. Problems.—

(1) How many times greater is 96 than 42?
(2) What is the ratio of 3 to 42?
(3) What is the ratio of 10 to 5?
(4) What is the ratio of 25 to 80?
(5) How many times greater than 78 is 100?
(6) What is the ratio of $\frac{1}{2}$ to 3?
(7) What is the ratio of $\frac{1}{8}$ to $\frac{1}{3}$?

Answers.—(1) $2\frac{2}{7}$ times. (2) 1 : 14. (3) 2 : 1. (4) $\frac{5}{16}$. (5) $1\frac{11}{39}$ greater. (6) 1 : 6. (7) $\frac{3}{8}$.

PROPORTION

404. A **proportion** is an *expression* of the *equality* of two equal ratios; the sign of proportion being the double colon (::). The sign of equality (=) is sometimes used.

405. A proportion must have *four terms.*

406. The **terms** of proportion are called the *antecedents* and the *consequents.*

407. The *first* and *third* terms are called the **antecedents** the *second* and *fourth* terms are called the **consequents.** As in the proportion 2:4::3:6, 2 and 3 are the antecedents and 4 and 6 are the consequents.

408. The *first* and *last* terms are called the **extremes** and the *second* and *third* terms the **means.** As in the proportion 2:4::3:6, 2 and 6 are the *extremes* and 4 and 3 are the *means.*

PRINCIPLES.—(1) *The product of the means and the product of the extremes are equal.*

(2) *The product of the means divided by either extreme equals the other extreme.*

(3) *The product of the extremes divided by either mean equals the other mean.*

409. In every proportion there are at least *two causes* and *two effects.* The effects are proportional to the causes. To illustrate; if 5 men can dig 9 rods of ditch in 2 days, how many rods can 10 men dig in the same time?

410. The reasoning is as follows. Here we have *two causes* and *two effects,* the first cause is 5 men, the effect is 9 rods of ditch in 2 days. This is the *first* effect and the result of the *first* cause. The *second* cause is 10 men, and the unknown effect is the *second* effect. It now follows, that 5 men as *a cause,* have the *same relation* to 10 men as a cause, that 9 rods of ditch as *an effect* have to the unknown effect.

411. The unknown term of a proportion may be expressed by the letter x.

412. An example in proportion would give *three* of the terms leaving the *other* term to be found, thus, $2:4::3:x$. The problem is then solved on the principle of proportion. The product of the means divided by the known extreme equals the other extreme. Then in this case $4\times3\div2=x$, or 6.

413. PROBLEM.—$25 : 45 :: 35 : x$.

SOLUTION.— (1) $45\times35\div25=x$.

$$(2)\quad \frac{\overset{9}{\cancel{45}}\times\overset{7}{\cancel{35}}}{\underset{5}{\cancel{25}}}=63\quad \textit{Ans.}$$

· EXPLANATION.—Whereas, the product of the means of a proportion divided by one of the extremes is equal to the other extreme, therefore in the proportion $25:45::35:x$, x equals 45 times 35 divided by 25. The problem may be solved by writing the two means above a horizontal line and writing the known extreme under the line as in (2) and employing cancellation. First dividing 25 and 35 by 5 we get 5 and 7. Then dividing 45 by 5 we get 9; finally multiplying 9 by 7 we obtain 63.

414. RULE.—*Multiply the two means together and divide the product by the given extreme.*

415. PROBLEMS.—(1) $30 : 25 :: 26 : x$. (2) $49 : 56 :: 35 : x$. (3) $63 : 40 :: 90 : x$. (4) $35 : 49 :: 50 : x$. (5) $75 : 82 :: 40 : x$.

ANSWERS.—(1) $21\frac{2}{3}$. (2) 40. (3) $57\frac{1}{7}$. (4) 70. (5) $43\frac{11}{15}$.

EXAMINATION QUESTIONS AND PROBLEMS

(1) What is, (*a*) involution; (*b*) evolution?

(2) Define, (*a*) power; (*b*) root.

(3) What is the value of 7^6?

(4) What is the value of 27^3?

(5) What is the square root of 65,536?

(6) What does $\sqrt{5{,}764{,}801}$ equal?

(7) Express $\sqrt{\frac{3}{4}}$ as a decimal.

(8) Find the square root of 9.150625.

(9) Find the value of $\sqrt{.009801}$.

(10) A square field contains 490 A. What is the length in rods of each side?

(11) What is the ratio of 1,320 feet to 1 mile?

(12) What is the ratio of 12 pounds to 8 ounces?

(13) Supply the unknown extreme in the proportion $12:8::8:x$.

(14) Supply the missing mean in the proportion $25:35::x:56$.

(15) If a man can do a piece of work in 10 days, working 6 hours a day, how long will it take him to do the work if he works 8 hours a day?

ANSWERS TO EXAMINATION PROBLEMS

(1) ?

(2) ?

(3) 117,649.

(4) 19,683.

(5) 256.

(6) 2,401.

(7) .866 +.

(8) 3.025.

(9) .099.

(10) 280 rods.

(11) 1 : 4.

(12) 24 : 1.

(13) 5⅓.

(14) 40.

(15) 7½ days.

ALGEBRA

DEFINITIONS AND NOTATIONS

1. That which gives the idea of size or extent is called **magnitude.**

2. A **quantity** is a definite amount of some magnitude actually expressed as measured in terms of some unit.

Thus, the distance between two cities is a measurable magnitude; but when we say the distance is 100 miles we express the magnitude as a definite quantity.

3. **Mathematics** treats of the relation between magnitudes.

4. **Arithmetic** is a branch of mathematics in which the relations between the magnitudes are expressed by means of *numbers.*

5. **Algebra** is a branch of mathematics which differs from Arithmetic in that it employs *letters* of the alphabet in addition to the *numbers* of Arithmetic to express the relation between magnitudes. The letters are used as symbols to represent numbers or quantities.

6. Figures used as symbols to represent numbers are called **numerical expressions**; and letters used as symbols to represent numbers are called **literal expressions.**

Thus 9 books is a numerical expression and x books is a literal expression.

In the expression x books the number that x represents is not known but in any given problem it can be determined.

7. The number that a letter represents is called its **value,** or more frequently its **numerical value.** Thus, if in the literal expression above x stands for the number 7, then 7 is the *value,* or *numerical value* of x.

SIGNS USED IN ALGEBRA

8. The principal signs used in Algebra are the same as those used in Arithmetic. They are divided into four general classes.

(1) **Signs** of **operation** which show that addition, subtraction. multiplication, or division is to be performed.

(2) **Signs** of **relation** which express a comparison between two quantities.

(3) **Signs** of **aggregation** which indicate that a group of symbols is to be treated like a single quantity.

(4) **Signs** of **abbreviation.**

SIGNS OF OPERATION

9. The **signs** of **addition** $+$, **subtraction** $-$, **multiplication** $\times$, and **division** $\div$, are the same signs as those used in Arithmetic. Thus $a + b$ means that b is to be added to a and is read a plus b; $a - b$ means that b is to be subtracted from a and is read a minus b; $a \times b$ means that a is to be multiplied by b and is read a times b; $a \div b$ means that a is to be divided by b and is read a divided by b.

10. In Algebra the sign $\times$ is commonly omitted. Thus $a \times b$, which is read a times b, may be written ab and be read ab.

11. NOTE.—The sign $\times$ is the *only sign* of *operation* that is *ever omitted.*

12. Sometimes a dot $(\cdot)$ is used in place of the sign $\times$. Thus $a \times b$ may be written $a \cdot b$ and is read a times b.

13. The sign $\div$ is used less in Algebra than in Arithmetic. The quotient $a \div b$ is usually expressed by the fractional form $\frac{a}{b}$ which is read a divided by b.

14. NOTE.—In Arithmetic the results of the operations $6+3$, $6-3$, 6×3, and $6 \div 3$ can be expressed by a single quantity, but this is not always possible in Algebra. $a+b$, $a-b$, ab and $\frac{a}{b}$ indicate that in order to add, subtract, multiply or divide two quantities, such as a and b, they must be expressed in the above forms, unless definite values are assigned to them, in which case the results can be expressed by a single quantity as in Arithmetic.

SIGN OF RELATION

15. The **equality sign** $=$, is read equals, or is equal to, and means that the numbers or quantities preceding and following the sign have the same value.

Thus $5 + 3 = 8$ and is read 5 plus 3 equals 8. $x + y = m$ and is read x plus y equals m.

If $x = 3$ and $y = 4$ then $m = 7$.

SIGNS OF AGGREGATION

16. The **signs** of **aggregation**, the **parenthesis** $(\quad)$, **brackets** $[\quad]$, **braces** $\}\quad\{$, and **vinculum** $\overline{\qquad\quad}$, are the same as those used in Arithmetic and have the same meaning.

Thus $(a + b) \times c$, or $(a + b)c$, means that b is to be added to a and the sum multiplied by c; and is read, the expression $a + b$ multiplied by c.

17. The parenthesis is used in such expressions as $a \times (-b)$ and $a + (-b)$ to avoid confusion between the signs $\times$ and $-$ or $+$ and $-$. If $-b$ were written first no parenthesis would be required, thus $-b \times a$ and $-b + a$.

18. An **expression** within a parenthesis is often called a **parenthetical expression.**

SIGNS OF ABBREVIATION

19. The principal **signs** of **abbreviation** are $\therefore$ read therefore, or hence; and .... read so on.

FACTORS

20. When a number is considered as the product of two or more numbers each of these numbers is called a **factor** of the given number.

Thus 6 is the *product* of 2 and 3. 2 and 3 are therefore called the *factors* of 6. a and b are the factors of the quantity ab.

COEFFICIENTS

21. In the product, any factor which may be considered the multiplier is called the **coefficient** of the product of the remaining factors.

In the product $7ab$, 7 is the *coefficient* of ab; or $7a$ is the coefficient of b; or $7b$ is the coefficient of a. Usually, however, the *first* factor is considered the *coefficient*. Thus in the product xy, x is considered the coefficient of y.

22. Letters used as coefficients are called **literal coefficients**; and Arabic numerals used as coefficients are called **numerical coefficients.**

Thus in $9ax$, 9 is the *numerical coefficient* of ax. In ax, a is the *literal coefficient* of x. The numerical coefficient of any literal quantity as ax, or x, is understood to be 1. When 1 is the numerical coefficient it is not written.

MULTIPLES

23. The product obtained by multiplying a quantity either by itself or by any other quantity is called a **multiple** of the quantity.

Thus 9 and 12 are multiples of 3.
12 is a multiple of 2, 3, 4, and 6.
$4xy$ is a multiple of 4, x, and y.
The given quantity is thus a *factor* of the multiple.

POWERS

24. A **power** of a quantity is the product obtained by using the quantity a number of times as a factor.

Thus 32 is the 5th power of 2, as $2 \times 2 \times 2 \times 2 \times 2 = 32$.

25. The operation of forming a power is called **involution.**

26. The quantity used as a factor is called the **base** or **root** of the power.

27. The number of times that the base is used as a factor is called the **degree** of the power and is indicated by a number called the **exponent** or **index** of the power, which is written in small characters a little above and to the right of the base. In the expression $a \times a \times a$, a is the base, and the power is said to be the third power, or the power is of the third degree (since the base is used three times as a factor) and the degree is indicated by the index ^{3} thus a^3.

28. A power of the second degree is called the **square,** and a power of the third degree is called the **cube.**

Thus x^2 is read x square, and y^3 is read y cube.

29. When the exponent is higher than 3 as in x^4, x^5, etc., the quantities x^4, x^5, etc., are usually read x to the **fourth power,** x to the **fifth power,** etc.

30. The second power is said to be higher than the first, the third higher than the second, etc.

31. Distinction between **coefficient** and **exponent.** In $3x$ the coefficient 3 is a *multiplier* of x and means that x is *added together* 3 times. Thus $3x = x + x + x$. But in x^3, x is *its own multiplier* and the exponent 3 means that x is taken 3 times *as a factor.* Thus $x^3 = x \times x \times x$.

ROOTS

32. A **root** of a power is that quantity which when used a given number of times as a factor produces the power.

33. A **root,** then, is one of the *equal* factors of a quantity.

For example, the factors of 36 are 2 and 18; 3 and 12; 4 and 9; 6 and 6. 6 is said to be a *root* of 36.

34. The operation of finding a root is the reverse of finding a power and is called **evolution.** If the quantity has two equal factors, (as $9 = 3 \times 3$) each factor is called a **square root;** if the quantity has three equal factors (as $8 = 2 \times 2 \times 2$) each factor is called a **cube root.** If the quantity has more than three equal factors each factor is called the **fourth root,** the **fifth root,** and so on.

35. In expressing a root the sign $\sqrt{\ }$ called the **radical sign** is used. For all roots higher than square root a number to indicate which root is to be taken, called the **index** of the root is written over the sign.

Thus $\sqrt{25}$ means the square root of 25. $\sqrt[3]{27}$ means the cube root of 27. $\sqrt[4]{x}$, $\sqrt[5]{x}$, and so on, mean the fourth root of x, the fifth root of x, and so on. In $\sqrt[3]{a^2 + 2ay - y^2}$ the radical sign includes the whole expression $a^2 + 2ay - y^2$. That is the operations under the sign must be performed first and then the cube root taken of the result.

ALGEBRAICAL EXPRESSIONS

36. An **algebraical expression** is any number or quantity written with one or more algebraical symbols.

Thus a, $2cd$, $xy - 2z$ are algebraical expressions.

37. A **term** is an algebraical expression consisting of one symbol, or, of two or more symbols when not separated by the signs $+$ or $-$.

Thus a, $2cd$, $2xy \cdot z$, $3xy \div z^2$ are terms.

38. In the expression $2x + y - z$ the quantity is considered as a succession of operations upon the individual terms, $2x$, y, and z. But since the expression $2xy \cdot z$ and $3xy \div z^2$ are usually written $2xyz$ and $\frac{3xy}{z^2}$, these quantities are considered as completed operations and are therefore regarded as terms. Hence any quantity in the form of $x + y$ or $x - y$ is an *expression* and any quantity in the form of xy or $\frac{x}{y}$ is a *term.*

39. Terms having the *same letters* with the *same exponents* are called **similar terms.**

Thus, $7ab^3a^2$, $3ab^3a^2$ are similar terms. Terms having different literal coefficients may be considered similar terms. Thus the terms ab^3x^2 and cb^3x^2 having the factor b^3x^2 in common may be called similar terms.

40. An algebraical expression of *one term* is called a **simple expression,** or **monomial.**

Thus x, $7x^2y$, $\frac{3a}{4b}$, are monomials.

41. An algebraical expression of *two* or *more terms* is called a **compound expression,** or **polynomial.**

Thus, $3ab + 4c$, $2x - 3y + z^2$, $a^3 + 3a^2b + 3a^3b^3$ are polynomials.

42. A polynomial of *two terms* is called a **binomial.**

Thus, $a^2 - b^2$, $8x^3 + 27y^2$ are binomials.

43. A polynomial of *three terms* is called a **trinomial.**

Thus, $a^2 - 2ab + b^2$, $4x^2 + 12xy + 9y^2$ are trinomials.

44. A term used to denote an increase is called a **positive term** and is preceded by the + sign; a term used to denote a decrease is called a **negative term** and is preceded by the — sign.

Thus $+ a^3b^2$ is a positive term, and $-3x^2y$ is a negative term.

45. The + sign is often omitted before a positive term. Thus a^3b^2 means the same as $+ a^3b^2$.

46. The result obtained by putting a particular value for each of the letters of an expression and performing the operations indicated is called the **numerical value** of the expression.

Thus if x has the value 2, and y the value 3, in the expression $x^2 + 2xy + y^2$, then the *numerical value* of the expression is $2^2 + 2 \cdot 2 \cdot 3 + 3^2 = 4 + 12 + 9 = 25$.

47. The act of putting value for a letter in an expression is called **substitution.**

Thus 2 was substituted for x and 3 for y in the expression in Art. 46.

48. **Problem.—Assuming $x = 3$ find the numerical value of $2x^3$.**

Solution.—$2x^3 = 2 \cdot 3^3 = 2 \cdot 27 = 54$, ***Ans.***

Explanation.—We substitute the given value of x which is 3 in the expression $2x^3$ and get $2 \cdot 3^3$. We then simplify this expression by finding the third power of 3, which is 27, and multiplying it by 2, getting 54, the result.

49. Problem.—Assuming $x = 4$ and $y = 3$ find the numerical value of $3x^2 - 6y$.

Solution.—$3x^2 - 6y = 3 \cdot 4^2 - 6 \cdot 3 = 3 \cdot 16 - 6 \cdot 3 = 48 - 18 = 30$, *Ans.*

Explanation.—Substituting the given values of x and y in the expression $3x^2 - 6y$ we get $3 \cdot 4^2 - 6 \cdot 3$. We now simplify the expression. First we raise 4 to the second power as indicated by the exponent 2, getting 16 in the place of 4^2; second we multiply 3 by 16 and 6 by 3 as indicated by the signs of multiplication getting 48 and 18 as products; lastly we subtract 18 from 48 as indicated by the sign of subtraction getting 30 as the numerical value of the given expression.

50. Rule.—*Put the number expressing the value of each letter in the place of the letter. Simplify the resulting expressions.*

51. Problems.—Find the numerical value of the following expressions in which $a = 2$, $b = 3$, $x = 4$ and $y = 5$:

(1) $7ax$. (2) $8b^2$.
(3) $2xy^2$. (4) $a + bx$.
(5) $3b^2 - xy$. (6) $2a^2b + x^2$.
(7) $a^2b - b^2$ (8) $a^2x + ay^2 + b$.
(9) $a^2x^2 + by - y^2$. (10) $\frac{7xy}{2y + x}$.
(11) $\sqrt{5xy}$.
(13) $\sqrt[3]{a^3b + 3x - 3b}$. (12) $\sqrt{b^2y - 5x}$.
(15) $\sqrt[3]{4y^2 - (3y^2 - a)}$.
(17) $6a + 4b - 4x$. (14) $\frac{ab^2}{x^2y}$.
(19) $x^2 - 2^x$. (16) b^2axy^2.
(20) $abxy - a^2y + b^2x - \frac{x^2y}{ab + x}$. (18) $\frac{3ab}{b} + \frac{2ax}{x - a}$.

Answers.—(1) 56. (2) 72. (3) 200. (4) 14. (5) 7. (6) 40. (7) 3. (8) 69. (9) 54. (10) 10. (11) 10. (12) 5. (13) 3. (14) $\frac{9}{40}$. (15) 3. (16) 1,800. (17) 8. (18) 14. (19) 0. (20) 128.

PARENTHESIS*

52. The Parenthesis preceded by a + sign. In the expression $10 + (4 + 5 — 3)$ it makes no difference whether the operations within the parenthesis are performed first and the result then added to 10, or the parenthesis removed first and the operations then performed in their order from left to right.

Thus by the first process, performing the operations within the parenthesis first, we obtain $4 + 5 — 3 = 6$; then adding this result to 10 we obtain $10 + 6 = 16$; and by the second process performing the operations in their order we obtain $10 + 4 + 5 — 3 = 16$. Hence $10 + (4 + 5 — 3) = 10 + 4 + 5 — 3$. Or in algebraical symbols $a + (b — c + d) = a + b — c + d$.

53. Rule.—*If an expression within a parenthesis be preceded by a + sign, the parenthesis may be removed and the operations performed in their order from left to right.*

54. The Parenthesis preceded by a — sign. In the expression $12 — (4 + 2)$ the same results will be obtained whether 4 is first added to 2 and the result then subtracted from 12; or whether 4 is first subtracted from 12 and then 2 is subtracted from the result. By the first process $12 — (4 + 2) = 12 — 6 = 6$; and by the second process $12 — (4 + 2) = 12 — 4 — 2 = 6$. This will be clear by considering that the expression $4 + 2$ included within the parenthesis means that both 4 and 2 are to be subtracted from 12. It evidently makes no difference whether 4 and 2 are added together and this sum subtracted in one operation, or whether 4 is subtracted first and 2 afterwards. Similarly $12 — (4 — 2) = 12 — 4 + 2$. Or in algebraical symbols $a — (b + c) = a — b — c$ and $a — (b — c) = a — b + c$.

55. Rule.—*If an expression within a parenthesis is preceded by a — sign the parenthesis may be removed provided the sign*

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*All rules for the parenthesis apply to the other signs of aggregation also.

of each term within the parenthesis be changed. The operations may then be performed in their order.

56. **Problem.**—Remove parenthesis and combine, $4 - (2 + 1)$.

Solution.— $4 - (2 + 1) = 4 - 2 - 1 = 1$, *Ans.*

Explanation.—We first remove the parenthesis including $2 + 1$. As this parenthesis is preceded by a $-$ sign we must change the signs of 2 and 1; this makes the whole expression $4 - 2 - 1$. Carrying out the operations indicated by the signs we get 1 as a result.

57. **Problems.**—Remove the parentheses and combine:

(1) $8 - (6 - 2)$. (2) $8 + (6 - 2)$.
(3) $9 - (4 + 2)$. (4) $8 + (6 + 3)$.
(5) $7 - (5 - 3)$. (6) $8 - (6 + 2)$.
(7) $(11 - 6) - (7 - 4)$. (8) $(10 - 4) - (2 + 3)$.
(9) $(16 - 6) + (8 - 6)$. (10) $(7 + 3) - (4 - 2)$.

Answers.—(1) 4. (2) 12. (3) 3. (4) 17. (5) 5. (6) 0. (7) 2. (8) 1. (9) 12. (10) 8.

58. **Problem.**—Remove parenthesis from the expression $(a + x) - (y - b)$. Assuming $a = 3$, $b = 2$, $x = 5$ and $y = 4$, find the numerical value.

Solution.— $(a + x) - (y - b) = a + x - y + b$. $a + x - y + b = 3 + 5 - 4 + 2 = 6$, *Ans.*

Explanation.—We must first remove the parenthesis from the expression. The parenthesis including the $a + x$ can be left out at once as it is preceded by a $+$ sign (understood). To remove the parenthesis including the $y - b$ we must change the signs of the terms within as the parenthesis is preceded by a $-$ sign. The whole expression then becomes $a + x - y + b$. We next substitute the given numerical values for the letters and simplify, getting 6 as a result.

59. **Problems.**—Remove parentheses; assuming $a = 3$, $b = 2$, $x = 5$, $y = 4$, find the numerical value:

(1) $(a + b) + (x - a)$. (2) $(y - b + x) - a$.
(3) $(3y + b^2) - (5b - x^2)$. (4) $2y^2 - [4x - ab + bx]$.
(5) $x^2y - (b^2x + ay^2 - 6y)$. (6) $xy^2 - (3x^2 - abx - ay)$.
(7) $10x - \{a^2y - ab + xy\}$. (8) $(abxy - ab) - (y + bx)$.
(9) $(xy + ab) - \overline{ay - bx}$. (10) $(a^2 + x^2) - (b^2 - y^2)$.

Answers.—(1) 7. (2) 4. (3) 31. (4) 8. (5) 56. (6) 47. (7) 0. (8) 100. (9) 24. (10) 46.

60. To insert parenthesis. If we remove the parenthesis from the expression $8+(5+2)$ we get $8+5+2$. The only difference in these two expressions is that one contains the parenthesis while the other does not. So if we wish to insert a parenthesis in the expression $8+5+2$ we can at once get the expression $8+(5+2)$ without making any other change. Or in algebraical symbols $a+b+c=a+(b+c)=(a+b)+c$.

If we remove the parenthesis from the expression $8-(5+2)$ we get $8-5-2$. In this case when we remove the parenthesis we have also to change the signs of the quantities within, because the parenthesis is preceded by a — sign. Hence, if we wish to insert a parenthesis in the expression $8-5-2$ which will include the last two terms we must change the signs of the terms we put within the parenthesis since the parenthesis is to be preceded by a — sign. We get then in this case $8-5-2=8-(5+2)$. Or in algebraical symbols $a-b+c=a-(b-c)$.

61. Rule.—*To insert parenthesis.* (1) *If the first term to be within the parenthesis is preceded by the + sign insert the parenthesis without changing any sign.* (2) *If the first term to be within the parenthesis is preceded by a — sign, change the signs of all the quantities which come within the parenthesis.*

62. Problems.—Insert parentheses so that each parenthesis will include the last three terms:

(1) $a+b+c+d$. (2) $x+y+z-w$.
(3) $x-y-z+w$. (4) $a+b-c+d$.
(5) $a-c-b-d$. (6) $m-n-p+q$.

Answers.—

(1) $a+(b+c+d)$. (2) $x+(y+z-w)$.
(3) $x-(y+z-w)$. (4) $a+(b-c+d)$.
(5) $a-(c+b+d)$. (6) $m-(n+p-q)$.

63. The parenthesis preceded by a coefficient. The expression $6(4-2+3)$ means that the operations within the parenthesis are to be performed first and the result then multiplied by 6; or, that each term within the parenthesis is to be multiplied, in turn, by 6 and the operations are to be performed afterwards. By the first process $6(4-2+3)=6\times 5=30$, and by the second process $6(4-2+3)=6\times 4-6\times 2+6\times 3=24-12+18=30$. Using the algebraical symbols this means that $a(x-y+z)=ax-ay+az$. The quantity 6 in the expression $6(4-2+3)$ is called a *coefficient* of the parenthetical expression.

64. The operation of changing $a(x-y+z)$ to the form $ax-ay+az$ is called *expanding* the expression $a(x-y+z)$.

65. **Problem.—Find the product and combine, $4(y^2-bx)-3(a+x)$.**

Solution.— $4(y^2-bx)-3(a+x)=(4y^2-4bx)-(3a+3x)=4y^2-4bx-3a-3x$, *Ans.*

Explanation.—We first multiply each term within the first parenthesis by 4 and then each term within the second parenthesis by 3. This gives $(4y^2-4bx)-(3a+3x)$. We next remove the parenthesis changing the signs of the terms in the second because it is preceded by a — sign. This gives $4y^2-4bx-3a-3x$ as a result.

66. Rule.—*Multiply each term within the parenthesis by the coefficient. Remove the parenthesis and combine if possible.*

67. **Problems.—Assuming a, b, x and y to have the same values as in Art. 59, find the numerical values; substitute before removing parentheses:**

(1) $6(ab+x)$.
(2) $4(ay-b)$.
(3) $a(y+x)$.
(4) $b^2(2x-a)$.
(5) $5a(y-b)$.
(6) $2ab^2-b(by-x)$.
(7) $5y-b^2\{y \quad 6a\}$.
(8) $3ay-x(b+2)$.
(9) $(y-a)b+(x^2-5a)$.
(10) $(2a^2+y^2)-b[x-3y]$.
(11) $(a^2-b^2)+(2y-a)x$.
(12) $2\,\overline{3x+y}-(x^2-2y)$.

Answers.—(1) 66. (2) 40. (3) 27. (4) 28. (5) 30. (6) 18. (7) 76. (8) 16. (9) 12. (10) 48. (11) 30. (12) 21.

68. A parenthetical expression divided by a quantity. In finding the quotient of $(6+4)\div 2$ the same result will be obtained whether we first add 6 and 4 and divide their sum by 2, or first divide 6 by 2 then 4 by 2 and add the two quotients.

By the first process $(6+4) \div 2 = 10 \div 2 = 5$; and by the second process $6 \div 2 + 4 \div 2 = 3 + 2 = 5$. Also $(12 - 4) \div 2 = 12 \div 2 - 4 \div 2 = 4$. Or, in algebraical symbols $(a+b) \div c = a \div c + b \div c$; and $(a-b) \div c = a \div c - b \div c$; or, as usually written, $\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$ and $\frac{a-b}{c} = \frac{a}{c} - \frac{b}{c}$.

69. **Problem.—Perform the indicated operations and find the numerical value of $(a+y) \div x$, assuming $a=3$, $y=4$ and $x=5$.**

Solution.—(1) $(a+y) \div x = (3+4) \div 5 = 7 \div 5 = 1\frac{2}{5}$, *Ans.*

Explanation.—In this case we first substitute the given values of the letters, this gives $(3+4) \div 5$. We then combine the terms within the parenthesis getting 7 and lastly divide by 5 getting $1\frac{2}{5}$ as the result.

Solution.—(2) $(a+y) \div x = \frac{a}{x} + \frac{y}{x}$

then $\frac{a}{x} + \frac{y}{x} = \frac{3}{5} + \frac{4}{5} = 1\frac{2}{5}$, *Ans.*

Explanation.—We first divide each term within the parenthesis by the divisor x. This gives $\frac{a}{x} + \frac{y}{x}$. We next substitute the given numerical values and combine getting $1\frac{2}{5}$ as the result.

70. Rule.—*Divide each term of the parenthetical expression in turn by the given quantity and connect the quotients with the given signs.*

71. **Problems.—Assuming a, b, x and y to have the same values as in Art. 59, perform the indicated operations and find the numerical value:**

(1) $(y-b) \div a$. (2) $(2y+x) \div x$.
(3) $(x^3-x) \div b$. (4) $(x^2+b-a) \div a$.
(5) $(5x-2b+a) \div b$. (6) $(y+2b) \div b$.
(7) $(x^2-y) \div b$. (8) $(xy-x) \div a$.
(9) $(b^2x+y) \div ab$. (10) $(b^2y+ax^2) \div x^2$.

Answers.—(1) $\frac{2}{3}$. (2) $2\frac{3}{5}$. (3) 10. (4) 8. (5) 12. (6) 4. (7) $10\frac{1}{2}$. (8) 5. (9) 4. (10) $3\frac{18}{25}$.

POSITIVE AND NEGATIVE QUANTITIES

72. At the end of a week a man having $100 receives $12 and pays a debt of $10. He then has $102 or $2 more than he had at first. If he had paid a debt of $15 he would then have had $97, or $3 less than he had at first. In the first case there was an increase of his money; and in the second case a decrease of his money.

73. A quantity which stands for an increase is considered a positive quantity, and a quantity which stands for a decrease is considered a negative quantity.

74. By Art. 44 the + sign is used to indicate a positive quantity and the — sign to indicate a negative quantity. In the problem in Art. 72, 12 is a positive quantity since it stands for an increase, or something received. 10 and 15 may be considered as negative quantities, since they each stand for a decrease or something given away. 2 is clearly a positive quantity and 3 a negative quantity.

75. The two problems may be solved thus; $+12-10=+2$, and $+12-15=-3$, or written in the usual form of addition or subtraction in Arithmetic,

$$\begin{array}{r} +12 \\ -10 \\ \hline +2 \end{array} \quad \text{and} \quad \begin{array}{r} +12 \\ -15 \\ \hline -3 \end{array}$$

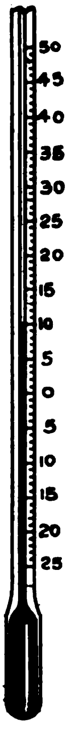
That is, the result of adding 12 and subtracting 10 is equivalent to adding 2; and the result of adding 12 and subtracting 15 is equivalent to subracting 3.

76. Suppose a man had only $12 in his possession; after paying $10 he would have $2 in his possession; but in order to pay $15 he would have been obliged to borrow $3 and would have then been $3 in debt.

77. Anything on hand or in one's possession is considered a positive quantity; and debts are considered negative quantities.

In this case, therefore, the problems would be stated as above, and we find that the result of subtracting 10 from 12 is + 2 and the result of subtracting 15 from 12 is — 3.

78. The **thermometer.** A good illustration of positive and negative quantities is afforded by the thermometer. A certain point on the stem is marked zero, 0, and above and below this point the stem is marked into divisions at equal distances apart called degrees, the degrees being numbered as shown in the illustration.



Suppose now the mercury stands at zero. If the temperature increases the mercury rises above zero. If the temperature decreases the mercury falls below zero. As before noted, any quantity that stands for an increase is a positive quantity and is indicated by the + sign preceding it. Any quantity that stands for a decrease is a negative quantity and is indicated by the — sign preceding it. Hence, degrees above zero may be written + and degrees below zero may be written —.

79. Consider the following problems: The temperature rises or falls in the following cases.

(1) Falls 20° from 10° above zero.
(2) Rises 25° from 5° below zero.
(3) Falls 20° from 5° below zero.
(4) Rises 15° from 25° below zero.

We can easily determine the resulting temperatures to be in (1) 10° below zero. (2) 20° above zero. (3) 25° below zero. (4) 10° below zero. The problems are solved in Algebra as follows:

Explanation and Solution.—(1) Since the temperature falls or decreases the operation is subtracting 20° from 10° above zero. 10° is therefore the minuend and is a positive quantity. There is a decrease of temperature hence 20° is a negative quantity. The resulting temperature, 10° below zero, is a negative quantity. Hence the operation of subtracting 20° from 10° above zero may be stated $+10° - 20° = -10°$, or, as usually written,

$$\begin{array}{r} +10° \\ -20° \\ \hline -10°, \end{array} \quad \textit{Ans.}$$

NOTE.—The forms $+10° - 20° = -10°$ and $\begin{array}{r} +10° \\ -20° \\ \hline -10° \end{array}$ are read, plus 10 degrees minus 20 degrees equals minus 10 degrees.

EXPLANATION AND SOLUTION.—(2) Since the temperature rises or increases the operation is adding 25° to 5° below zero. 5° below zero is a negative number. 25° is clearly a positive quantity, and the result, 20° above zero, is a positive quantity. Hence the operation may be stated $-5° + 25° = +20°$, or,

$$\begin{array}{r} -5° \\ +25° \\ \hline +20°, \end{array} \quad \textit{Ans.}$$

EXPLANATION AND SOLUTION.—(3) Since the temperature falls or decreases, the operation is subtracting 20° from 5° below zero. 5° below zero is therefore the minuend and is a negative quantity. 20° is a negative quantity; and the result, 25° below zero, is a negative quantity. Hence the operation may be stated: $-5° - 20° = -25°$ or,

$$\begin{array}{r} -5° \\ -20° \\ \hline -25°, \end{array} \quad \textit{Ans.}$$

EXPLANATION AND SOLUTION.—(4) Since the temperature rises or increases the operation is adding 15° to 25° below zero. 25° below zero is a negative quantity, and 15° is a positive quantity; and the result, 10° below zero, is a negative quantity. Hence, the operation may be stated $-25° + 15° = -10°$, or,

$$\begin{array}{r} -25° \\ +15° \\ \hline -10°, \end{array} \quad \textit{Ans.}$$

80. **Problems.**—Find the resulting temperature in the following cases; explain and state the problems as in Art. 79:

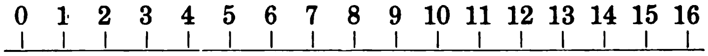
The temperature

(1) Falls 25° from 68° above zero.
(2) Rises 37° from 11° above zero.
(3) Rises 28° from 7° below zero.
(4) Falls 17° from 3° above zero.
(5) Falls 8° from 6° below zero.
(6) Rises 9° from 14° below zero.

Answers.—(1) + 43°. (2) + 48°. (3) + 21°. (4) — 14°. (5) — 14°. (6) — 5°.

POSITIVE AND NEGATIVE NUMBERS

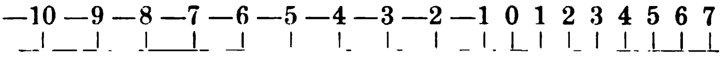
81. The series of numbers, 1, 2, 3, 4, 5, 6, 7, 8, 9, etc., can be represented by drawing a straight line, and, marking the left extremity 0, successively applying a unit of measure toward the right. Thus:



This is called the **natural series** of numbers. In this series, any number to the left is numerically less or arithmetically less than any number to the right.

82. Subtraction of a larger, from a smaller number.

Since subtraction is taking away a number of units from another number of units it follows that as the subtrahend increases the remainder or difference decreases. This we see in the series of subtractions $9-7=2$; $9-8=1$, and $9-9=0$. Therefore increasing the subtrahend 1 more unit decreases the difference 1 more unit. Now if we wish to subtract 10 from 9 we cannot do it in any real arithmetical sense but we found in Art. 77 that the expression $9-10$ has a definite mathematical meaning. If we think of the natural series of numbers as formed by increasing the number of units taken, then any number in this series represents an addition of that number of units to 0, and is therefore a positive number and can be indicated by +. Hence $9-10$ may be written $+9-10$. But $+9-10$ represents an increase of 9 followed by a decrease of 10 which is equivalent to a decrease of 1. Therefore $9-10=-1$, and in the same way $9-11=-2$, and $9-12=-3$, and so on. Hence it is clear that the results, -1, -2, -3 and so on, must be represented by a new series of numbers which is formed by successively applying the unit of measure toward the left from 0. Thus:



83. The complete series thus formed is called the **scalar series** of numbers. The numbers to the right of 0 will be spoken of as the **positive branch**; and the numbers to the left of 0 as the **negative branch** of the scalar series. If no sign be written before a number the + sign is understood. The sign — must *never* be omitted if it is intended.

84. Since the numbers in the negative branch denote a decrease, any number to the left of another, or the number numerically the larger, denotes the larger decrease and is therefore the smaller quantity considering the whole scalar series. Any number in the series is said to be algebraically larger than any number to the left, and algebraically smaller than any number to the right.

85. Distinction between arithmetical sum and difference and algebraical sum and difference.

Of the numbers + 5 and — 5 the former represents an *increase* of 5 units and the latter a *decrease* of 5 units. Considering the whole scalar series then, + 5 and — 5 stand for *different things* and are *unequal.* Since Arithmetic does not employ the negative branch of the scalar series, + 5 and — 5 both represent the *same number* of units and are therefore said to be arithmetically equal. That is, + 5 and — 5 are *algebraically unequal,* but *arithmetically equal.*

86. By the **algebraical difference** between two scalar numbers is meant the number of units that must be counted in order to go from one to the other, and the direction taken determines the sign of the difference.

The **arithmetical difference** is the difference obtained by subtracting the smaller number from the larger *regardless* of their signs.

Thus, in order to obtain the algebraical difference between — 2 and + 3 we begin at — 2 and count in the direction toward + 3 until we arrive at the latter number. The algebraical difference is found to be 5. The arithmetical difference is 1.

87. When two scalar numbers are algebraically added, we begin at the number added to and count as many units as there are in the number added in the direction indicated by its sign.

Thus in order to add 8 to -5 we begin at -5, count 8 units toward the right or positive direction, and arrive at 3. The algebraical sum is therefore 3. The arithmetical sum is 13.

The algebraical sum of more than two scalar numbers of like or unlike signs means the result of all the operations performed in their order from left to right.

88. The expressions **numerical sum** and **numerical difference** are often used instead of **arithmetical sum** and **arithmetical difference.**

ADDITION OF SCALAR NUMBERS

89. The addition of a **positive number to a positive number** is the same in Algebra as in Arithmetic. The sign of the sum will always be +. Or in algebraical symbols $+a+b=+(a+b)$.

90. Addition of a **positive number to a negative number** or of a **negative number to a positive number** is illustrated by the following problems.

PROBLEMS.—(1) Add 15 to -8.

SOLUTION.—

$$\begin{array}{r} -8 \\ \underline{15} \\ +7, \end{array} \quad \textit{Ans.}$$

EXPLANATION.—By Art. 87, to add 15 to -8 we begin at -8 and count 15 units in the + direction (towards the right). Counting 8 units towards the right brings us to 0 and we have 7 units more to count, so we arrive at $+7$ finally.

(2) Add -7 to 4.

SOLUTION.—

$$\begin{array}{r} 4 \\ \underline{-7} \\ -3, \end{array} \quad \textit{Ans.}$$

EXPLANATION.—By Art. 87, to add -7 to 4 we begin at 4 and count 7 units in the negative direction (toward the left).

Counting 4 units toward the left brings us to 0 and we have 3 units more to count, so we arrive at -3 finally.

91. In each of the problems in Art. 90, it will be seen that the result is numerically equal to the arithmetical difference of the numbers to be added. This is true whenever two quantities with unlike signs are added. It will also be seen that the result has the sign of the larger one of the addends.

92. Rule.—*To add two numbers that have unlike signs find the numerical difference and prefix the sign of the larger number.*

93. Problems.—Add:

$$(1)\ \begin{array}{r}1\\-6\\\hline\end{array}\quad(2)\ \begin{array}{r}12\\-4\\\hline\end{array}\quad(3)\ \begin{array}{r}-18\\9\\\hline\end{array}\quad(4)\ \begin{array}{r}-20\\27\\\hline\end{array}\quad(5)\ \begin{array}{r}67\\-43\\\hline\end{array}\quad(6)\ \begin{array}{r}-98\\49\\\hline\end{array}$$

$$(7)\ \begin{array}{r}102\\-37\\\hline\end{array}\quad(8)\ \begin{array}{r}46\\-126\\\hline\end{array}\quad(9)\ \begin{array}{r}-58\\127\\\hline\end{array}\quad(10)\ \begin{array}{r}234\\-689\\\hline\end{array}$$

Answers.—(1) -5. (2) 8. (3) -9. (4) 7. (5) 24. (6) -49. (7) 65. (8) -80. (9) 69. (10) -455.

94. Addition of a **negative number to a negative number.** If a man gives away \$10 on Monday and \$8 on Tuesday his wealth is decreased \$18. Thus a decrease added to another decrease is a still larger decrease and the sum or total decrease is clearly the arithmetical sum of the two numbers. Hence the problem above is stated $-10-8=-18$. Or in algebraical symbols $-a-b=-(a+b)$.

95. Rule.—*To add a negative number to another negative number find the numerical sum of the two numbers and prefix the $-$ sign to the result.*

96. Problem.—Add -5 to -7.

Solution.—

$$\begin{array}{r}-5\\-7\\\hline -12,\ \textit{Ans.}\end{array}$$

Explanation.—Since to add a negative number to a negative number their numerical sum must be taken, we first add 5 to

7 getting 12. As the numbers are both negative their sum 12 will be negative, so a — sign must be prefixed, giving — 12 as the result.

97. Problems.—Add:

$$\begin{array}{rrrrrrrrrr} (1) & -28 & (2) & -17 & (3) & -144 & (4) & -269 & (5) & -428 \\ & \underline{-64} & & \underline{-79} & & \underline{-278} & & \underline{-794} & & \underline{-2689} \end{array}$$

Answers.—(1) — 92. (2) — 96. (3) — 422. (4) — 1063. (5) — 3117.

98. Problems.—Find the algebraical sum of 24; — 16, 32, — 14 and 9.

$$\text{Solution.—}\begin{array}{rrrrrr} (1) & 24 & (2) & -16 & (3) & 65 \\ & 32 & & \underline{-14} & & \underline{-30} \\ & \underline{9} & & -30 & & 35,\ \textit{Ans.} \\ & 65 & & & & \end{array}$$

Explanation.—It makes no difference in what order the operations are performed, therefore; (1) We find the sum of the positive numbers 24, 32 and 9. This is 65. (2) We find the sum of the negative numbers — 16 and — 14 which is — 30. (3) We add 65 and — 30 getting 35 as the algebraic sum.

99. Rule.—*To find the algebraical sum of more than two scalar numbers, find the sum of the positive numbers and of the negative numbers separately, then find the algebraical sum of the two results.*

100. Problems.—Find the algebraical sum:

$$\begin{array}{rrrrrrrrrrrr} (1) & 31 & (2) & -47 & (3) & -16 & (4) & -76 & (5) & 27 & (6) & -16 \\ & -64 & & 64 & & -80 & & 98 & & 64 & & -41 \\ & \underline{82} & & \underline{-28} & & \underline{29} & & \underline{-81} & & \underline{-37} & & \underline{-68} \end{array}$$

$$\begin{array}{rrrrrrrr} (7) & 49 & (8) & -17 & (9) & -17 & (10) & 201 \\ & 64 & & -103 & & -68 & & -149 \\ & \underline{-207} & & \underline{91} & & \underline{-28} & & \underline{-89} \end{array}$$

Answers.—(1) 49. (2) — 11. (3) — 67. (4) — 59. (5) 54. (6) — 125. (7) — 94. (8) — 29. (9) — 113. (10) — 37.

101. The double use of the signs + and —.

In Arithmetic the problem $\begin{array}{r}10\\ \underline{+\,6}\end{array}$ means that 6 units are to be added to 10 and $\begin{array}{r}10\\ \underline{-6}\end{array}$ means that 6 units are to be subtracted from 10. But in Algebra the signs + and — in the problems $\begin{array}{r}10\\ \underline{+6}\end{array}$ and $\begin{array}{r}10\\ \underline{-6}\end{array}$ signify the positive number 6 and the negative number 6 respectively, and the sum of the two numbers may be taken or their difference. Hence in Algebra it must be stated whether the operation is to be addition or subtraction.

SUBTRACTION OF SCALAR NUMBERS

102. By Art. 86 in order to subtract one scalar number from another we begin at the subtrahend and count in a + or — direction, as the case may be, until we arrive at the minuend. The number of units counted is the difference and the direction determines the sign.

103. In subtracting scalar numbers six cases arise:

(1) Subtracting a smaller positive number from a larger positive number.

(2) Subtracting a larger positive number from a smaller positive number.

(3) Subtracting a positive number from a negative number.

(4) Subtracting a negative number from a positive number.

(5) Subtracting a smaller negative number from a larger negative number.

(6) Subtracting a larger negative number from a smaller negative number.

104. These six cases can be illustrated by the following problems. Subtract:

$$\text{(1)}\ \begin{array}{r}+10\\ \underline{+6}\\ +4,\end{array}\ \textit{Ans.}\qquad \text{(2)}\ \begin{array}{r}+4\\ \underline{+6}\\ -2,\end{array}\ \textit{Ans.}\qquad \text{(3)}\ \begin{array}{r}-4\\ \underline{+3}\\ -7,\end{array}\ \textit{Ans.}$$

$$\text{(4)}\ \begin{array}{r}+3\\ \underline{-4}\\ +7,\end{array}\ \textit{Ans.}\qquad \text{(5)}\ \begin{array}{r}-8\\ \underline{-6}\\ -2,\end{array}\ \textit{Ans.}\qquad \text{(6)}\ \begin{array}{r}-6\\ \underline{-8}\\ +2,\end{array}\ \textit{Ans.}$$

In these examples, as written, the signs all indicate the kind of number, showing in each case whether the number is positive or negative. The operation is *stated* in words *to be subtraction.* We may express the operation by placing a — sign before the subtrahend and enclosing the subtrahend in parenthesis to avoid confusion of signs, thus:

$$(1)\ \begin{array}{r} +10 \\ \underline{-(+6)} \end{array} \qquad (2)\ \begin{array}{r} +4 \\ \underline{-(+6)} \end{array} \qquad (3)\ \begin{array}{r} -4 \\ \underline{-(+3)} \end{array}$$

$$(4)\ \begin{array}{r} +3 \\ \underline{-(-4)} \end{array} \qquad (5)\ \begin{array}{r} -8 \\ \underline{-(-6)} \end{array} \qquad (6)\ \begin{array}{r} -6 \\ \underline{-(-8)} \end{array}$$

Now removing the parentheses we find that the signs of the subtrahends are changed and *adding* algebraically according to Arts. 89, 92, and 95 we get:

$$(1)\ \begin{array}{r} +10 \\ \underline{-6} \\ +4, \end{array}\ \textit{Ans.} \qquad (2)\ \begin{array}{r} +4 \\ \underline{-6} \\ -2, \end{array}\ \textit{Ans.} \qquad (3)\ \begin{array}{r} -4 \\ \underline{-3} \\ -7, \end{array}\ \textit{Ans.}$$

$$(4)\ \begin{array}{r} +3 \\ \underline{+4} \\ +7, \end{array}\ \textit{Ans.} \qquad (5)\ \begin{array}{r} -8 \\ \underline{+6} \\ -2, \end{array}\ \textit{Ans.} \qquad (6)\ \begin{array}{r} -6 \\ \underline{+8} \\ +2, \end{array}\ \textit{Ans.}$$

The results thus obtained by *adding* are the same as those obtained by *subtracting* before the signs of the subtrahends were changed.

105. RULE.—*In order to subtract any number from any other number in Algebra change the sign of the subtrahend, and solve the problem as in addition.*

106. PROBLEM.—Subtract — 8 from 13.

SOLUTION.—

$$\begin{array}{r} 13 \\ \underline{-8} \\ 21, \end{array}\ \textit{Ans.}$$

EXPLANATION.—Considering the sign of the subtrahend changed we have + 8. This added to + 13 gives + 21 as the result.

107. **Problems.**—Subtract the lower numbers from the upper, applying the rule in Art. 105:

(1)	(2)	(3)	(4)	(5)	(6)
27	—18	46	75	—83	—63
—31	13	—64	—98	—41	—19

(7)	(8)	(9)	(10)
—79	121	—237	647
—97	—247	478	—129

Answers.—(1) 58. (2) —31. (3) 110. (4) 173. (5) —42. (6) —44. (7) 18. (8) 368. (9) —715 (10) 776.

ADDITION AND SUBTRACTION

108. The sum of 4 bushels, 6 bushels, and 7 bushels is 17 bushels, or, written in the usual form,

$$\begin{array}{r l}
4 & \text{bushels} \\
6 & \text{bushels} \\
\underline{7} & \text{bushels} \\
17 & \text{bushels}, \quad \textit{Ans.}
\end{array}$$

Using the abbreviation b for bushels we write,

$$\begin{array}{r l}
4 & b \\
6 & b \\
\underline{7} & b \\
17 & b, \quad \textit{Ans.}
\end{array}$$

Likewise the sum of 8 dollars, 13 dollars, 21 dollars may be found, using d for dollars,

$$\begin{array}{r l}
8 & d \\
13 & d \\
\underline{21} & d \\
42 & d, \quad \textit{Ans.}
\end{array}$$

Using any letter whatsoever, as x, for any quantity we find the sum of $6x$, $18x$, $12x$, thus

$$\begin{array}{r l}
6x & \\
18x & \\
\underline{12x} & \\
36x, & \textit{Ans.}
\end{array}$$

That is, the coefficient of the literal quantity in the sum, is the sum of the coefficients of the literal quantities added.

109. In order to subtract 8 bushels from 12 bushels we may write

$$\begin{array}{r l}
12 & b; \\
\underline{8} & \underline{b} \\
4 & b, \quad \textit{Ans.}
\end{array}$$

or, in general, to subtract $8x$ from $12x$.

$$\begin{array}{r l}
12x & \\
\underline{8x} & \\
4x, & \textit{Ans.}
\end{array}$$

That is, the coefficient of the literal quantity in the difference, is the difference of the coefficients of the literal quantities in the minuend and in the subtrahend.

110. It is as impossible to add $8b$ and $5d$ and express the result in one term as it is to add 8 bushels and 5 dollars and express the result in one term.

It is impossible to combine into one term two or more terms unless they are alike or similar. (Art. 39.)

111. Problem.—Add $5b$, $4b$, $-3b$, $-8b$.

Solution.—

$$\begin{array}{rl}
5b & \\
4b & \\
-3b & \\
\underline{-8b} & \\
-2b, & \textit{Ans.}
\end{array}$$

Explanation.—Since the terms are similar terms they can be combined by adding the numerical coefficients and prefixing the sum to the literal part. The sum of 5 and 4 is 9; and of -3 and -8 is -11. Then 9 plus -11 equals -2. Prefixing -2 to b we get $-2b$ as a result.

112. Rule.—*To add or subtract simple monomial expressions add or subtract the numerical coefficients and prefix the result to the common literal part of the terms.*

113. Problems.—(*a*) Add; (*b*) Subtract the lower from the upper expression in all but the first 5 problems:

$$\begin{array}{rrrrrrrrrr}
(1) & 7a & (2) & 6x & (3) & -14y & (4) & -8z & (5) & 9y\\
& 5a & & 3x & & 7y & & -11z & & -6y\\
& 4a & & -9x & & -21y & & -12z & & -4y\\
& \underline{10a} & & \underline{2x} & & \underline{9y} & & 19z & & \underline{8y}\\
& & & & & & & \underline{13z} & &
\end{array}$$

$$\begin{array}{rrrrrrrrrr}
(6) & 17y^2 & (7) & 21ab & (8) & 6abx^2 & (9) & ay^2 & (10) & 57c\\
& \underline{29y^2} & & \underline{-17ab} & & \underline{13abx^2} & & \underline{-7ay^2} & & \underline{24c}
\end{array}$$

$$\begin{array}{rrrrrrrr}
(11) & -67d & (12) & -19m & (13) & 84y & (14) & -143p\\
& \underline{49d} & & \underline{-43m} & & \underline{-75y} & & \underline{-98p}
\end{array}$$

Answers.—(*a*) (1) $26a$. (2) $2x$. (3) $-19y$. (4) z. (5) $7y$. (6) $46y^2$. (7) $4ab$. (8) $19abx^2$. (9) $-6ay^2$. (10) $81c$. (11) $-18d$. (12) $-62m$. (13) $9y$. (14) $-241p$. (*b*) (6) $-12y^2$. (7) $38ab$. (8) $-7abx^2$. (9) $8ay^2$. (10) $33c$. (11) $-116d$. (12) $24m$. (13) $159y$. (14) $-45p$.

ADDITION AND SUBTRACTION OF POLYNOMINALS CONTAINING SIMILAR AND DISSIMILAR TERMS

114. Problem.—Add $x^2 - 2xy + y^2$ and $a - 3xy - d$.

Solution.—

$$\begin{array}{lllll} x^2 & -\,2xy & +\,y^2 & & \\ & -\,3xy & & +\,a & -\,d \\ \hline x^2 & -\,5xy & +\,y^2 & +\,a & -\,d, \quad \textit{Ans.} \end{array}$$

Explanation.—We rewrite so that none but similar terms shall be in the same vertical column as shown in the solution, and then add each column separately as in Art. 112.

115. Problem.—In the problem, Art. 114, subtract the lower from the upper expression.

Solution.—

$$\begin{array}{lllll} +\,x^2 & -\,2xy & +\,y^2 & +\,0 & +\,0 \\ +\,0 & -\,3xy & +\,0 & +\,a & -\,d \\ \hline +\,x^2 & +\,xy & +\,y^2 & -\,a & +\,d, \quad \textit{Ans.} \end{array}$$

Explanation.—We first supply missing places with *0* and then subtract. In the first column *0* from x^2 leaves x^2; in the fourth column $+ a$ from *0* gives $-a$; and in the fifth column $-d$ from *0* is $+d$ (Art. 112). The other columns present no difficulties. Performing the operations we obtain the result, $x^2 + xy + y^2 - a + d$.

116. In the problem, add $3x^2 - 4x^3 + 2x + 1$ to $3 - 4x + 8x^3$, each expression should be arranged in a regular order beginning either with the terms containing the highest or lowest exponent of some one letter and writing the successive terms so that the exponents of the letter will decrease if the highest is taken first, or increase if the lowest is taken first, regularly.

Thus, the first expression in this problem should be written either (1) $-4x^3 + 3x^2 + 2x + 1$, or (2) $1 + 2x + 3x^2 - 4x^3$.

When written as in (1) it is said to be arranged according to the **descending powers** of the letter x, and when written as in (2), according to the **ascending powers** of the letter.

117. Problems.—Add:

(1)
$$\begin{array}{l} 9a^2 - 4ay + 6y^2 \\ \underline{7a^2 - 9ay + 4y^2} \end{array}$$

(2)
$$\begin{array}{r} -a^2 - 2b^2 \\ \underline{5a^2 - 9b^2} \end{array}$$

(3)
$$\begin{array}{l} b^2 + 3c \\ \underline{b^2 + 2c} \end{array}$$

(4)
$$\begin{array}{llll} -\,6x^3 & -\,x^2y & +\,xy^2 & -\,y^3 \\ \hline -\,3x^3 & -\,4x^2y & -\,7xy^2 & -\,3y^3 \\ \hline \end{array}$$

(5) $2a^2 - 4a - 3$ to $2a^3 + 3a^2 + 4a - 2$.

(6) $1 - x^2 + x$ to $1 - 2x + 3x^2 - 4x^3$.

(7) $8x^3 - 16x^4 + 24x^5 - 32x^6$, $-4x + 8x^2 - 12x^3 + 16x^4$, and $3 - 6x + 9x^2 - 12x^3$.

(8) $3a^n - 2a^{n-3}b - 3a^{n-2}b^2$ to $a^n - a^{n-3}b + a^{n-2}b^2$.

(9) $3d + 8b$ to $6x - 5y$.

(10) $x^2 - xy + y^2$ to $x^3 + y^3$.

Answers.—(1) $16a^2 - 13ay + 10y^2$. (2) $4a^2 - 11b^2$. (3) $2b^2 + 5c$. (4) $-9x^3 - 5x^2y - 6xy^2 - 4y^3$. (5) $2a^3 + 5a^2 - 5$. (6) $2 - x + 2x^2 - 4x^3$. (7) $3 - 10x + 17x^2 - 16x^3 + 24x^5 - 32x^6$. (8) $4a^n - 3a^{n-3}b - 2a^{n-2}b^2$. (9) $6x - 5y + 3d + 8b$. (10) $x^3 + x^2 - xy + y^2 + y^3$.

118. **Problems.**—In problems 1, 2, 3, and 4 in Art. 117, subtract the lower from the upper expression, and in problems 5, 6, 8, 9 and 10 subtract the first from the last.

Answers.—(1) $2a^2 + 5ay + 2y^2$. (2) $-6a^2 + 7b^2$. (3) c. (4) $-3x^3 + 3x^2y + 8xy^2 + 2y^3$. (5) $2a^3 + a^2 + 8a + 1$. (6) $-3x + 4x^2 - 4x^3$. (8) $-2a^n + a^{n-3}b + 4a^{n-2}b^2$. (9) $6x - 3d - 5y - 8b$. (10) $x^3 - x^2 + xy - y^2 + y^3$.

EXAMINATION QUESTIONS AND PROBLEMS

(1) (a) In what respect is Algebra like Arithmetic? (b) In what respect different?

(2) What is the difference between a coefficient and an exponent?

(3) What is the difference between power and root? Give an example of each.

(4) What is substitution?

Assuming $x = 4$, $y = 5$, and $z = 3$, find the numerical values of the following expressions:

(5) $\frac{x^2z}{y}$.

(6) $x^2 + (y^2 - z^2)$.

(7) $x^2 - (y + z)$.

(8) $\sqrt[3]{y^3 - (xyz + 1)}$.

Assuming $a = 1$, $b = 2$, $c = 3$, and $d = 4$, find the value of the following expressions:

(9) $(a + b) - (-c + d)$.

(10) $(c + d + a)c$.

(11) $(bc + d) \div (b + c)$.

(12) Remove the signs of aggregation from this expression: $a + b - [x + y - (a + b) - x]$.

HINT.—Remove parenthesis first.

(13) Add 25, -8, 125, 9, -43.

(14) Add $5ab$, $8ab$, $-17ab$, $4ab$.

(15) Subtract 825 from 417.

(16) Subtract $-36xy$ from $17xy$.

(17) Add $x^3 + 3x^2 + x + 1$, $2 - 3x^2 + x^3 - 4x$, $3x^2 - 2x^3 + 8x - 4$, and $4x^3 - 4 + 8x^2 - x$.

(18) Subtract the sum of $a - b - c$, $b - a - c$, and $-a + c - b$ from $a + b + c$.

(19) From $3ab + 4cy$ subtract the sum of $3cy - 4ab$ and $ab + cy$.

(20) Give a concrete illustration of the use of positive and negative numbers.

ANSWERS TO EXAMINATION PROBLEMS

(1) ?

(2) ?

(3) ?

(4) ?

(5) $9\frac{3}{8}$.

(6) 32.

(7) 8.

(8) 4.

(9) 2.

(10) 24.

(11) 2.

(12) $a + b - x - y + a + b + x$, or, combining similar terms, $2a + 2b - y$.

(13) 108.

(14) 0.

(15) -408.

(16) $53xy$.

(17) $4x^3 + 11x^2 + 4x - 5$.

(18) $2a + 2b + 2c$.

(19) $6ab$.

(20) ?

MULTIPLICATION AND DIVISION

LAWS OF SIGNS IN MULTIPLICATION

119. When the multiplicand is *positive* and the multiplier is *positive.*

A man receives $5 a day. How will his financial condition in 3 days, compare with his present financial condition?

$3 \times (+5)$ means that an increase of 5 units is to be taken or added 3 times. Then the product $3 \times (+5) = +5 + 5 + 5 = +15$.

120. When the multiplicand is *negative* and the multiplier is *positive.*

A man spends $5 a day. How will his financial condition in 3 days, compare with his present financial condition?

$3 \times (-5)$ means that a decrease of 5 units is to be taken 3 times. Hence $3 \times (-5) = -5 - 5 - 5 = -15$.

121. When the multiplicand is *positive* and the multiplier is *negative.*

A man receives $5 a day. How did his financial condition 3 days ago, compare with his present financial condition?

$-3 \times (+5)$ means that 5 units are to be decreased 3 times, that is, 5 units are to be counted 3 times in the negative direction. Hence the result is 15 units counted in the negative direction, or -15. Hence $-3 \times (+5) = -15$.

122. When the multiplicand is *negative* and the multiplier is *negative.*

A man spends $5 a day for 20 days. How did his financial condition 3 days ago, compare with his present financial condition?

At the present time he has spent $100; three days ago he had spent $85; he was therefore $15 better off three days ago; therefore $-3 \times (-5)$ is equivalent to counting 5 units three times in the positive direction, beginning at -100 and arriving at -85. Hence $-3 \times (-5) = 3 \times (+5) = +15$.

123. Using general symbols the results in Arts. 119-122 may be stated thus:

(1) $(+a)\times(+b)=+ab.$
(2) $(-a)\times(+b)=-ab.$
(3) $(+a)\times(-b)=-ab.$
(4) $(-a)\times(-b)=+ab.$

Comparing (1) and (4) we find we have a + product when the signs of the factors are both + or both —.

Comparing (2) and (3) we find we have a — product when one of the factors is + and the other is —.

124. Laws of Signs in Multiplication. (1) *If two factors have like signs the sign of the product is +.*

(2) *If two factors have unlike signs the sign of the product is —.*

LAWS OF SIGNS IN DIVISION

125.

(1) Since $(+a)\times(+b)=+ab$; then $+ab\div(+b)=+a.$
(2) Since $(-a)\times(+b)=-ab$; then $-ab\div(+b)=-a.$
(3) Since $(+a)\times(-b)=-ab$; then $-ab\div(-b)=+a.$
(4) Since $(-a)\times(-b)=+ab$; then $+ab\div(-b)=-a.$

Comparing (1) and (3) we find we have a + quotient when the signs of the dividend and divisor are both + or —.

Comparing (2) and (4) we find we have a — quotient when the dividend is — and the divisor +, or the dividend + and the divisor —.

126. Laws of Signs in Division. (1) *If the dividend and divisor have like signs the sign of the quotient is +.*

(2) *If the dividend and divisor have unlike signs the sign of the quotient is —.*

127. The laws of signs in Multiplication and Division may be briefly stated as follows:

Like signs give + and unlike signs give —.

128. Problems.—Multiply:

(1) 4 by (— 7). (2) — 6 by 9. (3) — 11 by (— 13).
(4) 17 by — (41). (5) — 23 by (— 62).

Divide:

(6) 63 by (—9). (7) —88 by 11. (8) —356 by (—4).
(9) 729 by 9. (10) —256 by 16.

ANSWERS.—(1) —28. (2) —54. (3) 143. (4) —697. (5) 1426. (6) —7. (7) —8. (8) 89. (9) 81. (10) —16.

INDEX LAW IN MULTIPLICATION

129. Since the exponent indicates the number of times a quantity is used as a factor (whence *aaaaa* is written a^5, and *aaa* is written a^3), it follows that $a^5 \times a^3 = aaaaa \times aaa$.

Now in $aaaaa \times aaa$ we have the product obtained by taking *a* 5 times as a factor multiplied by the product obtained by taking *a* 3 times as a factor. There must, therefore, be $5 + 3$ factors in the final product, or $a^5 \times a^3 = a^{5+3} = a^8$. It is evident also that the product of $a^5 \times a^4$ is a^9; of $a^5 \times a^5$ is a^{10}, of $a^6 \times a^4$ is a^{10}, of $a^7 \times a^4$ is a^{11}, and so on. The product of *a* with any exponent, as a^m, multiplied by *a* with any exponent, as a^n, will be a^{m+n}. In like manner, since $a^m \times a^n \times \dots . a^b = (a^m \times a^n \times \dots .)a^b$, then $a^m \times a^n \times \dots . a^b = a^{m+n+\dots . b}$.

130. INDEX LAW IN MULTIPLICATION. *The exponent of a letter in the product is equal to the sum of the exponents of that letter in the different factors of the product.*

131. PROBLEM.—Find the product of $x^{3+m} \cdot x^{2+n}$.

SOLUTION.—$x^{3+m} \cdot x^{2+n} = x^{(3+m)+(2+n)} = x^{5+m+n}$, *Ans.*

EXPLANATION.—Adding the exponents $3 + m$ and $2 + n$ in the usual form we obtain the exponent of *x* in the product, which is $5 + m + n$. Writing this in the proper place we get x^{5+m+n} as the result.

132. PROBLEMS.—Find the product of:

(1) $x^{2n} \cdot x^{3n}$. (2) $a^6 \cdot a^{3n}$.
(3) $y^{2b-q} \cdot y^{-b+2q}$. (4) $x^4 \cdot x^{2+a^2}$.
(5) $b^2 \cdot b^3 \cdot b$. (6) $x \cdot x^2 \cdot x^4 \cdot x^{-1}$.

ANSWERS.—(1) x^{5n}. (2) a^{6+3n}. (3) y^{b+q}. (4) x^{6+a^2}. (5) b^6. (6) x^6.

INDEX LAW IN DIVISION

133. If it be required to divide a^5 by a^3 we may write $\frac{a^5}{a^3}=\frac{aaaaa}{aaa}=aa=a^2$.

Now the expression $\frac{aaaaa}{aaa}$ is a fraction having a as a factor 5 times in its numerator and 3 times in its denominator; therefore, if we cancel the 3 factors in the denominator with 3 equal factors in the numerator, thus $\frac{\cancel{a}\cancel{a}\cancel{a}aa}{\cancel{a}\cancel{a}\cancel{a}}$, we have left, a used as a factor twice in the numerator, or a^2. Hence $a^5 \div a^3 = a^2$. But $a^2 = a^{5-3}$. Hence $a^5 \div a^3 = a^{5-3} = a^2$.

$$\frac{a^6}{a^3}=a^{6-3}=a^3. \quad \frac{a^7}{a^3}=a^{7-3}=a^4 \text{ and so on.}$$

The quotient of a with any exponent, as a^m, divided by a with any exponent, as a^n, will be a^{m-n}.

134. INDEX LAW IN DIVISION. *The exponent of a letter in the quotient is equal to the difference obtained by subtracting the exponent of that letter in the divisor from the exponent of that letter in the dividend.*

135. PROBLEM.—Divide x^{2p+q} by x^{p-q}.

SOLUTION.—$x^{2p+q} \div x^{p-q} = x^{(2p+q)-(p-q)} = x^{p+2q}$, *Ans.*

EXPLANATION.—Subtracting the exponent $p-q$ from $2p+q$ in the usual form we obtain the exponent of x in the quotient, which is $p+2q$. Whence $x^{2p+q} \div x^{p-q} = x^{p+2q}$.

136. PROBLEMS.—Divide:

(1) m^{4a} by m^{2a}. (2) x^{3m} by x^{2-m}.
(3) y^{c+d} by y^{-c+2d}. (4) a^{2x} by a^{2y}.
(5) b^{x^2} by b^{y^2}.

ANSWERS.—(1) m^{2a}. (2) x^{4m-2}. (3) y^{2c-d}. (4) a^{2x-2y}. (5) $b^{x^2-y^2}$.

MULTIPLICATION OF MONOMIALS

137. Since the product of any given number of factors is obtained by multiplying the first factor by the second, this product by the third, and the product thus obtained by the fourth, and so on, and since the order of the factors is immaterial we can express the product $abcd\ldots.$ in the form $a \times b \times c \times \times d \times \ldots.$; or $ab \times c \times d \times \ldots.$; or $ab \times cd \times \ldots.$; or $abc \times d \times \ldots.$; or as the product of the different factors in any order or combination.

138. RULE.—*To find the product of two or more monomials find the continued product of all the different factors in the monomials.*

139. PROBLEMS.—(1) Find the product of $2ax^2y$ and $3by^2$.

SOLUTION.—$2ax^2y \times 3by^2 = 2 \cdot 3 \cdot a \cdot b \cdot x^2 \cdot y^2 \cdot y = 6abx^2y^3$, *Ans.*

EXPLANATION.—Since the order of the factors is immaterial we write the continued product of the factors, writing first the numerical factors 2 and 3, next the literal factors in alphabetical order. After having written the continued product of all the factors we combine as much as possible, multiplying the numerical factors and the like literal factors.

(2) Find the product of $-2a^2y$, $3bxy$, and $-cxy^2$.

SOLUTION.—$(-2a^2y) \times (3bxy) \times (-cxy^2) = (-2a^2y \cdot 3bxy) \times (-cxy^2) = (-6a^2bxy^2) \times (-cxy^2) = 6a^2bcx^2y^4$, *Ans.*

EXPLANATION.—Since the product of these monomials is equal to the product of any two multiplied by the third, we find the product of the first two which is $-6a^2bxy^2$. We then multiply this product by the third monomial, getting $6a^2bcx^2y^4$ as the final product.

(3) Multiply $-x$ by $-y$ by $-z$.

SOLUTION.—

$$(-x) \cdot (-y) = +xy$$
$$(+xy) \cdot (-z) = -xyz, \quad \textit{Ans.}$$

EXPLANATION.—Multiplying the first two terms, not forgetting to apply the law of signs, we get $+xy$. Multiplying this product by the third term we get $-xyz$.

If we continue by multiplying by a fourth negative factor $-w$ the product will be $+xyzw$. A fifth negative term will make the product $-$ again; a sixth $+$; and so on alternately.

140. Law of Signs for the product of more than two factors: *An odd number of negative factors makes the sign of the product —; and an even number of negative factors makes the sign of the product +.*

141. From the problems in Art. 139 we see that the processes of finding the product of two or more monomials has three distinct steps:

(1) *Determine the sign of the product.*

(2) *Multiply together the numerical coefficients, if any, of the different monomials and write the result as the coefficient of the product.*

(3) *Annex to the numerical coefficient all the different letters, giving to each letter an exponent equal to the sum of the exponents of that letter in the different monomials.*

142. Problems.—Find the product of:

(1) $\begin{array}{r}-5ab\\2ab\\\hline\end{array}$ (2) $\begin{array}{r}4xy\\-2xy\\\hline\end{array}$ (3) $\begin{array}{r}6x^2\\-3x\\\hline\end{array}$ (4) $\begin{array}{r}-4abx^2\\-2a^2bx^2y\\\hline\end{array}$

(5) $\begin{array}{r}7axyz^3\\2xy^3z\\\hline\end{array}$ (6) $\begin{array}{r}a^2y\\bx^4\\\hline\end{array}$ (7) $\begin{array}{r}x^{1+n}\\y^{2+m}\\\hline\end{array}$ (8) $\begin{array}{r}-a^{m+n}\\-a^{m+n}\\\hline\end{array}$

(9) $\begin{array}{r}x^{m+3}\\x^{m-5}\\\hline\end{array}$ (10) $\begin{array}{r}-a^xb^y\\-a^xb^y\\\hline\end{array}$ (11) $\begin{array}{r}x^my^{2n}\\-x^my^n\\\hline\end{array}$ (12) $\begin{array}{r}-2x^2y^n\\3x^my^{2m}\\\hline\end{array}$

Answers.—(1) $-10a^2b^2$. (2) $-8x^2y^2$. (3) $-18x^3$. (4) $8a^3b^2x^4y$. (5) $14ax^2y^4z^4$. (6) a^2bx^4y. (7) $x^{1+n}y^{2+m}$. (8) a^{2m+2n}. (9) x^{2m-2}. (10) $a^{2x}b^{2y}$. (11) $-x^{2m}y^{3n}$. (12) $-6x^{m+2}y^{2m+n}$.

DIVISION OF MONOMIALS

143. Since the product $-15a^3b^5x^2$ may be obtained by multiplying the monomials $-3a^2b^3x^2$ and $5ab^2$ it follows that $\frac{-15a^3b^5x^2}{5ab^2} = -3a^2b^3x^2$.

(1) We obtain the — sign of the quotient from the law of signs in division.

(2) We obtain the numerical coefficient, 3, of the quotient by dividing the numerical coefficient, 15, of the dividend by the numerical coefficient, 5, of the divisor. The result thus far obtained is $\frac{-3a^3b^5x^2}{ab^2}$.

(3) We divide a^3 by a and obtain the exponent 2 of a in the quotient by subtracting 1, the exponent of a in the divisor from 3 the exponent of a in the dividend. The result now obtained is $\frac{-3a^2b^5x^2}{b^2}$.

(4) Dividing by b^2 in the same way we obtain as the result of this step $-3a^2b^3x^2$, the required quotient as all the operations have been performed.

Since the divisor contained no factor x this factor in the dividend is not divided and hence its exponent remains unchanged in the quotient.

If the divisor contains any factor as y, not contained in the dividend, as the indicated division $\frac{-15a^3b^5x^2}{5ab^2y}$, the result of the fourth step would have been $\frac{-3a^2b^3x^2}{y}$. Since there is no factor y in the dividend, we cannot obtain another factor in the quotient by continuing the operations in the usual way, and we therefore allow the result to stand, an indicated division.

144. From the problem given above we see that the process of finding the quotient of two monomials has three distinct steps:

(1) *Determine the sign of the quotient.*

(2) *Divide the numerical coefficient of the dividend, if any, by the numerical coefficient of the divisor and write the result as the coefficient of the quotient.*

(3) *Annex to the numerical coefficient all the letters of the dividend giving to each letter an exponent obtained by subtracting the exponent of that letter in the divisor from the ex-*

ponent of that letter in the dividend. If the dividend contains any factor not contained in the divisor, its exponent will not be changed in the quotient. If the divisor contains any factor not contained in the dividend, the division by this factor must be indicated in the quotient.

145. If the dividend and divisor contain equal factors these factors may be cancelled by drawing lines through them.

146. PROBLEM.—Simplify, $\frac{3abx^2yz^3}{4ax^3yz}$.

SOLUTION.— $\frac{3abx^2yz^3}{4ax^3yz} = \frac{3\cancel{a}b\cancel{x^2}\cancel{y}\overset{z^2}{\cancel{z^3}}}{4\cancel{a}\underset{x}{\cancel{x^3}}\cancel{y}\cancel{z}} = \frac{3bz^2}{4x}$, *Ans.*

EXPLANATION.—a and y are equal factors in both dividend and divisor and may therefore be cancelled. Since x^2 is a factor of x^2 and x^3 these two quantities may be cancelled, placing an x under the x^3 this being the quotient of $x^3 \div x^2$. Since z is a factor of z and z^3 they may be cancelled and be expressed as shown. The remaining factors can be grouped and written thus, $\frac{3bz^2}{4x}$. Since the dividend contains no factors contained in the divisor the operation can be carried no further.

147. PROBLEMS.—Perform the indicated division.

(1) $\frac{27b^8x}{9x}$ (2) $\frac{96m^3c}{-8m^2}$ (3) $\frac{-29y^7z^2}{29y^5}$

(4) $\frac{60x^4y^5}{12x^2y^6}$ (5) $\frac{-a^2b^5x^4}{ab^2x^2}$ (6) $\frac{-51bcx^2}{17bx^3}$

(7) $\frac{225a^2b^5}{25b^6c}$ (8) $\frac{45x^2y^8}{-5x^3y}$ (9) $\frac{4b^2x^4y^5}{7b^5x^3y}$

(10) $\frac{-3ab^2c^3d^4}{-a^2b^3cd^4}$ (11) $\frac{49a^3x^2z^4}{-7ax^2z^8}$ (12) $\frac{-35a^{4m+1}}{7a^{4m-2}}$

(13) $\frac{85x^{a+4}}{17x^2}$ (14) $\frac{75a^mb^n}{15a^nb^m}$ (15) $\frac{84x^py^q}{-7x^py^{-q}}$

Answers.—(1) $3b^2$. (2) $-12mc$. (3) $-y^2z^2$. (4) $\frac{5x^3}{y}$. (5) $-ab^3x^2$. (6) $\frac{-3c}{x}$. (7) $\frac{9a^2}{bc}$. (8) $\frac{-9y^2}{x}$. (9) $\frac{4xy^4}{7b^3}$. (10) $\frac{3c^2}{ab}$. (11) $-7a^2z$. (12) $-5a^3$. (13) $5x^{a+2}$. (14) $5a^{m-n}b^{n-m}$. (15) $-12y^{2q}$.

MULTIPLICATION OF POLYNOMIALS BY MONOMIALS

148. **Problem.**—Multiply $3x^2 - 3xy + 6y^2$ by $-2xy$.

Solution.—

$$\begin{array}{l} 3x^2 - 3xy + 6y^2 \\ -2xy \\ \hline -6x^3y + 6x^2y^2 - 12xy^3, \quad \textit{Ans.} \end{array}$$

Explanation.—We first multiply $3x^2$ by $-2xy$ and proceed toward the right giving each product its proper sign.

149. **Rule.**—*To multiply a polynomial by a monomial, multiply each term of the polynomial from left to right by the monomial and connect the terms of the product with the proper signs.*

150. **Problems.**—Multiply:

(1) $1 + 2a + a^2$ by $2a$. (2) $x^2 - 2xy + y^2$ by $-xy$.
(3) $ax^n + by^m$ by cx^2. (4) $x^3 - 3x^2y + 5xy^2 - 7y^3$ by x^2y.
(5) $-3x^2y^3 - 5x^3y^2 + 7x^4y + 6x^5$ by $-4x^4y^3$.

Answers.—(1) $2a + 4a^2 + 2a^3$. (2) $-x^3y + 2x^2y^2 - xy^3$. (3) $acx^{n+2} + bcx^2y^m$. (4) $x^5y - 3x^4y^2 + 5x^3y^3 - 7x^2y^4$. (5) $12x^6y^6 + 20x^7y^5 - 28x^8y^4 - 24x^9y^3$.

MULTIPLICATION OF POLYNOMIALS BY POLYNOMIALS

151. **Problem.**—Multiply $3x^2 - 2xy + y^2$ by $2x^3y^2 - 3x^2y^3 + xy^4$.

Solution.—

$$\begin{array}{rrrrrl} 3x^2 & -\ 2xy & +\ y^2 & & & \\ 2x^3y^2 & -\ 3x^2y^3 & +\ xy^4 & & & \\ \hline 6x^5y^2 & -\ 4x^4y^3 & +\ 2x^3y^4 & & & \ldots\ldots\ldots\ldots\textit{partial product } (1) \\ & -\ 9x^4y^3 & +\ 6x^3y^4 & -\ 3x^2y^5 & & \ldots\ldots\ldots\textit{partial product } (2) \\ & & +\ 3x^3y^4 & -\ 2x^2y^5 & +\ xy^6 & \ldots\textit{partial product } (3) \\ \hline 6x^5y^2 & -\ 13x^4y^3 & +\ 11x^3y^4 & -\ 5x^2y^5 & +\ xy^6, & \textit{Ans.} \end{array}$$

Explanation.—Beginning with $3x^2$ we multiply each term of the multiplicand in order by $2x^3y^2$. Then we perform the same operation with the remaining terms of the multiplier in order, writing similar partial products in their own vertical column.

We then add the vertical columns of partial products, placing the proper sign before each sum and thus obtain the complete product.

152. Rule.—*Multiply each term of the multiplicand, working from left to right, by each term of the multiplier in the same order and connect the partial products with the proper signs. Add the similar partial products. The sum will be the product required.*

153. Note.—In arranging the polynomials for multiplication, the multiplicand and multiplier should both be written according to the descending or ascending powers of the same letter. Then the similar terms of the partial products will be easily arranged in vertical columns.

154. Problems.—Multiply:

(1) $a^2 + 2ab + b^2$ by $a - b$. (2) $a^2 + ab + b^2$ by $a - b$.
(3) $x^2 - xy + y^2$ by $x + y$. (4) $x^4 + x^2 + 1$ by $x^2 - 1$.
(5) $a + b + c$ by $a - b - c$. (6) $3x^2 + 3x + 1 + x^3$ by $x - 1$.
(7) $x^2 + 3x - 1$ by $x^3 - 2x - 2$. (8) $1 - 2x - x^2$ by $1 - x^2 + 2x$.
(9) $a^3 + 4a^2 + 5a - 24$ by $a^2 + 4a + 5$.
(10) $x^{n-1} - x^{n-2} - x^{n-3}$ by $x + 1$.

Answers.—(1) $a^3 + a^2b - ab^2 - b^3$. (2) $a^3 - b^3$. (3) $x^3 + y^3$. (4) $x^6 - 1$. (5) $a^2 - b^2 - c^2 - 2bc$. (6) $x^4 + 2x^3 - 2x - 1$. (7) $x^5 + x^4 - 3x^3 - 6x^2 - 4x + 2$. (8) $1 - 6x^2 + x^4$. (9) $a^5 + 8a^4 + 26a^3 + 16a^2 - 71a - 120$. (10) $x^n - 2x^{n-2} - x^{n-3}$.

DIVISION OF POLYNOMIALS BY MONOMIALS

155. Problem.—Divide $6a^4b^3x^2 - 8a^3b^2x^3 + 4a^2x^4$ by $2a^2x$.

Solution.—$\dfrac{6a^4b^3x^2 - 8a^3b^2x^3 + 4a^2x^4}{2a^2x} = (a)\ \dfrac{6a^4b^3x^2}{2a^2x} - \dfrac{8a^3b^2x^3}{2a^2x} + \dfrac{4a^2x^4}{2a^2x}$
$= (b)\ 3a^2b^3x - 4ab^2x^2 + 2x^3$, *Ans.*

Explanation.—We first indicate the division of each term of the dividend, as in (*a*) determining the signs and then actually perform the division, determining the coefficients and exponents for each term according to Art. 144, getting the result shown in (*b*). After some practice the indicated divisions shown in (*a*) may be omitted.

156. RULE.—*Divide each term of the dividend in order from left to right by the divisor and connect the partial quotients by the proper signs.*

157. PROBLEMS.—Divide:

(1) $4x^6 - 12x^9$ by $4x^5$.
(2) $4x^6 - 6x^5y + 8x^4y^2$ by $2x^3$.
(3) $9x^4y - 6x^3y^2 + 12x^2y^3 - 15xy^4$ by $3xy$.
(4) $-7x^5 + 5x^4y - 6x^3y^2 - 8x^2y^3 + 4xy^4 - 9y^5$ by $-xy$.
(5) $3x^m - 2x^{m+1} + 5x^{m+2} - x^{m+3}$ by $-x^3$.

ANSWERS.—(1) $x - 3x^4$. (2) $2x^3 - 3x^2y + 4xy^2$. (3) $3x^3 - 2x^2y + 4xy^2 - 5y^3$. ,(4) $\frac{7x^4}{y} - 5x^3 + 6x^2y + 8xy^2 - 4y^3 + \frac{9y^4}{x}$. (5) $-3x^{m-3} + 2x^{m-2} - 5x^{m-1} + x^m$.

DIVISION OF POLYNOMIALS BY POLYNOMIALS

158. If the product of $a - b$ multiplied by $2a + b$ is $2a^2 - ab - b^2$ then $a - b$ is contained in $2a^2 - ab - b^2$ $2a + b$ times. If only the product, $2a^2 - ab - b^2$, and the multiplicand, $a - b$ are given, we wish to find by what number $a - b$ is to be multiplied to produce $2a^2 - ab - b^2$; that is, how many times $a - b$ is contained in $2a^2 - ab - b^2$. Therefore $2a^2 - ab - b^2$, the former product, is now the dividend, and $a - b$ the former multiplicand is now the divisor, and $2a + b$, the former multiplier will be the quotient.

The relation between these numbers in multiplication and the reverse process of division is shown thus:

Multiplication		Division
multiplicand...........	$a - b$	.........*divisor*
multiplier	$2a + b$	.........*quotient*
	$2a^2 - 2ab$	
	$ab - b^2$	
product	$2a^2 - ab - b^2$	...*dividend*

159.

$$
\begin{array}{l}
3x^2 - 2xy + y^2 \\
\underline{2x^2 + 3xy - y^2} \\
6x^4 - 4x^3y + 2x^2y^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots (1) \\
\qquad\quad 9x^3y - 6x^2y^2 + 3xy^3 \ldots\ldots\ldots\ldots (2) \\
\qquad\qquad\quad \underline{- 3x^2y^2 + 2xy^3 - y^4} \ldots\ldots (3) \\
6x^4 + 5x^3y - 7x^2y^2 + 5xy^3 - y^4
\end{array}
$$

The product is the sum of the three partial products (1), (2), and (3), obtained by multiplying the multiplicand by the three terms of the multiplier in turn from left to right. Hence the first term of each of the partial products (1), (2), and (3), is obtained by multiplying $3x^2$, the first term of the multiplicand, by $2x^2$, $3xy$, and $-y^2$, the three terms of the multiplier, respectively.

We will now explain the reverse operation, namely division. As explained in Art. 158, the product $6x^4 + 5x^3y - 7x^2y^2 + 5xy^3 - y^4$ becomes the dividend and the multiplicand $3x^2 - 2xy + y^2$ becomes the divisor. In dividing, the work is arranged thus:

$$
\begin{array}{ll}
(4)\ldots\ldots 6x^4 + 5x^3y - 7x^2y^2 + 5xy^3 - y^4) & \underline{3x^2 - 2xy} + y^2 \ldots\ldots (5) \\
(6)\ldots\ldots \underline{6x^4 - 4x^3y + 2x^2y^2} & 2x^2 + 3xy - y^2 \ldots\ldots (11) \\
(7)\ldots\ldots\ldots + 9x^3y - 6x^2y^2 + 5xy^3 & \\
(8)\ldots\ldots\ldots + \underline{9x^3y - 9x^2y^2 + 3xy^3} & \\
(9)\ldots\ldots\ldots\ldots\ldots\ldots - 3x^2y^2 + 2xy^3 - y^4 & \\
(10)\ldots\ldots\ldots\ldots\ldots - \underline{3x^2y^2 + 2xy^3 - y^4} &
\end{array}
$$

The divisor (5) is placed at the right of the dividend (4) with a short line between. The quotient (11), when obtained, will be placed under the divisor.

The first step in the operation is to divide the first term of the dividend, $6x^4$, by the first term of the divisor, $3x^2$, which gives $2x^2$. The $2x^2$ is the first term of the quotient because, as can be seen from the multiplication above, the $6x^4$ is the product of the first term of the multiplicand (now divisor) by the first term of the multiplier (now quotient). Therefore we place $2x^2$ as the first term of the quotient.

The second step in the solution is to multiply the complete divisor (5) by the first term of the quotient. This gives the quantity (6) (which should be written as shown) which is the same as the partial product (1) in the multiplication.

The third step in the solution is to subtract (6) from the dividend, getting the remainder (7). It will be noticed that not all of the remainder is written in (7), but only as many terms as there are terms in the divisor. This remainder is equal to the sum of the partial products (2) and (3) and the first term is the same as the first term of partial product (2). The first term of (2) is the product of the first term of the multiplicand (divisor) by the second term of the multiplier (quotient); hence the fourth step in the solution is to divide the first term of the remainder (7) by the first term of the divisor (5). This gives $+3xy$, the second term of the quotient.

The fifth step is to multiply the complete divisor by the second term of the quotient, which gives (8), the same as partial product (2). Sixth, subtracting (8) from (7) gives (9) which is the same as partial product (3). The first term of (3) is the product of the first term of the multiplicand (divisor) by the third term of the multiplier (quotient). Hence, to get the third term of the quotient we divide the first term of (9) by the first term of (5). We then multiply and subtract as before. There is no remainder after the last subtraction so the divisor is complete.

160. RULE.—*To divide a polynomial by a polynomial.*

(1) *Arrange the terms of dividend and divisor according to the ascending or descending powers of some letter.*

(2) *Divide the first term of the dividend by the first term of the divisor and write the result as the first term of the quotient.*

(3) *Multiply the complete divisor by the first term of the quotient and subtract the result from the dividend.*

(4) *Regard the remainder as a new dividend and divide it by the first term of the divisor, writing the result as the second term of the quotient.*

(5) *Multiply the divisor by the second term of the quotient and subtract the result from the new dividend.*

(6) *Continue in this way until there is no remainder or until the first term of the remainder will not contain the first term of the divisor. If the first term of the remainder will not contain the first term of the divisor express the division by writing the remainder over the divisor and use the expression so obtained as the last term of the quotient.*

161. PROBLEMS.—Divide:

(1) $x^2 - x - 20$ by $x - 5$.
(2) $x^2 + 7x + 12$ by $x + 3$.
(3) $a^2 - 3a - 18$ by $a - 6$.
(4) $xy - y^2 + 2x^2$ by $x + y$.
(5) $c^2 - 11c + 10$ by $c - 10$.
(6) $20x^2y - 25x^3 - 18y^3 + 27xy^2$ by $6y - 5x$.
(7) $a^3x^3 - a^2x^2b - abx + b^2$ by $ax - b$.
(8) $4y^4 - 9y^2 + 6y - 1$ by $2y^2 + 3y - 1$ (see Art. 115).
(9) $a^4 - 4a^3x + 6a^2x^2 - 4ax^3 + x^4$ by $a^2 - 2ax + x^2$.
(10) $4x^6 - 12x^3y + 9y^2$ by $2x^3 - 3y$.

ANSWERS.—(1) $x + 4$. (2) $x + 4$. (3) $a + 3$. (4) $2x - y$. (5) $c - 1$. (6) $5x^2 + 2xy - 3y^2$. (7) $a^2x^2 - b$. (8) $2y^2 - 3y + 1$. (9) $a^2 - 2ax + x^2$. (10) $2x^3 - 3y$.

SIMPLE EQUATIONS

162. An **equation** is an algebraic statement indicating the equality of two expressions.

Thus the statement that $3x + 2$ and $2x + 4$ have the same value, is written $3x + 2 = 2x + 4$.

We can easily determine by inspection that the value of x that makes the expressions $3x + 2$ and $2x + 4$ equal, is 2. Thus substituting 2 for x, the equation becomes $3 \cdot 2 + 2 = 2 \cdot 2 + 4$; or $6 + 2 = 4 + 4$; or $8 = 8$.

Any other value of x, as 3, renders the equation an absurdity. Thus, substituting 3 for x, the equation becomes $3 \cdot 3 + 2 = 2 \cdot 3 + 4$; or $9 + 2 = 6 + 4$; or $11 = 10$, an untrue statement.

163. The *unknown quantities* in an equation are usually expressed by the last letters of the alphabet, such as x, y, or z, the *known quantities* by the first letters of the alphabet, such as a, b, or c, and also by *numbers.*

164. An equation in which all the known quantities are Arabic numerals is called a **numerical equation.**

165. An equation which, when reduced to its simplest form contains only the first power of the unknown quantity, is called a **simple equation,** or an equation of the first degree.

Thus $7x + 5 = 10x - 4$, and $ax + b = c$, are *simple equations* in x.

166. Equations may contain more than one unknown quantity, as, $x + 4y = 6y - 2x$.

167. That part of an equation on the *left* of the sign of equality is called the **first member** or **left side** of the equation; and that part of the equation on the *right* of the sign of equality is called the **second member** or **right side** of the equation.

168. The **solution** of an equation consists in finding the value of the unknown quantity.

169. It is convenient in solving any equation to bring all the terms containing the *unknown* quantity to the *left* side of the equation, and all the *known* terms to the *right* side of the equation.

This process is called **transposing** the **terms.**

170. In solving equations we make use of the following statements called **axioms,** the truth of which is admitted to be self-evident.

Axiom 1. *If equal numbers be added to equal numbers their sums are equal.*

Axiom 2. *If equal numbers be subtracted from equal numbers their remainders are equal.*

Axiom 3. *If equal numbers be multiplied by equal numbers their products are equal.*

Axiom 4. *If equal numbers be divided by equal numbers their quotients are equal.*

171. From the nature of an equation it is evident that whatever *operation* is *performed* on *one side must be performed* on the *other side.*

SOLUTION OF A SIMPLE NUMERICAL EQUATION

172. Problem.—Solve, $5x - 4 = 12 + x$.

Solution and Explanation.—4 can be removed from the *left* side of the equation by adding 4 to this side, since $-4 + 4 = 0$. But in order not to destroy the equation we must also add 4 to the *right* side. Performing these operations the equation becomes $5x - 4 + 4 = 12 + x + 4$. (*Axiom 1.*)

x can be removed from the right side of the equation by subtracting x from this side. But we must also subtract x from the left side of the equation. Performing these operations the equation becomes $5x - 4 + 4 - x = 12 + x + 4 - x$. (*Axiom 2.*)

Reducing the equation to its simplest form we find that the known quantities on the left side cancel each other, and also the unknown quantities on the right side cancel each other, therefore, canceling, we obtain $5x - x = 12 + 4$, or $4x = 16$. If we now divide the left side of the equation, $4x$, by 4 we shall obtain x. But we must also divide the right side 16, by 4. Performing these operations we obtain, $\frac{4x}{4} = \frac{16}{4}$, or $x = 4$. (*Axiom* 4.)

173. Verification. If, when the value found for x is substituted for x in the original equation, the equation reduces to an identity, the equation is said to be **verified.** Thus substituting 4 for x in the equation, $5x - 4 = 12 + x$, we obtain $5 \cdot 4 - 4 = 12 + 4$; or $20 - 4 = 12 + 4$; or $16 = 16$; we therefore know that the equation has been correctly solved.

174. From the above problem it is evident that the following statements are true:

(1) *If a number appears on one side of an equation with the + sign, it can be removed from that side by subtraction, and will therefore appear on the other side with the — sign.*

(2) *If a number appears on one side of an equation with the — sign, it can be removed from that side by addition, and will therefore appear on the other side with the + sign.*

175. RULE.—*In order to transpose any number from one side of an equation to the other, remove the number from its original position and place it on the other side of the equation with its sign changed.*

176. PROBLEM.—Solve $15 - 21x - 32 - 5x = 68 - 10 - 4x + 3x$.

SOLUTION.— $15 - 21x - 32 - 5x = 68 - 10 - 4x + 3x$.

$$-21x - 5x + 4x - 3x = 68 - 10 - 15 + 32.$$
$$-25x = 75$$
$$x = -3, \quad \textit{Ans.}$$

EXPLANATION.—Writing the unknown quantities on the left side in their order, and transposing the unknown quantities on the right side in their order, the left side of the equation becomes $-21x-5x+4x-3x$. Writing the known quantities on the right side in their order and transposing the known quantities on the left side in their order, the right side of the equation becomes $68-10-15+32$. Hence the original equation becomes $-21x-5x+4x-3x=68-10-15+32$. Reducing to the simplest form we obtain $-25x=75$. Dividing both sides of the equation by -25 we obtain $x=-3$.

177. NOTE.—The left side of the equation, $-25x=75$, can be made $+$ by multiplying both sides by -1. (*Axiom 3.*) We then obtain $25x=-75$. Whence $x=-3$. This is often a convenient step but it is not necessary.

178. RULE.—(1) *To solve a simple equation, transpose the unknown quantities on the right side of the equation to the left side, and the known quantities on the left side of the equation to the right side, changing the signs from $+$ to $-$ or from $-$ to $+$ of all terms transposed.*

(2) *Combine all the unknown quantities into one term and the known quantities into one number.*

(3) *Divide both sides of the equation by the coefficient of the unknown letter.*

179. PROBLEMS.—**Find the value of x.**

(1) $5x+2=27$.
(2) $4x+7=6x+1$.
(3) $3x-7=2x+2$.
(4) $7x-12=3x+4$.
(5) $x-2=2x-7$.
(6) $x-3=15-2x$.
(7) $7x-8=6x+2$.
(8) $60-3x+6x=-11-7x+77+4x$.
(9) $6(x-5)+2x=8x-2(x+10)$.

HINT.—**Expand the terms containing parentheses as the first step in the solution.**

(10) $9(13-x)-4x=5(21-2x)+9x$.
(11) $7(3x-6)+5(x-3)=77-4(17-x)$.
(12) $8(3x-2)-7x-5(12-3x)+28=8(3x+2)-32$.

ANSWERS.—(1) 5. (2) 3. (3) 9. (4) 4. (5) 5. (6) 6. (7) 10. (8) 1. (9) 5. (10) 1. (11) 3. (12) 4.

STATEMENT AND SOLUTION OF PROBLEMS

180. The **statement** of a **problem** consists in expressing in algebraic symbols the conditions of a problem that are expressed in common language.

In solving a problem containing one unknown quantity it is necessary to find two expressions that have the same value and write them equal to each other.

181. PROBLEMS STATED AND SOLVED. (1) Three times a certain number is equal to the number increased by 12. Find the number.

ANALYSIS AND SOLUTION.—The words "a certain number," "the number," and still more definitely the words "find the number," suggest a definite value to be found. The object of the problem is to find this "certain number." It is therefore the unknown quantity and must be represented by a symbol. We therefore say:

Let $x =$ the number. Using symbols for words and phrases, we write (1) the figure 3 for "three." We may write (2) the sign of multiplication ×, for "times," though this will ordinarily be omitted. For the words "a certain number" we write (3) the letter x, the symbol already adopted for the unknown quantity. We write the symbol = for the words "equal to." The left side of the equation is now complete and has the form $3x =$.

To find an expression of equal value for the right side we continue reading the problem. For the words "the number" we write the letter x. "Increased by" means "made larger" or "added to" and we therefore write the sign of addition, +. The number 12 follows the sign +. The right side of the equation is now complete and has this form: $x + 12$. The equation is now stated $3x = x + 12$.

The equation can now be easily solved.

The complete statement and solution of the problem should be in the following form:

Let $x =$ the number.

Then $3x =$ three times this number

and $x + 12 =$ the number increased by 12.

But by the problem these last two expressions are equal.

$\therefore\ 3x = x + 12.$ (1)

Transposing $3x - x = 12$ (2)

Combining $2x = 12$ (3)

Dividing by 2, $x = 6$ (4)

Therefore the number is 6.

VERIFICATION. Substitute 6 in the original equation (1) $3 \cdot 6 = 6 + 12.$

$18 = 18$, the solution is therefore correct.

(2) The sum of two numbers is 36, and twice the greater exceeds three times the smaller, by 2. Find the numbers.

ANALYSIS AND SOLUTION.—The words "Find the numbers" suggest two unknown quantities, "the greater number" and "the smaller." If one of them is known, however, the other can be easily determined, so only one symbol is required. x, then, may stand for either "the greater number" or "the smaller." It can be told only by trial which method will be the more convenient. We will therefore write,

Let x = the greater number.

If 36 is the sum, and one of the numbers is given, the other number is the difference between 36 and the given number. Hence, we write

$36 - x$ = the smaller number.

The words "twice the greater number" are equivalent to "two times the greater number." Hence, we write,

$2x$ = twice the greater number.

Since $36 - x$ = the smaller number; three times the smaller number $= 3(36 - x)$. Hence we write $3(36 - x)$ = three times the smaller number. The problem expresses a relation between twice the greater number and three times the smaller; and this relation must be expressed in the form of an equation. Hence we write,

$2x = 3(36 - x)$ as a partial statement of these conditions. There is now left only the number 2, a known quantity, which will be placed on the right side of the equation. Its sign is to be determined. Reading the word "exceeds," we know that the left side of our incomplete equation is greater in value than the right side, and the excess is 2. Hence we must add 2 to the right side to make it equal to the left side. The complete equation will therefore be $2x = 3(36 - x) + 2$.

The complete statement and solution of the problem should be in the following form:

Let $x =$ the greater number,
Then $36 - x =$ the smaller number,
$2x =$ twice the larger number,
and $3(36 - x) =$ three times the smaller number.

$$\therefore 2x = 3(36 - x) + 2 \qquad (1)$$

Expanding and removing the parenthesis,

$$2x = 108 - 3x + 2 \qquad (2)$$

Transposing, $2x + 3x = 108 + 2$ (3)
Combining, $5x = 110$ (4)
Dividing by 5, $x = 22$ (5)
Whence $36 - x = 14$ (6)

Therefore the numbers are 22 and 14.

VERIFICATION. Substitute 22 in the original equation.

$2 \cdot 22 = 3(36 - 22) + 2$
$44 = 3(14) + 2$
$44 = 42 + 2$
$44 = 44$, the solution is therefore correct.

(3) A man is now twice as old as his son. 10 years ago he was three times as old as his son. Find the age of each.

ANALYSIS AND SOLUTION.—

Let $x =$ the number of years in the son's age now,
Then $2x =$ the number of years in the father's age now,
$x - 10 =$ the number of years in the son's age 10 years ago,
and $2x - 10 =$ the number of years in the father's age 10 years ago.

Two relations are expressed between the son's and the father's ages. The first sentence of the problem expresses an equality between their ages now. Hence we write the expression for the father's age equal to twice the expression for the son's age.

$2x = 2(x)$ or $2x = 2x$, an identity from which no solution can be obtained.

The second sentence of the problem expresses an equality between the father's age 10 years ago and three times the son's age 10 years ago. Hence, we write the expression for the father's age 10 years ago equal to three times the son's age 10 years ago.

$$2x - 10 = 3(x - 10) \qquad (1)$$

Expanding and removing the parenthesis,

$$2x - 10 = 3x - 30 \qquad (2)$$

Transposing, $2x - 3x = -30 + 10$ (3)

Combining, $-x = -20$ (4)

Dividing by -1, $x = 20$ (5)

$$2x = 40 \qquad (6)$$

Therefore the son's age is 20 and the father's age is 40.

182. Problems.—(1) The sum of two numbers is 60 and the greater is 4 times the lesser. Find the numbers.

(2) The difference between two numbers is 12 and their sum is 76. Find the numbers.

(3) Three times a certain number is equal to the number increased by 24. Find the number.

(4) The sum of two numbers is 63, and the smaller equals the larger diminished by 33. Find the numbers.

(5) A man is four times as old as his son; in 18 years he will be only twice as old. Find the age of each at the present time.

(6) A, B, and C buy a cottage for $3,000. B pays twice as much as A, and C pays twice as much as A and B together. How much does each pay?

Hint.—Let x = the number of dollars A pays.

(7) Two men start from the same place and travel in opposite directions, one 25 miles a day and the other 40 miles a day. In how many days will they be 780 miles apart?

(8) Two men start from the same place and travel in the same direction, one 25 miles a day and the other 40 miles a day. In how many days will they be 225 miles apart?

(9) Two trains start at the same time from points 228 miles apart and travel toward each other, one at the rate of 45 miles an hour and the other at the rate of 33 miles an hour. In how many hours will they meet?

(10) Twelve men agreed to buy a camp together, but two declined to take their share and each of the others had to pay \$5 more for his share. What was the cost of the camp?

HINT.—Let x = the number of dollars each would have had to pay at first.

ANSWERS.—(1) 12, 48. (2) 44, 32. (3) 12. (4) 15, 48. (5) 9 years, 36 years. (6) A pays \$333$\frac{1}{3}$; B, \$666$\frac{2}{3}$; C, \$2,000. (7) 12 days. (8) 15 days. (9) 2$\frac{12}{13}$ hours. (10) \$300.

FORMULAS

183. A **Formula** is a statement in algebraic symbols expressing a relation between two or more magnitudes.

184. An example of a simple formula is the one expressing the relation between rate, time, and distance. As rate is the distance a body moves in a given time, for example, the number of feet per second, or the number of miles per hour, the *distance* a body moves in a given time will be the product of the *rate* multiplied by the *time*. Expressed in an equation, *Rate* × *Time* = *Distance*, or, using symbols, $R\ T = D$.

185. Any formula expressed as a simple equation can be solved where all but one of the quantities are known. Thus, if two of the quantities R, T and D are given, the third can be determined. There can then arise three problems.

(1) Given R and T to find D.

(2) Given R and D to find T.

(3) Given T and D to find R.

(1) If R and T are given, D is the unknown quantity. Writing this on the left side of the formula, it becomes $D = R\ T$.

(2) If R and D are given, T is the unknown quantity and this should be written alone on the left side of the formula. To find the value of T we write the original or fundamental formula, $RT = D$. Since R is a factor or multiplier we remove it from the left side of the formula by performing with it the reverse operation, that of division, and obtain $\frac{RT}{R} = T$; also dividing the right side by R we obtain $\frac{D}{R}$; therefore $T = \frac{D}{R}$. (*Axiom 4*).

(3) If T and D are given, R is the unknown quantity and this should be written alone on the left side of the formula. To find the value of R we write as before $RT = D$. Now T

is to be removed from the left side of the formula, and as it is a factor or multiplier, we divide both sides of the formula by it and obtain $R = \frac{D}{T}$, (*Axiom 4*).

186. Problems.—(1) A train moves at a speed of 44 miles an hour. How far will it go in 9 hours?

Solution and Explanation.—The quantities given are rate, *R*, and time, *T*. The distance, *D*, is required. We therefore write the formula (1) for distance, $D = RT$.

Substituting for the known quantities, *R* and *T*, their given values 44 and 9, the formula becomes,

$$D = 44 \times 9 = 396 \text{ mi., } \textit{Ans.}$$

(2) How long will it take a train moving at a speed of 45 miles an hour to go 495 miles?

Solution and Explanation.—The quantities given are rate, *R*, and distance, *D*. The time, *T*, is required. We therefore write the formula (2) for time, $T = \frac{D}{R}$.

Substituting the known values for *D* and *R* the formula becomes $T = \frac{495}{45} = 11$ hr., *Ans.*

(3) In 13 hours a train travels 520 miles. How many miles an hour does it go?

Solution and Explanation.—The quantities given are time, *T*, and distance, *D*. The rate, *R*, is required. We therefore write the formula (3) for rate, $R = \frac{D}{T}$. Substituting the known values for *D* and *T* the formula becomes $R = \frac{520}{13} = 40$ mi., *Ans.*

187. Ohms law in electricity states that the current is directly proportional to the electro-motive force and inversely proportional to the resistance.

This is stated in the formula $I = \frac{E}{R}$, in which *I* is the current measured in amperes; *E* is the electro-motive force (often written E. M. F.) measured in volts; and *R* is the resistance measured in ohms. The formula may be stated in words thus, $\textit{amperes} = \frac{\textit{volts}}{\textit{ohms}}$.

As in the formula $D = RT$, three problems may arise.

(1) Given E and R to find I.

(2) Given R and I to find E.

(3) Given E and I to find R.

(1) If E and R are given, the formula stands unchanged, $I = \frac{E}{R}$.

(2) If R and I are given, E must be be written alone on the left side of the formula. Since R in the formula $I = \frac{E}{R}$ is a divisor, it must be removed by performing the reverse operation, multiplication. Multiplying the right side by R we obtain $\frac{E \cdot R}{R} = E$; multiplying the left side by R we obtain RI; therefore $RI = E$, (*Axiom 3*), or $E = RI$.

(3) If E and I are given, R must be written alone on the left side of the formula. R is removed from the right side of the formula as in (2); whence $RI = E$. Since R is the required quantity, I must be removed from the left side. I is a factor and must therefore be removed by division. Dividing the left side by I we obtain $\frac{RI}{I} = R$; dividing the right side by I we obtain $\frac{E}{I}$; therefore $R = \frac{E}{I}$, (*Axiom 4*).

188. Problems.—(1) **The E. M. F. of an electrical current is 110 volts. The resistance is 220 ohms. Find the current.**

Solution and Explanation.—The quantities given are E. M. F., E, and resistance, R. The current, I, is required. We therefore write the formula $I = \frac{E}{R}$.

Substituting the known values for E and R, the formula becomes $I = \frac{110}{220} = \frac{1}{2}$ or .5 ampere, *Ans.*

(2) **A copper wire is carrying a 5 ampere current. The resistance of the wire is 2 ohms. How many volts are necessary to produce the current?**

Solution and Explanation.—The quantities given are current, I, and resistance, R. The voltage, E, is required. We therefore write, $E = RI$.

Substituting the known values for R and I, the formula becomes $E = 2 \times 5 = 10$ volts, *Ans.*

(3) An electric lamp circuit carries a current of 1.5 amperes under a pressure of 120 volts. What is the resistance of the circuit?

Solution and Explanation.—The quantities given are current, I, and voltage, E. The resistance, R, is required. We therefore write $R = \frac{E}{I}$.

Substituting the known values for E and I, the formula becomes $R = \frac{120}{1.5} = 80$ ohms, *Ans.*

EXAMINATION QUESTIONS AND PROBLEMS

(1) What is the law for signs in multiplication and division?

(2) What is the index law in multiplication and division?

(3) Multiply $a + b + c$ by $a + c$.

(4) Multiply $a^2 - ab + b^2$ by $a^2 + ab + b^2$.

(5) Multiply $(x - y)(x + y)(x^2 + y^2)$.

(6) Divide $x^3 + 4x^2 - 9x - 36$ by $x + 3$.

(7) Divide $x^4 - 4x^3 + 5x^2 - 4x + 1$ by $x^2 - 3x + 1$.

(8) Divide $a^6 + 6a^5b + 15a^4b^2 + 20a^3b^3 + 15a^2b^4 + 6ab^5 + b^6$ by $a^2 + 2ab + b^2$.

(9) Find the products: (*a*) $(x + 1)(x - 1)$. (*b*) $(x^2 + y)(x^2 + y)$. (*c*) $(a + b^2)(a - b^2)$.

(10) Find the results: (*a*) $\frac{a^3 - b^3}{a - b}$. (*b*) $\frac{x^4 - y^4}{x^2 + y^2}$.

(11) Solve for x: $(x - 2)(x - 2) - 7 = (x - 3)(x - 3)$.

(12) Solve for x: $3(x - 5) + 4(x - 4) = 5(x + 4)$.

(13) In a certain city 40,004 votes were cast. A received 5,000 more than half as many as B. How many did each receive?

(14) The E. M. F. of a group of 4 cells is 4 volts and the resistance of the circuit is 1.5 ohms. What will be the current?

(15) What voltage is required to maintain a current of 1.4 amperes through a resistance of 2.5 ohms?

(16) Find the value of x: $25 - x = 4x - 2x - 5$.

(17) The sum of two numbers is 83. The difference between them is 13. Find the numbers.

(18) 12 battery wells are to be washed with a solution containing 25 parts acid, 10 parts soda, 3 parts potash, 62 parts

water. If it takes 3 gallons of the solution to wash one well, how much water will be needed?

(19) A gas increases in volume an amount equal to $\frac{1}{491}$ of its volume at 32° F. for every increase of 1° F. in temperature. What was the volume of the gas at 32° F. if its volume at 37° F. is 248 cu. ft.?

(20) The resistance of a certain coil is 500 ohms, that of another is 2,000 ohms. The formula for finding the joint resistance of two coils in multiple is $R(r + r') = rr'$ where R is the unknown resistance, and r and r' are the resistances of the two coils joined in multiple. Find the joint resistance of these coils when so connected.

ANSWERS TO EXAMINATION PROBLEMS

(1) ?

(2) ?

(3) $a^2 + ab + 2ac + bc + c^2$.

(4) $a^4 + a^2b^2 + b^4$.

(5) $x^4 - y^4$.

(6) $x^2 + x - 12$.

(7) $x^2 - x + 1$.

(8) $a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$.

(9) (*a*) $x^2 - 1$. (*b*) $x^4 + 2x^2y + y^2$. (*c*) $a^2 - b^4$.

(10) (*a*) $a^2 + ab + b^2$. (*b*) $x^2 - y^2$.

(11) $x = 6$.

(12) $x = 25\frac{1}{2}$.

(13) A, 16,668; B, 23,336.

(14) $2\frac{2}{3}$ amperes.

(15) 3.5 volts.

(16) $x = 10$.

(17) 48, 35.

(18) 22.32 gallons.

(19) 245.5 cu. ft.

(20) 400 ohms.

DRAWING

INTRODUCTION

It is the aim of this course to furnish sufficient instruction to enable the student to understand and make practical drawings as required in signal work.

The various parts of the subject are arranged in the order in which they should be studied. Exercises are provided to afford practice and to illustrate the principles presented in the text. Directions are placed at the beginning of each exercise telling clearly what is to be done.

In outline the treatment is as follows:

A description of instruments and materials required and notes on their use.

General instruction in methods.

Exercises affording practice in the use of instruments.

Simple geometrical definitions and construction.

Working drawings.

GENERAL METHODS

INSTRUMENTS AND MATERIALS

1. **Equipment:** The following outfit is as small as is consistent with good work. It is described in detail in the succeeding articles.

ITEM	SIZE
Drawing board	20″×24″
T square	30″
45° triangle	7″
30°×60° triangle	8″
Irregular curve	
Triangular Architect's scale	12″
Triangular Engineer's scale	12″
Compasses	5½″
Dividers	5″
Bow pencil	3½″
Bow pen	3½″
Bow dividers	3½″
Ruling pen	5″
Drawing paper	9″×24″
Tracing cloth	9″×24″

4H pencil 6H pencil Tacks Waterproof black ink
Pencil eraser Ink eraser Printing pens Penholder

2. Drawing Board: The drawing board should be of soft white pine or white wood ⅞′ thick, 24″ long and 20″ wide. It is advisable to have cleats at the ends or back to prevent warping. The upper or working surface is required to be smooth and flat. One of the short edges is used as a working edge and must be smooth and straight. It should be tested with a straight edge from time to time as the accuracy of the work will be affected if it is not true. This working edge is always placed at the left side.* It should be marked plainly and care taken never to use the other edges. Fig. 1 shows a suitable board.

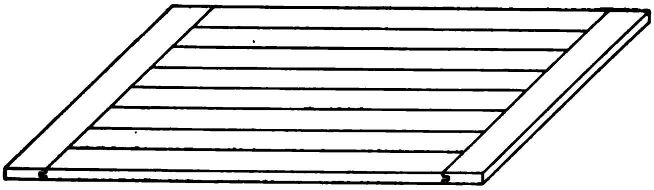


Fig. 1

3. Paper: A drawing paper with a fairly smooth surface that will take ink and stand considerable erasing is required. All drawings are to be made on one standard size of sheet measuring overall 9″×24″.

4. Tacks: For fastening the paper and the tracing cloth to the drawing board either thumb tacks or one-ounce tacks of copper or iron may be used. Thumb tacks are somewhat easier to put in place and remove, while the small copper or iron tacks possess the advantage that they can be pushed down almost even with the surface of the paper and thus interfere very little with the movement of the T square or triangles.

5. T Square: The T square consists of a blade fastened to a head by means of a clamp or screws. For the work of this course, a plain wooden T square of the latter type with the blade permanently fixed to the head will answer, see Fig. 2. The inner edge of the head and working edge of the blade must be smooth and straight. The accuracy of the drawing depends primarily on the good condition of these edges together with the working edge of the drawing board. All should be examined frequently.

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*A left handed person should place the working edge at the right side.

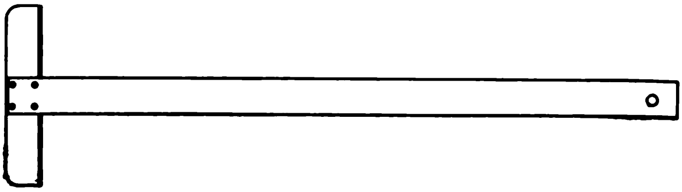

Fig. 2

In using the T square, the head should be held firmly against the working edge with the left hand, the blade resting on top of the board. It is advisable to keep the left hand near the head when moving the T square up and down, or holding it in position while drawing a line. The upper or working edge of the blade is the only one used. Lines may be drawn from left to right by moving the pencil or ruling pen along this edge and are called horizontal lines, see Fig. 3. Any one of such a series of lines is at every point along its length at the same distance from any other line of the same series and they are known for this reason as parallel lines. The head of the T square must never be used in contact with any but the working edge of the board.

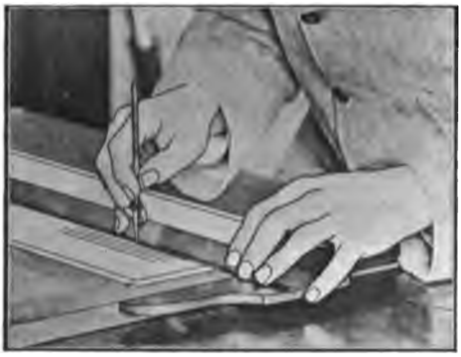

Fig. 3

6. **Triangles:** Triangles are made from various materials such as wood, hard rubber, celluloid and metal. For most work two triangles are sufficient. One of these called a *45 deg. triangle*

is shown in Fig. 4. This has two angles of 45 deg. and one of 90 deg. The other, shown in Fig. 5, is called a *30 deg. by 60 deg. triangle* and has one angle of 30 deg., one of 60 deg., and the third of 90 deg.

Triangles are usually used with the T square in straight line work, for the purpose of drawing lines making angles of 90 deg., 75 deg., 60 deg., 45 deg., 30 deg., or 15 deg. with the horizontal lines drawn along the blade. The T square is first moved to the required position by the left hand (see Art 5). The triangle is moved along the blade to the proper place with the right hand. Then by sliding the left hand along the blade toward the triangle, the head of the T square may be held firmly against the working edge of the board and the hand brought to a position where the first and second fingers can be used to keep the triangle in place while the right hand is used for drawing the line, see Fig. 6. Figs. 7 and 8 show the various positions of the triangles for drawing lines as mentioned above.

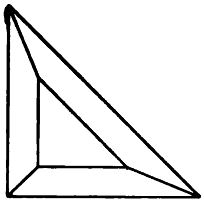

Fig. 4

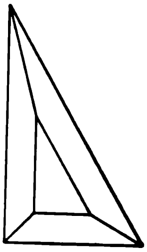

Fig. 5

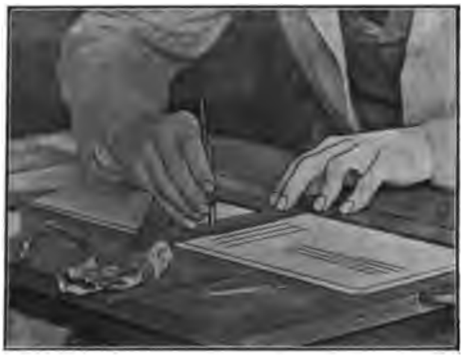

Fig. 6

Lines at 90 deg. to other lines are said to be perpendicular to them. When lines are perpendicular to horizontal lines they

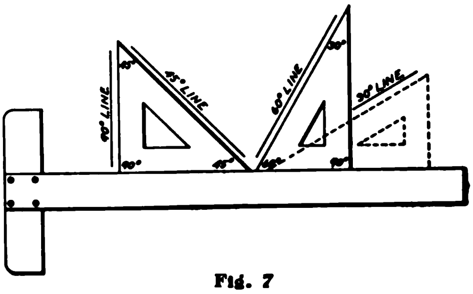

Fig. 7

are called vertical.

When the triangle is moved along the blade by means of the first two fingers, keeping the T square in place with the thumb and other fingers, a series of lines may be drawn using the same edge. Each one of these lines is, at every point along its length, the same distance from any other line of the series. They are therefore said to be parallel.

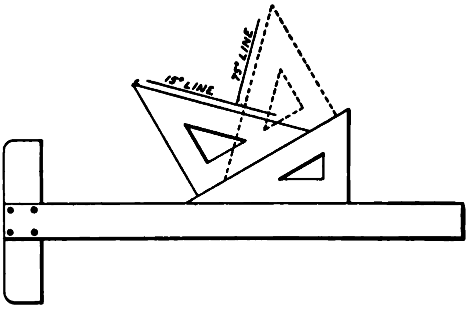

Fig. 8

In some cases it is necessary to use the triangles without the T square. Parallel lines may be drawn by holding one triangle firmly and moving the other along its edge by means of the first and second fingers of the left hand as shown in Fig. 9.

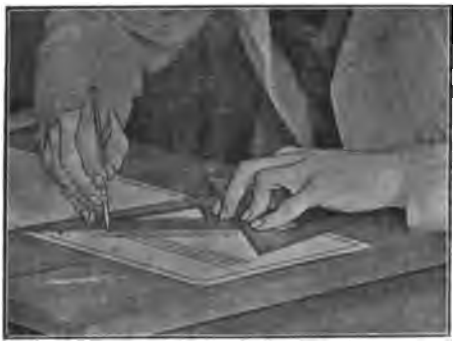

Fig. 9

Fig. 10 illustrates a method of drawing a line perpendicular to another. The 30° × 60° triangle is first placed so that its edge is parallel to the given line. The edge should not be placed directly on the line, but at a slight distance from it, which allows the eye to determine readily whether or not the edge is everywhere at the same distance away. The 45° triangle is now placed against one of the other two edges of the 30°×60° and held firmly while the latter is moved along it a distance of about ¼″ from the line. It is now held in position, the 45°

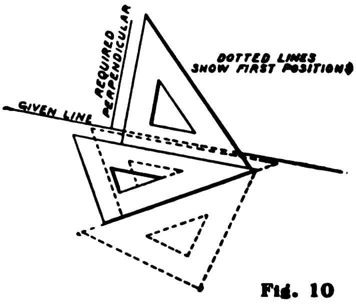

Fig. 10

triangle placed against the edge parallel to the given line, and the perpendicular, or a series of parallel perpendicular lines drawn, as shown. Similarly lines at angles of 30 deg., 45 deg., and 60 deg. may be drawn to a given line. The edges of the triangles should not be used near the ends as they are usually worn away and not accurate.

To prevent the working edges of the T square and triangles from casting shadows on the paper, the light should come from the upper left hand corner of the board.

7. **Pencils:** Pencils are made with leads of varying hardness and are graded accordingly. Two grades are sufficient for producing working drawings. They are usually selected to correspond with the texture of the paper, some papers having a grain that wears away the lead much faster than others. A 6H and 4H* will give good results with most drawing papers.

It is important that the pencils be correctly sharpened. Two forms of point are used, the wedge and conical. The wedge point is used for drawing lines and the conical for laying off dimensions from the scale, for lettering and sketching. Each pencil may be sharpened with the wedge point on one end and the conical on the other. The points should be neatly made after the manner shown in Fig. 11. The wood is removed with a knife and the lead dressed to form, either with a fine file or a piece of fine sand paper.

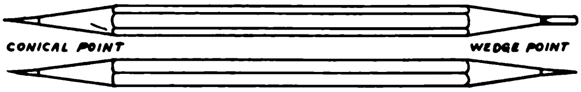

Fig. 11

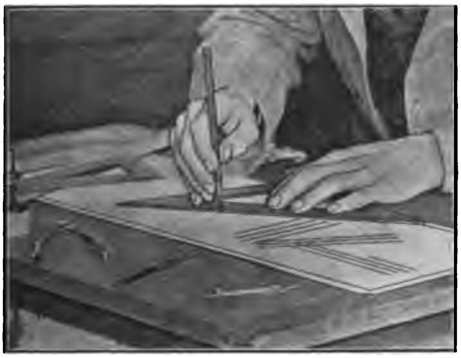

Fig. 12

In drawing lines using a guide, as the T square or triangle, the pencil should be held in the right hand in a nearly vertical position; see Fig. 12. The point should rest lightly on the paper with the flat side of the wedge against the guide and not fitted sharply into the angle made by the edge of the guide and the paper. The arm is held free from the body to allow an even easy movement. Lines are drawn from the left to the right and from the bottom to the top of the board, or in general, always away from the body.

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*6H is two grades harder than 4H.

The 6H pencil is best used for laying off dimensions and drawing fine lines, the 4H for lettering, sketching and general line work. The points must be sharpened frequently. The conical point for lettering may be somewhat blunt, allowing it to move over the paper smoothly, but the others, particularly that used for laying off dimensions, must be as fine as possible.

8. Scales: There are two forms of scales in general use, the flat and triangular. The triangular form is shown in Fig. 13.

Fig. 13

While the flat scale is more convenient to handle, the triangular is recommended for this work, since it affords a greater variety of graduations. Scales are ordinarily made of boxwood and in some cases are provided with a white facing on which the graduations are placed. The white surface aids in reading the divisions, but is not necessary.

The scale should be used only on the drafting board. If it is used for taking measurements from objects, such as a machine part, the edges soon become rough and the accuracy of the work is affected. For the same reason it should never be used as a guide or ruler in drawing lines. The points of dividers or compasses will soon destroy its accuracy, if they are allowed to touch the scale in taking off distances. The proper method is to place the scale so that its edge touches the line on which the distance is to be measured and then, by means of the sharp conical pointed pencil, make slight marks on the paper opposite the desired graduations. If it is desired to set the compasses or dividers, the proper distance should be transferred to the paper as above and the instrument set to these marks. If a series of measurements are to be laid off along the same line, the operation should be completed without moving the scale. For example, if it is desired to lay off spaces of ½″, ⅞″, ¾″ and 1 9/16″ from a given point on the line, lay the scale* with its edge touching the line and the graduation marked 0 opposite the given point. Place marks opposite divisions ½″, 1⅜″, 2⅛″ and 3 11/16″. This method gives the desired spaces and the result will be more accurate than if the scale was moved each time.

There are two styles of scales in general use called the "Architect's" and the "Engineer's."

The "*Architect's*" scale is used principally in making drawings of machines or machine parts, buildings, etc., of such a size that they can be easily read and placed on a sheet of convenient size. One edge marked 16, is divided into inches and the inches into sixteenths; it is used for making full size drawings. The use of the graduations on the other edges is for making drawings that are less than full size. For example, it might be required to show an object one-quarter size in order to get it on the drawing sheet. The graduations mostly used are given in the following table:

MARK ON SCALE	SIZE	DRAWING IS MARKED
16	Full size	Scale, full size
16	Half size	Scale, 6″ = 1′
3	Quarter size	Scale, 3″ = 1′
1½	One-eighth	Scale, 1½″ = 1′
1	One-twelfth	Scale, 1″ = 1′
¾	One-sixteenth	Scale, ¾″ = 1′
½	One-twenty-fourth	Scale, ½″ = 1′
⅜	One-thirty-second	Scale, ⅜″ = 1′
¼	One-forty-eighth	Scale, ¼″ = 1′
⅛	One-ninety-sixth	Scale, ⅛″ = 1′

In making a drawing half size it is usual to employ the edge marked 16, considering each half inch division equal to one inch.

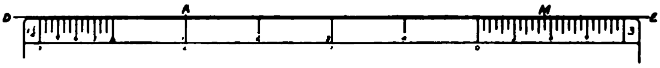

Fig. 14

The use of the other graduations is illustrated in Fig. 14. For example, it is required to lay off, to a scale of one-quarter size, a distance of 2′ 5½″ from a given point A on the line DE. Find the mark 3 denoting the proper graduation, (see table). Place the scale so that its edge touches the line DE with the graduated surface to be used toward the light. From the mark 3 extending along the scale there is a closely divided portion ending in a line marked 0. This portion represents *one* foot divided into inches and the inches into eighths.* Its actual length is *one-quarter* of a foot or 3″. 3″ beyond the 0 there is another line marked 1, 3″ beyond this a line marked 2 and at the other end of the scale a line marked 3. These lines are one-quarter of one, two, and three feet from the line marked 0 and therefore represent one, two, and three feet to the required scale. Now placing line 2 directly opposite point A and running along the scale to line 0 we get the required distance for 2′. Continuing into the closely divided part over five one inch divisions and one-half of the next inch division, and making a slight mark at M, we have laid off the required distance, 2′ 5½″.

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*Use full size graduations on "Architect's" scale, denoted by the number 16 appearing at the end or in the middle.

At the opposite end of the same edge on which the 3 appears is the mark 1½. This means that from this end the edge can be used for obtaining one-eighth size measurements. Fig. 15 shows 3′ 8″ laid off with this scale from B to N on the line F-G. The other scales are to be used similarly.

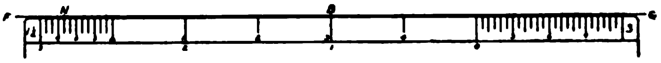

Fig. 15

A drawing may be made to a smaller scale than full size by using the full size graduations and, if one-quarter size is desired, mentally dividing each dimension by 4 and measuring off the result. This method should be avoided. It is much slower and not as accurate as the method described above.

The "*Engineer's*" scale is usually graduated with 10, 20, 30, 40, 50, and 60 divisions per inch of length and the corresponding scales marked with these numbers. Each division may be called a foot, or in some cases ten feet, so that with twenty divisions to the inch a drawing may be made on which one inch represents twenty feet, or two hundred feet as the case may be. This style of scale is used in laying out track plans, maps, etc. Fig. 16 shows a distance of 87′ 0″ measured from point C to point O on the line HJ, with a scale of 1″ = 10′.

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*On account of the small size of the illustration, the quarter size scale is divided into half inches only and the eighth size into inches, instead of having the actual scale graduations.

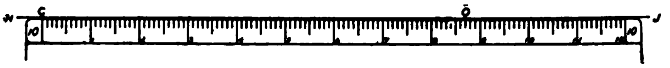

Fig. 16

9. **Compasses:** The excellence of this and following instruments made of metal lies in rigidity coupled with light weight, accurately made and smoothly working joints, durable screws, and well formed and properly hardened needle and pen points. In Fig. 17 is shown a standard pair of compasses with attachments. It consists of the instrument proper with fixed needle point and removable pencil, pen point and lengthening bar. The tongues on the removable pieces should fit their sockets accurately and tight enough to require a slight pull to remove them. The main joint should move with sufficient friction to keep the legs in place when once set and, at the same time, be free enough to allow of adjusting the compasses with the fingers of one hand. The joints or knees, near the middle of each leg, allow the legs to be bent so as to stand perpendicular to the paper when once the compasses are open.

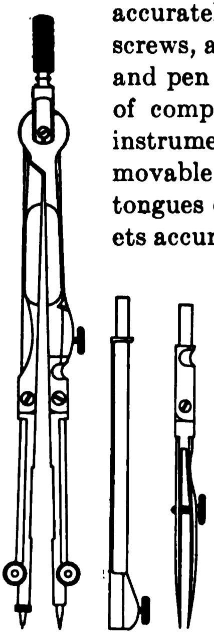

Fig. 17

The pen point should be put in place and the shouldered needle point moved in its socket till the end of the pen is about half way between the shoulder and point of the needle. The needle point should be set in this position and kept there. This is done with the compasses closed. The pencil point is now put in place and fitted with a 4H lead sharpened to a wedge point, see Fig. 18, only that the width of the point should be somewhat less than in a pencil. The lead is adjusted to the same position as the pen point, care being taken to have the flat of the

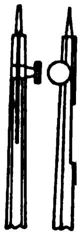

Fig. 18

wedge perpendicular to a line joining the needle and pencil points. Fig. 19 shows the different adjustments.

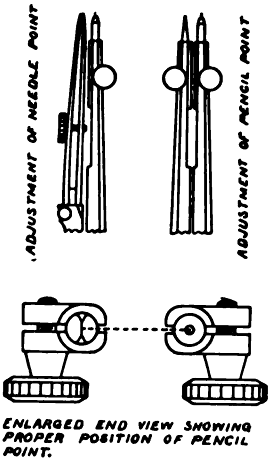
ADJUSTMENT OF NEEDLE POINT
ADJUSTMENT OF PENCIL POINT
ENLARGED END VIEW SHOWING PROPER POSITION OF PENCIL POINT.

Fig. 19

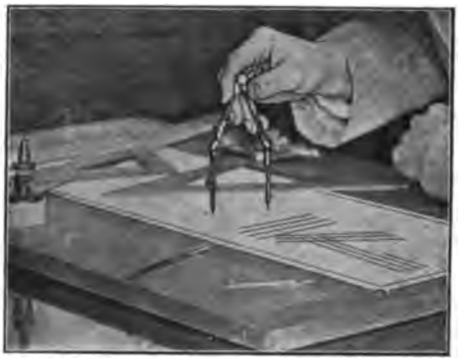

Fig. 20

In operation the compasses are held in the right hand. They are adjusted by the use of the thumb and first three fingers of that hand, see Fig. 20. After adjusting the points to the proper radius, the thumb and first two fingers take a position near the main joint, as in Fig. 21. The needle point is pressed lightly into the paper, just sufficient to prevent slipping, a light pressure put on the pen or pencil point and the compasses rotated from left to right, or clockwise. The leg holding the pen or pencil is inclined a little forward in the direction of motion. The legs should be bent so that both the needle and pencil or pen point will stand nearly perpendicular to the paper. The line should be stopped at the starting point, see Fig. 22. Figs. 21 and 22 also show the finger positions when starting and stopping the circle. Care should be taken to keep the

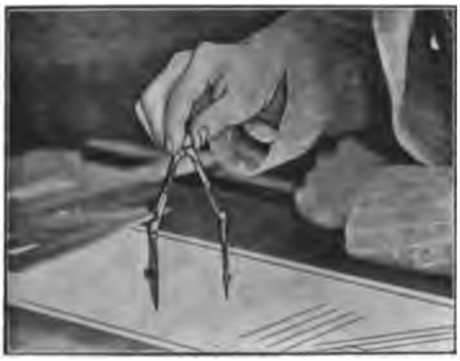

Fig. 21

center hole as small as possible. It may be enclosed within a small free hand circle so as to be readily found when wanted again. In placing the needle point in position it is often convenient to steady it with a finger of the left hand. The lengthening bar makes it possible to draw circles of a larger radius but when it is used considerable care is necessary to prevent the compasses from springing. (See Art. 12 with regard to care of the pen.)

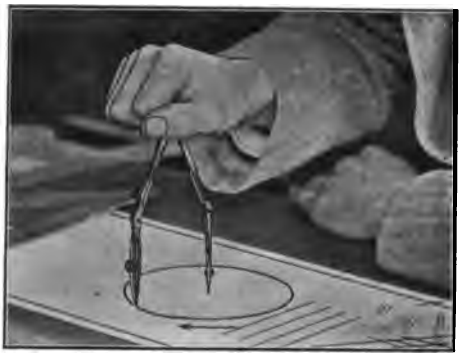

Fig. 22

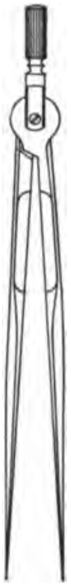

Fig. 23

10. Dividers: Fig. 23 shows a pair of dividers. It resembles the compasses with the exception that the legs have no joints and terminate in fine steel points.

With this instrument a measurement can be transferred from one part of a drawing to another or a line divided into a given number of equal parts. The method of this latter operation is illustrated in Fig. 24. Suppose the line AB is to be divided into seven equal parts. Starting at A as shown, with a distance between the points of the dividers as nearly as possible one seventh of AB, seven spaces are stepped off swinging the points around first in one direction and then in the other as indicated by the dotted lines. It will

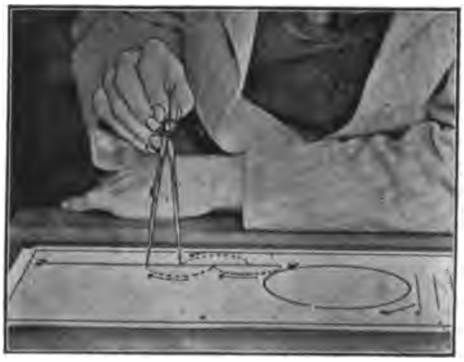

Fig. 24

probably be necessary to go through this operation two or three times, increasing or decreasing the distance between the points as required, before the last position will just meet point B.

The points of the dividers must be sharp. They should not be pressed into the paper until the correct spacing is found and then just sufficiently to mark the divisions. Dividers are sometimes provided with a spring and screw device in one of the legs for fine adjustment. This is known as a hair spring. It should be used carefully and the spring tension removed when the instrument is not in use.

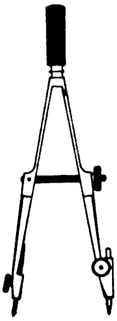

Fig. 25

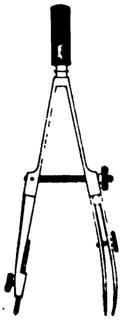

Fig. 26

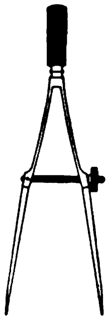

Fig. 27

11. Bow Instruments: Fig. 25 shows a *bow pencil;* Fig. 26, a *bow pen;* and Fig. 27 *bow dividers.* These instruments are used in making the small arcs, etc., for which the instruments described in Arts. 9 and 10 are not adapted. They are arranged to be set by means of a screw and thumb nut. To save wear on this screw, when making changes of any great amount, press the points together so as to remove the pressure from the nut. This allows the nut to be turned freely and rapidly by running the finger along it. The nut should always be run out when putting the instrument away, thus relieving the spring tension.

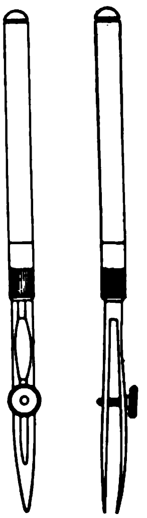

Fig. 28

12. Ruling Pen: This instrument is used for ink lines drawn with the aid of a guide. Fig. 28 illustrates front and side views. The ends of the blades should be rounded, of equal length and brought to an edge not quite sharp enough to cut the paper with light pressure.

The pen is filled by placing the quill from the ink bottle between the blades, $\frac{1}{4}$ in. of ink being sufficient. A greater amount may cause a blot by running out when the pen touches the paper. The ruling pen is held between the thumb and first and second fingers, as shown in Figs. 3, 6, and 9, and can be adjusted by turning the nut with the thumb and second finger to give any width of line within its limit. Its proper position is *vertical.* It should be moved with a free arm motion and only a light pressure on the paper and against the guide. Heavy pressure against the guide will spring the back blade and vary the width of line. If the pen is tipped away from or toward the guide both blades will not touch the paper, resulting in a ragged line. Ink should be kept off the *outside* of the blade points, as, if allowed to collect at the points, it will cause the ink to flow too freely from the pen thus producing a rough line, and if on the side next to the guide, will run onto it and from there to the paper. If the pen has not been used for a few minutes allowing the ink to dry slightly, it may sometimes be started by touching the end to a soft surface, such as a pen-wiper or a piece of chamois skin. Be sure that the pen is clean and bright when it is put away. Ink left on the blades is liable to corrode and roughen them.

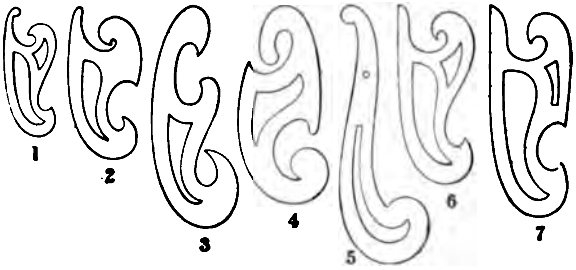

Fig. 29

13. Curves: These are of two general kinds. The first type, Fig. 29, known as *irregular curves* are used in drawing lines that are neither straight nor arcs of circles. The second, called *railroad curves*, two styles of which are shown in Fig. 30, are employed in drawing arcs of circles used in track work.*

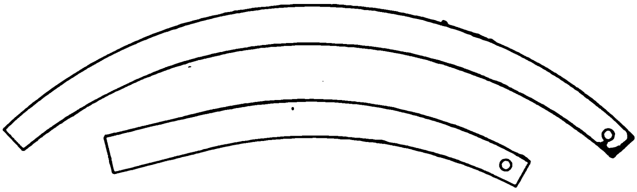

Fig. 30

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*If a railroad curve is not available, an irregular curve may be used in its place for the work of this course.

Ordinarily several points on an irregular curve, as A to H, Fig. 31, are known. As an aid a free hand curve may be first sketched through the points. Then starting at one end A, a curve or portion of a curve is selected that touches as many points as possible, say A to E. A line is then drawn along this curve to D. The curve is now moved to a new position touching points C to G and the line drawn from D to F and so on.

In inking the same method is followed, placing the curve so that the ink line will be drawn coincident with the pencil line and stopping the ink line a short distance before the edge of the curve and the pencil line begin to separate. This gives a smooth even effect to the resulting line.

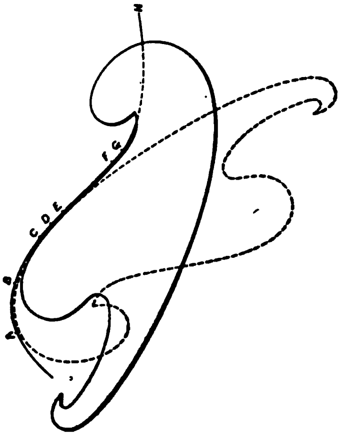

Fig. 31

Railroad curves are marked either with the radius or degree of curve, sometimes both.* Referring to the diagram, Fig. 32, the degree of curve is equal to the number of degrees of central angle subtended by a chord of 100 feet.

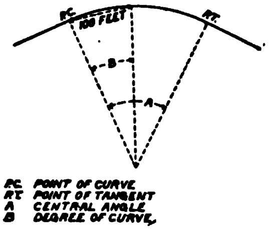

Fig 32

In further explanation of this diagram, it may be stated that a railroad is generally divided into 1,000 foot lengths starting from some given point, such as a terminal. Similarly the track plan or map is divided into 1,000 foot lengths which are indicated as follows. The starting point is marked 0, the first division point, 1+000 and the next division point, 2+000, etc. These points are called *chaining stations* and their numbers, *station numbers.* If a point occurs between chaining stations, for example, between 21+000 and 22+000, and is distant 475 feet from

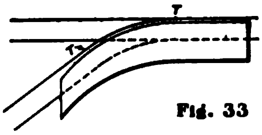

Fig. 33

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*A table is given in the appendix indicating the radius corresponding to the degree of curve for drawings made to different scales.

the lower station, its location is indicated by the station number 21+475. When working in the direction from a lower station number to a higher one, the point where the tangent* ends and the curve begins is the *point of curve*, indicated by the initials "PC". The point where the curve ends and the next tangent begins is the *point of tangent*, indicated by the initials "PT"; for instance, a 500 foot curve between stations 63+200 and 63+700 has "PC" at station 63+200 and "PT" at station 63+700.

In some instances the chaining stations occur every one hundred feet, being designated by the station numbers 1+00, 2+00, etc.

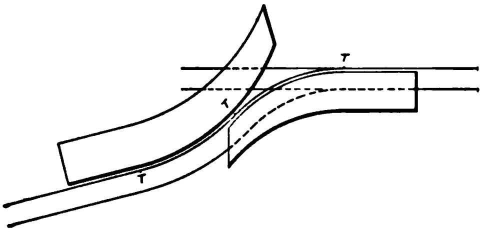

Fig. 34

Figs. 33, 34, and 35 illustrate the method of using these curves in making track plans. Fig. 33 shows a simple turnout with a curve of 9°, Fig. 34 a reverse curve of 9° and 7°, and Fig. 35 a cross-over with 7° curves. When a straight line and a curved line or two curved lines meet, but do not intersect or cut each other on being prolonged, they are said to be tangent. The point where they meet is called the *point of tangency*. In the diagrams the points of tangency are marked T.

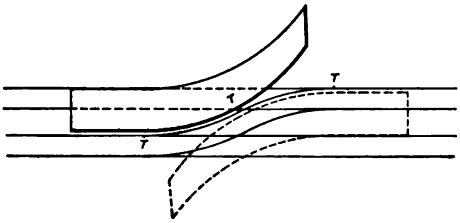

Fig. 35

14. **Protractors:** This class of instrument is used for laying off angles that can not be obtained with the triangles. A protractor, Fig. 36, reading to half degrees, meets all ordinary requirements. The center of the graduated arc is plainly marked.

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*Straight track. See definition of *tangent*, Art. 77.

In use the center of the protractor is made to coincide with the vertex of the angle to be laid off and the 180° divisions at the ends of the graduated arc, placed so that they come directly over the base line. The paper is now marked slightly, opposite the proper division, the protractor removed and the required line drawn through the vertex and the point just obtained, see Fig. 37.

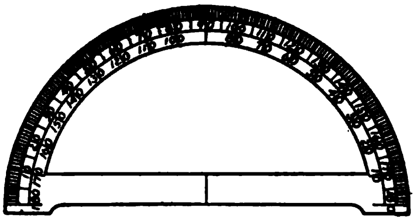


FIG. 36

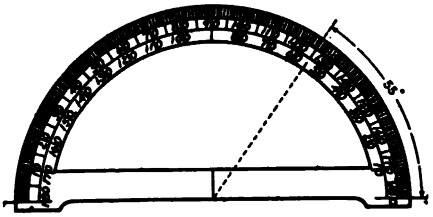


FIG. 37.

15. Erasers: In general three kinds are used known as the pencil, ink and sponge rubbers.

Erasing is accomplished by moving the rubber back and forth, pressing it lightly on the surface from which a mark is to be removed. Although it requires more time, the use of light pressure results in a better surface for subsequent work. Avoid rubbing in only one direction as it tends to make a trough in the paper. Use the ink eraser for ink lines both on paper and tracing cloth. The knife blade or steel eraser is used only to remove ends of ink lines where they have run over slightly. An erasing shield made of thin celluloid or metal, with various shaped openings, makes it possible to confine the erasing to a desired area. The sponge rubber is used for cleaning the surface of the paper after the drawing is completed. Care should be taken in using it to touch the ink lines as little as possible in order to preserve their sharp outline and clear appearance.

16. Tracing Cloth: It is sometimes necessary to make many duplicates of a drawing. This is mostly done by the process known as blue printing. (See Art. 29.) To make blue prints it is necessary to have the drawing on a partially transparent material. Tracing cloth answers this requirement. It is a grade of linen treated by a process that renders it translucent and provides a surface that will take ink lines.

In use, the tracing cloth is stretched smoothly over the pencil drawing and ink lines traced over the pencil lines, which can be readily seen through the cloth. Either the dull or glazed side may be used. It is preferable to use the glazed side since it is easier to erase from this surface. Before starting work, tracing cloth should be dusted over with powdered chalk and rubbed thoroughly with a cloth. The chalk dust should then be entirely removed with a brush or duster. If the surface of the cloth is not prepared in this manner, the ink, instead of making a full line, lays on the cloth in globules giving a ragged appearance.

17. **Ink:** Prepared ink is sold in small bottles having a quill or other device, for filling the pen, attached to the cork. Two apparently simple things should always be kept in mind. First, place the bottle where it will not be upset as, when overturned on a drawing, it usually spoils the work. Second, keep the cork in the bottle at all times since otherwise the ink rapidly thickens by evaporation. The waterproof ink is considered best for general work.

18. **Blue Print Paper:** Paper for commercial use comes in rolls wrapped for protection against light and moisture. Reference to a trade catalogue will provide information about grades of paper. Only enough should be purchased at a time to fill immediate needs as it spoils rapidly. Care must always be taken to exclude light and moisture from unused paper.

19. **Printing Pen:** An ordinary pen holder equipped with a No. 303 Gillott pen answers for the purpose of inking dimensions and of lettering.

GENERAL INSTRUCTIONS

20. Preparation of the Paper: The proper position of the drawing board is with one long edge toward the body and the working edge at the left side. The height of the board should be such that an easy working position is secured. It is often convenient to incline the board slightly by raising the back edge.

Remove dust and small particles from the board, T square and triangles with a brush or cloth. By repeating this operation frequently in the course of making a drawing, a much cleaner piece of work will result.

Place the paper in the center of the board, Fig. 38, and fasten at the corners with four tacks about $\frac{3}{16}''$ or $\frac{1}{4}''$ from the edge, proceeding as follows: After one tack is placed put the T square on the board, in the position shown by dotted lines, and swing the paper till the top edge is parallel to the working edge of the blade. Then place the other three tacks taking care to stretch the paper flat and thus avoid wrinkles. If a drawing is on the board for any length of time the moisture in the air may cause the paper to expand and wrinkle. In that case remove three of the tacks and stretch again testing some principle horizontal line with the working edge of the T square to make sure that the drawing is in the proper position.

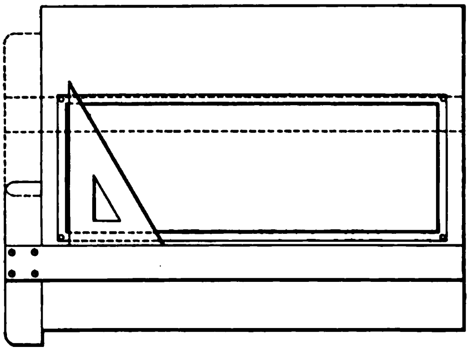


Fig. 38

The work has been arranged so that one size of sheet, 9″×24″, will meet all requirements. The margin is to be $\frac{1}{2}''$ wide making the size inside the border line 8″×23″.

After placing a sheet on the board with the top edge horizontal, try the left hand edge with the 30°×60° triangle to see if it is perpendicular to the top edge. If not, draw a line just inside of the left edge so that it can be properly trimmed when removed. Now with the scale horizontal, starting at this line, measure off ½″, 23″, and ½″. Draw vertical pencil lines through these points using the 30°×60° triangle. The last line should coincide with the right hand edge of the sheet, or fall inside of it, so that the sheet can be trimmed to the proper size when it is finally removed from the board. Similarly, starting at the top edge, lay off ½″, 8″, and ½″ and draw horizontal lines using the T square. The vertical and horizontal lines will meet and constitute the border and trimming lines.

21. Character of Lines: Pencil lines should be distinct, uniform and as fine as possible. They should be plain enough to allow of being easily read through tracing cloth, but care must be taken to keep the pencil point from cutting into the paper, making the line difficult to erase. Light full lines are frequently used on the pencil drawing in place of dotted or dash lines when the drawing is to be traced or inked, but if there is any chance of confusion they should be marked with a small (d) or other character to prevent errors.

Fig. 39 illustrates the various lines used in ink work. The *unshaded full line* is used for the visible lines of an object, that are not shaded.

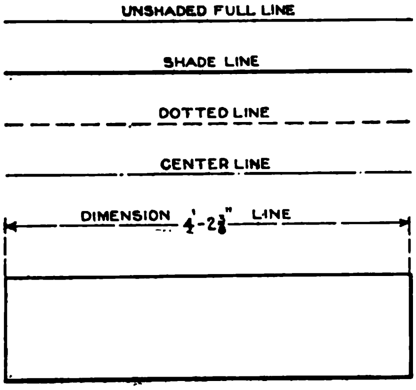


Fig. 39

Shade lines should be from two to three times the width of unshaded lines. They are used in some classes of work, such as drawings intended for display, to make the drawing indicate more clearly the form of an object.

Dotted lines are used for such lines of a drawing as are in back of other parts and can not be seen. This class of line should have the same width as unshaded full lines. Dotted lines are never shaded.

Center lines are made with a dash followed by one or two dots. The width of these lines should be about one-half that of the unshaded full lines.

Dimension lines are of the same width as center lines, but composed of a series of dashes and short spaces. They should end in arrow points and have a space for the dimension.

Extension lines are either dash or dotted lines of the same width as dimension lines. They are used when placing a dimension outside some portion of a drawing to indicate the limits of the dimension lines.

Border lines are best made of single lines about the width of shade lines.

In general all lines of a kind should be of uniform width, and the dots, dashes and spaces of uniform length. Ink lines should end squarely at the proper place. Junctions between lines should match exactly.

22. Shading: Shading is used in an endeavor, by making some lines heavier than others, to give the drawing an appearance of solidity or depth. The conventional method only will be treated. This calls for the shading of lower and right hand lines, Fig. 40, except in the case of openings where the upper and left hand lines are shaded as at A. Fig. 41 shows the shading of lines drawn at different angles. When there are two 45° lines on a drawing, one indicating an upper and the other a lower surface, it is best to shade one of them, the general practice being to shade the line indicating the lower surface.

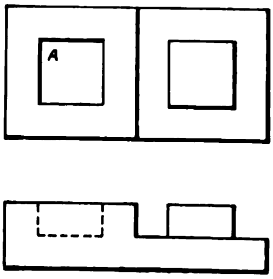


Fig. 40

Lines, such as AB, Fig. 42, that indicate the intersection of visible surfaces or those like CD, Fig. 43, that indicate the limit of a rounded surface, in this case a cylinder, are generally not shaded.

The extra width required for shading a line should be added on the outside if possible.

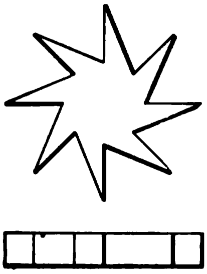


Fig. 41

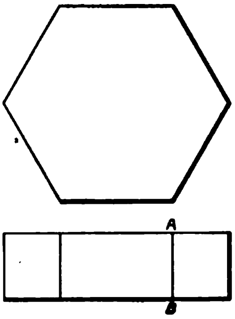


Fig. 42

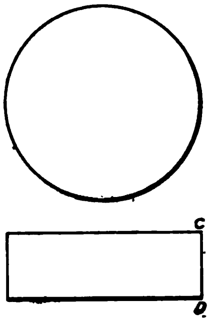


Fig. 43

The shading of circles is accomplished by taking a new center below and to the right (distance determined by trial) on the 45° diameter and, without changing the radius, drawing an arc on the outside if the circle is an outside edge, Fig. 44, or on the inside if it is an inside edge, Fig. 45. The shade lines of circles or arcs are the only ones that vary in width.

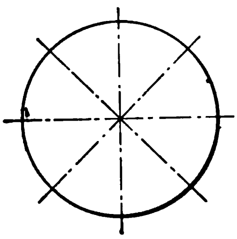


Fig. 44

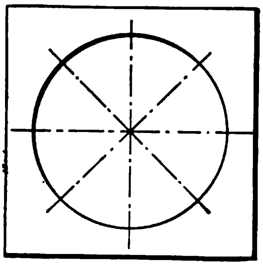


Fig. 45

23. **Sections:** In order to make a construction clear, it is sometimes necessary to show an object with a part toward the observer removed. Such a view is called a section and the surfaces are indicated by means of parallel lines drawn across them, usually at an angle of 45 deg. Surfaces of different parts meeting one another are made distinct by running the section lines at different angles, Fig. 46. Different kinds of material are indicated by different kinds of lines. Fig. 47 shows types of sectioning that are generally used for this purpose. The upper half of the wood section represents a section taken across

the grain and the lower, a section taken with the grain. In small sections for insulating material, instead of employing the section shown, *solid black* is frequently used.

In pencil work it is sufficient to draw section lines free hand, making the work as uniform as possible without spending much time. In ink work the lines should be a little narrower than the unshaded full lines, regularly spaced, and made with the triangles. The distance between the lines is governed by the area to be sectioned, the lines on large areas being from three to four times as far apart as those shown.

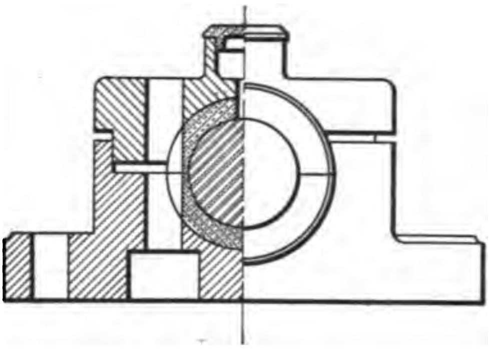


Fig. 46

If it is necessary to put a figure, letter, or note on a sectioned surface, a space for it should be left clear of section lines.

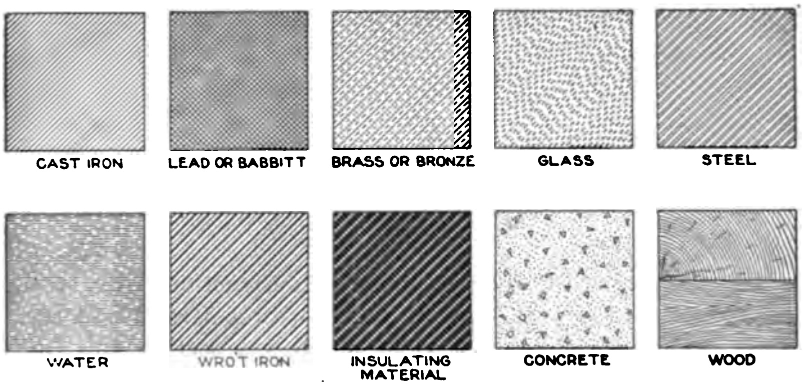


Fig. 47

To show a long piece, such as a shaft, and get it on a certain size sheet without reducing the scale, the method of leaving out the center portion is sometimes used. This is illustrated by Fig. 48 in which A shows the entire piece and B the way it may be drawn to save space.

24. Threads and Bolt Heads: Fig. 49 illustrates some *con-*

ventions that are used for screw threads. The pitch or number of threads per inch and other information should be given in a note. *Conventions* are used to save the time that would be required to draw the details. This illustration also shows methods of drawing square and hexagonal bolt heads and nuts, which, while not exact, answer for all ordinary work. When exact dimensions are required they may be obtained by reference to tables.

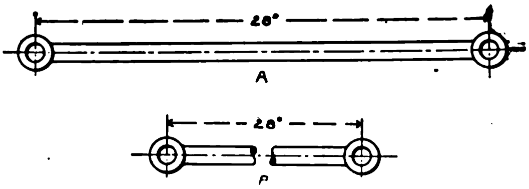
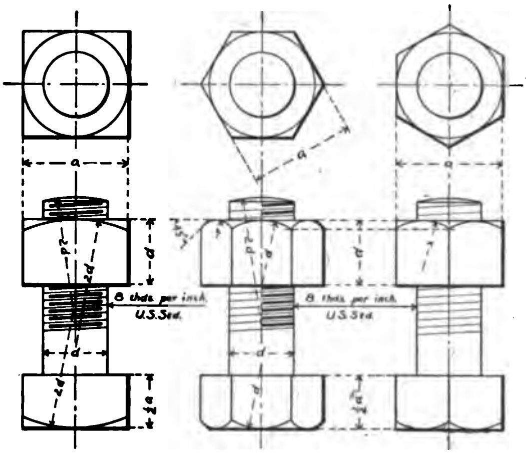


Fig. 48



a = 1½d + ⅛"
r and x to be determined by trial.

Fig. 49

25. Dimensioning: Working drawings are made exactly to scale so as to keep the proper proportion between the parts. It is necessary however, in order to prevent errors, to give all measurements by dimensions.

The lines used in dimensioning are described in Art. 21. As the lines are always drawn before the figures are placed, a suitable space must be left for the dimension. The points of the arrow heads should just touch the lines between which the measurement is made. In case room is lacking, the heads may be arranged in several ways as illustrated in Fig. 50. The arrow heads should be small and neatly made with the printing pen.

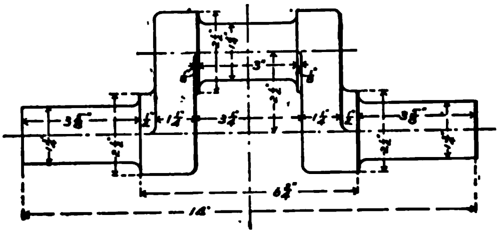


Fig. 50

In some cases the dimensions can be placed directly on the views, and in others it is best to place them outside the outline, running out extension lines to indicate the distances measured, as in Fig. 51. In dimensioning circles the diameters rather than the radii should be given, see view B, Fig. 51. Care should be taken not to crowd the dimensions. It is well to place those relating to a given part of an object in groups. Overall dimensions are desirable in most cases, view A, Fig. 51, in order to show at a glance the size of stock required to produce the piece, or the distance between two points, without having to add together the smaller dimensions.

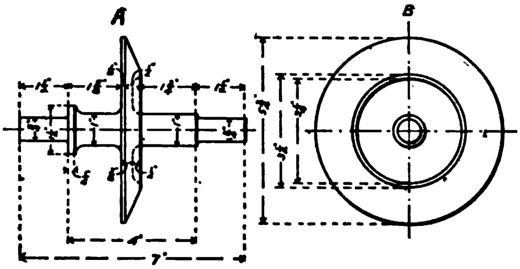


Fig. 51

When placing figures on horizontal dimension lines, the position of the figure to be easily read should be with its base toward the bottom of the board, and on vertical lines, toward the right hand edge. Fig. 52 shows positions on lines at other angles.

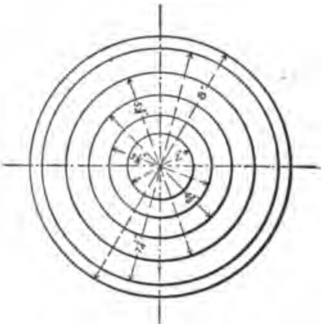


Fig. 52

26. Center Lines: Figs. 50, 51, and 52 convey some idea of the use of center lines. In pencil work these are the first lines drawn after the border lines and the views are constructed around them. They act as a sort of foundation on which to make a drawing, show the relation and symmetry of views, etc.

27. Lettering: An alphabet for practical work is given in Fig. 53. The large letters and figures are used for titles on drawings and when so employed should be made $\frac{1}{4}$ in. high and the lines forming them, $\frac{1}{32}$ in. thick. The smaller printing is used for dimensions, notes, etc., on drawings, and should be made with a single stroke of the pen, being $\frac{1}{16}$ in. high.

ABCDEFGHIJKLMNOPQRSTUVWXYZ&
1234567890
ABCDEFGHIJKLMNOPQRSTUVWXYZ&
1234567890

Fig. 53

All lettering should be neatly and plainly made with the conical pointed pencil or printing pen. If the letters of a word are placed fairly close together and considerable space left between words, capitals and punctuation used properly and the form, size and spacing made as uniform as possible, the lettering will appear neat and clear.

Notes are used to aid the reading of a drawing by designating materials to be used for certain parts and in explanation of conventions, etc. They should be clear and concise and provided with an arrow or other indicator to show to what they refer.

The title should be placed at the lower right hand corner of the sheet inside the border lines and should furnish information about the drawing for future reference. Some of the data usually given includes the name and location of the piece, scale, date, name of draftsman and number of drawing.

28. Tracing: The following remarks apply whether the inking is done directly on the paper or on tracing cloth. In either case the pencil work should be completed first. If a tracing is to be made the tracing cloth is stretched smoothly over the paper, fastened with tacks and the surface chalked as described in Art. 16.

Economy of time and better results are obtained if this part of the work is done in a systematic way. The following method is generally used. First ink all small arcs and circles, then larger arcs and circles, horizontal lines (starting at the top of the sheet), vertical lines (starting at the left hand side), lines at other angles, and irregular curves. Shade lines are now made heavy in the same order, putting the extra width on the outside. It may result in some saving of time to shade arcs and circles when they are first inked. Complete the work by inking the center and dimension lines, figures and lettering, section lines and title.

It is advisable, if possible, to complete a tracing the same day it is started, as the moisture in the air causes the surface to wrinkle. If the drawing is large, a small portion may be completed at a time and the cloth stretched smoothly when the work is taken up again. If this is carefully done, the extra space, due to the expansion of the cloth, can be made to come between views or where it will not disturb the accuracy of the drawing.

Good tracing cloth will allow one or two erasures to be made with the ink eraser. If more are attempted at the same place or the work done with a knife, a hole in the cloth is the usual result.

When completed, the tracing should be trimmed to the same size as the drawing. Avoid folding the tracing, bending it sharply, or allowing water to come in contact with it as these are the causes of white lines and spots that show on the blue prints.

29. Blue Printing: Blue prints are made as follows. The tracing is placed in a printing frame with the inked side against the glass. Then, using a dim light, a piece of blue print paper is placed with its sensitized side in contact with the tracing. The frame is placed so that a bright light, such as sunlight, can pass through the tracing to the paper.

After-the proper exposure, which varies according to the class of paper used, the paper is washed in clean water for from three to five minutes. If not washed sufficiently it is liable to fade. The prints are now hung up to dry and finally trimmed to the same size as the tracing. This method produces white lines on a blue background.

To produce blue lines on a white background the following method is ordinarily employed. First a negative is made in the same manner as the blue print just described with the exception that, in this case, the tracing is reversed so as to place the ink lines next to the sensitized surface of the paper, as this produces sharper lines on the negative. Instead of using ordinary blue print paper for this purpose, a special paper giving white lines on a brown background is employed. After printing, this requires washing in cleanwater, immersing in a fixing salt bath and again washing in water after which the negative is hung up to dry: Second a blue print is made from the negative in the ordinary manner except that the brown surface of the negative is placed next to the sensitized surface of the blue print paper. It takes considerably longer to print from the negative than from a tracing.

Blue prints generally shrink enough to be appreciably smaller than the tracing from which they were made. On account of this fact, scaling dimensions from blue prints should be avoided if possible.

30. Sketching: At times it is necessary to make a *free-hand* drawing (known as a *sketch*) of an object, from which a *working* drawing can be constructed at a later date. It should contain all the dimensions and information necessary, since a second opportunity to see the original object is not always available.

EXAMINATION QUESTIONS

(1) Why is it not advisable to draw vertical lines by using the T square with the head against the lower edge of the board?

(2) Why is it undesirable to allow ink to collect on the outside of the blade points of a ruling pen?

(3) What will be the probable result if the ruling pen is pressed against the T square when drawing a line?

(4) Why should the ruling pen be held in a vertical position when drawing lines?

(5) Why not use both edges of the T square blade when drawing horizontal lines?

(6) State as many operations as possible, in performing which the 30°×60° triangle could be used to advantage in making a drawing.

(7) Why is it bad practice to use a blunt pencil point?

(8) What are the effects of pressing heavily on a pencil when drawing lines?

(9) What is the difference between the "Engineer's" and the "Architect's" scales?

(10) What scale would you use, if required to draw an object measuring 13 ft. 4 in. overall, on a sheet of paper 23 in. long?

(11) If you were drawing a plan of a railroad yard 1,000 ft. long and wished to place it on a standard size sheet 23 in. long inside the border line, what scale would you use?

(12) If, in adjusting the lead in the compasses, the flat of the wedge is not placed at right angles to a line joining the points, how will it affect the lines drawn with this instrument?

(13) What is the object of bending the legs of the compasses to make the pencil, pen and needle points stand perpendicular to the paper?

(14) If the blades of the ruling pen are not placed parallel to the edge of the guide, what is the result when a line is drawn?

(15) If a circle can be drawn through any three points not in the same straight line, what is the name of the unbroken line that could be drawn through four points, through which neither a straight line nor a circle can be drawn?

(16) Name some angles which it is necessary to lay off with the protractor.

(17) After opening a roll of blue print paper and taking out the amount needed, what would you do with the balance of the paper to preserve it?

(18) When working at the drawing board, from what direction should the light come?

(19) If you were required to make a drawing of a long piece such as a piston rod with the piston head on one end and the cross head on the other, using a comparatively small sheet, what

method would you use in order to draw the ends sufficiently large to show details?

(20) Of two drawings made exactly to scale, one with and one without dimensions, which would it be proper to give to a workman? State your reasons.

(21) What is the object of giving overall dimensions?

(22) What is the difference between a working drawing and a sketch?

EXERCISES

31. Sheet 1. Divide the surface inside the border lines into twelve equal spaces by a horizontal and vertical lines, as in Fig. 54. Make these lines light so that they can be easily *erased* when the drawing is finished. Draw a square 3½ in. on a side in each of the spaces except No. 12, placing it in the exact center so that the margins on opposite sides will be alike. Now to afford practice in using the instruments, fill in the several spaces with the lines and figures described in the following paragraphs. Finish this drawing with ink lines.

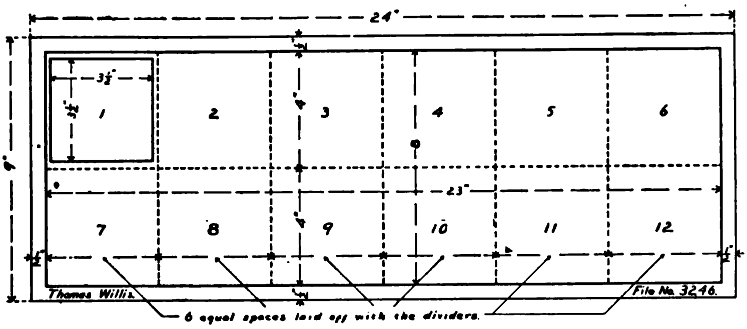


Fig. 54

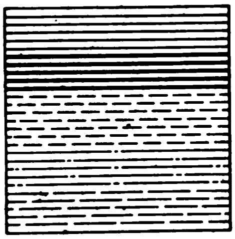


Fig. 55

Space 1. (Fig. 55) Divide the left hand vertical edge of the square into ⅛ in. divisions with the scale. Draw horizontal lines through these divisions making the first five unshaded full lines, the next five shade lines, then seven dotted lines, five center lines, and five dimension lines.

Space 2. Draw vertical lines in a manner similar to the horizontal lines in space 1.

Space 3. (Fig. 56) Divide into squares by horizontal and vertical lines ¼ in. apart. Then draw 45 deg. lines in both directions through the corners of the little squares. Careful accurate work is necessary to make the lines meet properly.

Space 4. (Fig. 57) Find the center of the square by drawing 45 deg. lines through the corners. Set the compasses to a radius of 1¾ in. and draw a circle around the above center. This circle should just meet, or be tangent to, each side of the square. Now draw a series of circles inside the first with radii of 1½ in., 1¼ in., 1 in., etc.

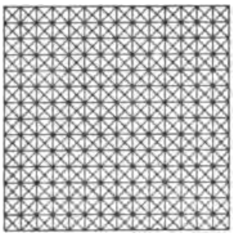


Fig. 56

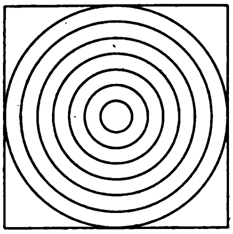


Fig. 57

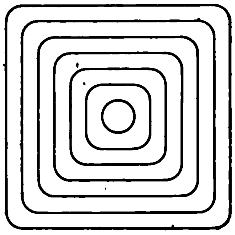


Fig. 58

Space 5. Draw dotted circles otherwise the same as space 4.

Space 6. (Fig. 58) Draw a series of squares inside each other with their sides ¼ in. apart, but instead of making the corners sharp, join the sides with arcs of circles taking a radius of ¼ in.

Spaces 7, 8, 9, 10, 11. Draw the pieces shown in Figs. 59, 60,

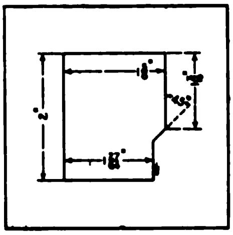


Fig. 59

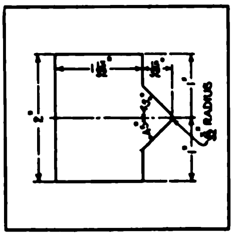


Fig. 60

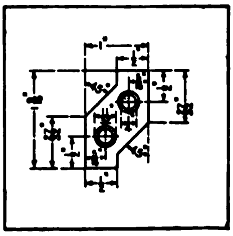


Fig. 61

61, 62, and 63 using the full size scale. Add center lines, dimensions, etc.

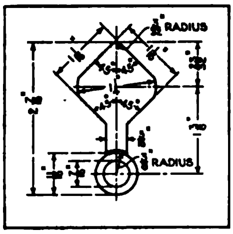


Fig. 62

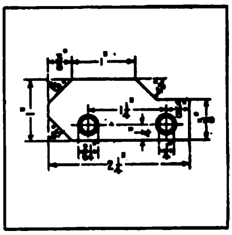


Fig. 63

Space 12. Place a title in this space of the form and wording shown in Fig. 64. Use this title on sheets 1, 2, and 3.

GENERAL METHODS
SCALE: FULL SIZE. MARCH 1, 1910.
DRAWING No 1

Fig. 64

32. Sheet 2. Divide this sheet into twelve spaces as was done with sheet 1. The sections shown in Fig. 47 are to be drawn on this sheet, the name of each section being printed beneath it as shown. In space 11, draw another cast iron section, using the 30°×60° triangle, the lines making an angle of 30 deg. with the *horizontal* (see Fig. 7). Owing to the larger area to be sectioned, the distance between the section lines should be about three times that shown. Place the title* in space 12. Finish with ink lines.

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*As this drawing is, of course, not to scale the words "scale: full size" should be omitted.

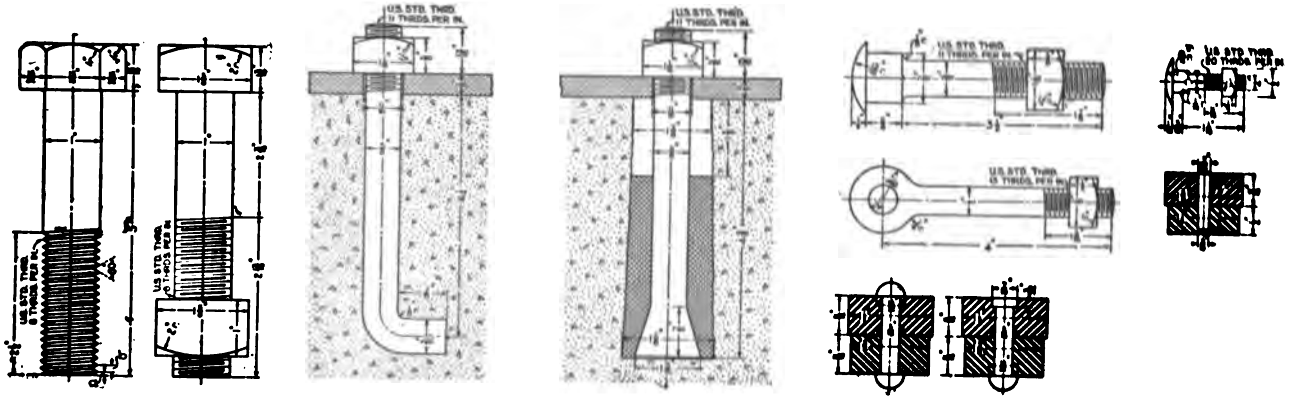

Fig. 66

33. Sheet 3. Draw the bolts, etc., illustrated in Fig. 65, using the full size scale. Arrange them so that the sheet will present a neat symmetrical appearance, remembering to reserve sufficient space in the lower right hand corner for the title. The value of *a* for the bolt shown at the left hand side of the sheet, should be *one-half* that of *b**. Finish with ink lines.

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*The letters *a* and *b*, also the value (60°) for the angle of the threads, should not appear on the drawing.

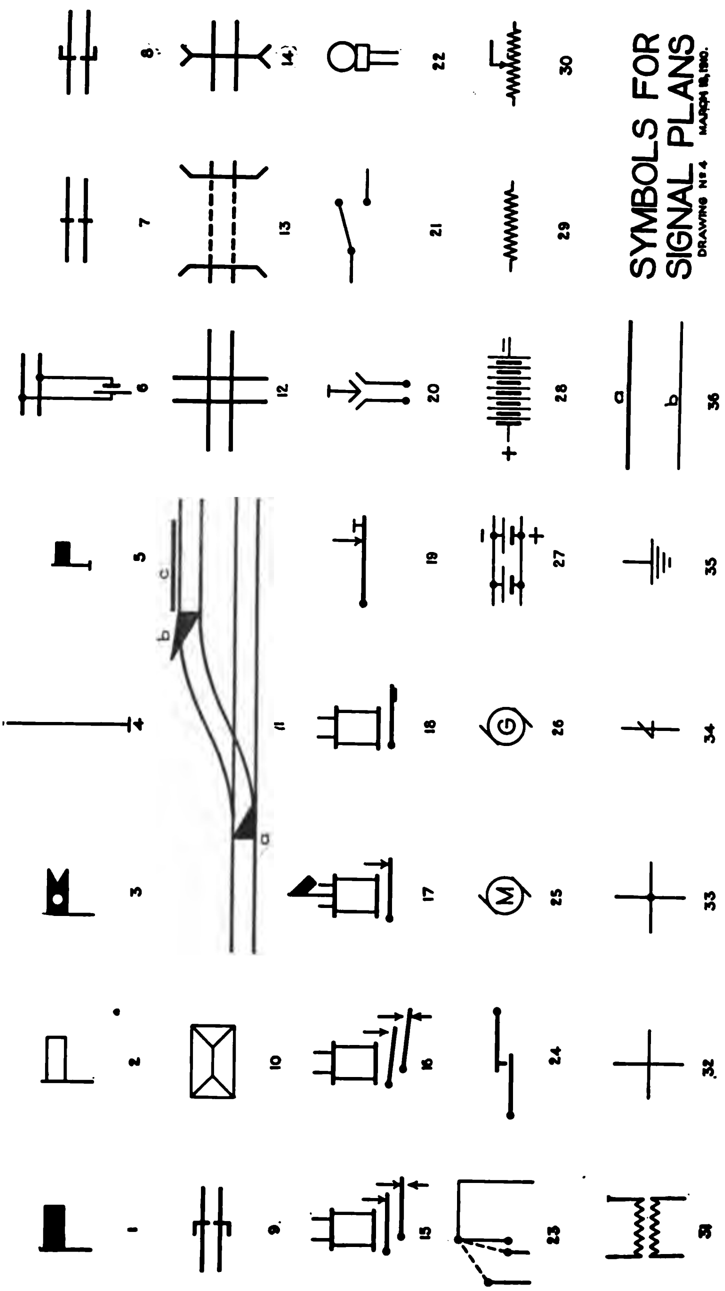
SYMBOLS FOR SIGNAL PLANS
DRAWING No. 4 MARCH 15, 1910.
a
b
c
M
G
+
-

Fig. 66

34. Sheet 4. Divide this sheet into ten equal parts by vertical lines and then into four equal parts by horizontal lines so as to make forty equal spaces. Reserve two spaces in the lower right hand corner for the title and draw in the others the symbols shown in Fig. 66. These symbols are a selection from those used in drawing diagrams and plans for signal work. They are employed as a standard short method of indicating different instruments, etc. No. 1 indicates a non-automatic interlocked two position home signal of the semaphore type; No. 2 an automatic two position home signal; No. 3 a distant power signal at mechanical interlocking plants (no track circuit); No. 4 a ground pole; No. 5 a one arm dwarf signal; No. 6 the location of a track battery; No. 7 insulated joints (track circuits in both directions); No. 8 insulated joints (track circuit on the right, no circuit on the left); No. 9 the reverse of No. 8; No. 10 an interlocking tower; No. 11*a* a switch; No. 11*b* a derail; No. 11*c* a detector bar*; No. 12 a railway crossing; No. 13 a tunnel; No. 14 a highway crossing; No. 15 a direct current relay, energized; No. 16 the same de-energized; No. 17 an indicator relay, semaphore type; No. 18 an electric lock; No. 19 a spring hand key, normally closed; No. 20 a floor push; No. 21 a circuit controller on lever of mechanical interlocker, normally open; No. 22 a bell; No. 23 a circuit controller on signal; No. 24 a track instrument contact, normally open; No. 25 a motor; No. 26 a generator; No. 27 battery cells in multiple; No. 28 battery cells in series; No. 29 a fixed resistance; No. 30 a variable resistance; No. 31 a transformer; No. 32 that wires cross but do not join; No. 33 wires join; No. 34 a lightning arrester; No. 35 a ground; No. 36*a* a common wire; and No. 36*b* an ordinary wire**.

The cut shows the correct sizes of the different symbols as related to each other, and the same relation should be observed in making the drawing. The symbols should be drawn three times the size shown in the illustration†. Under each symbol place its name in small lettering (Art. 27) instead of the number. Place the title in the space reserved and finish the sheet with ink lines.

The titles on sheets 4 to 7 inclusive should be as shown in the illustrations.

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*Note that No. 11 uses three spaces.

**See appendix for complete table of symbols.

†On other drawings the symbols should be of a size proportional to the size and character of the drawing.

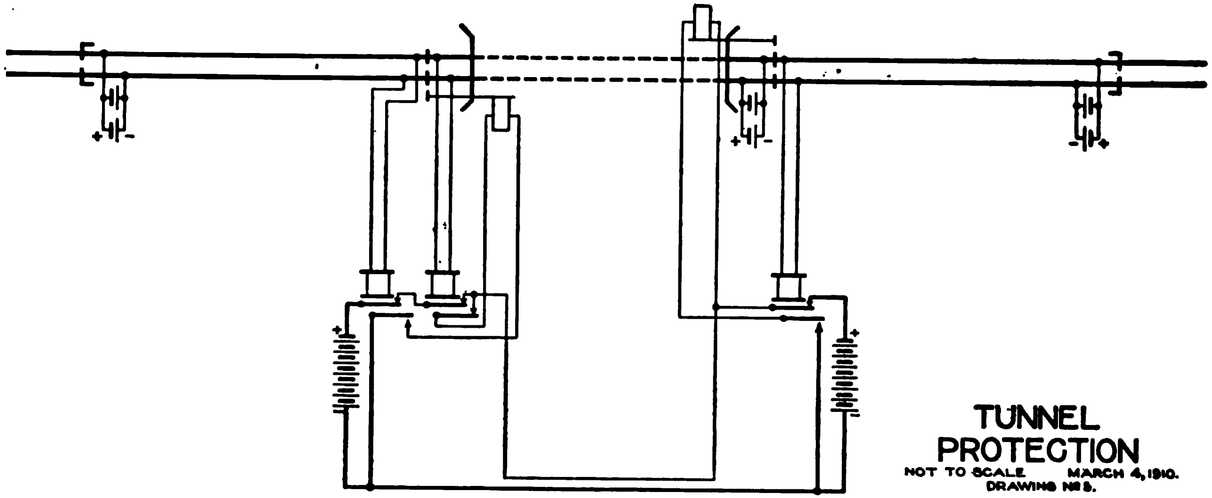

Fig. 67

35. Sheet 5. On this sheet draw the *circuit plan* shown in Fig. 67, making it twice as large as illustrated. It will be noted that this and the two following sheets make use of the symbols shown in Fig. 66. Some of the symbols are slightly modified from the ones shown in that illustration. For example, the track batteries show two cells instead of one, the semaphore blades are placed on top of the ground pole, and, where relay No. 15 is used, a contact point is left off. Finish with ink lines.

When drawing circuit plans, as few *angles* as possible should be made in the wires and they should cross each other *only* when necessary. This simplifies the reading of the plan when completed and furthermore it presents a neater appearance. Running the wires obliquely should also be avoided. Wires running parallel to one another in a group, should ordinarily be spaced ⅛ in. apart, in order to provide the necessary space in which to print the wire numbers or letters.

In some cases the arrangement of the wiring makes it desirable to run the controlling wires into the *sides* instead of the *top* of relays, indicators, etc.

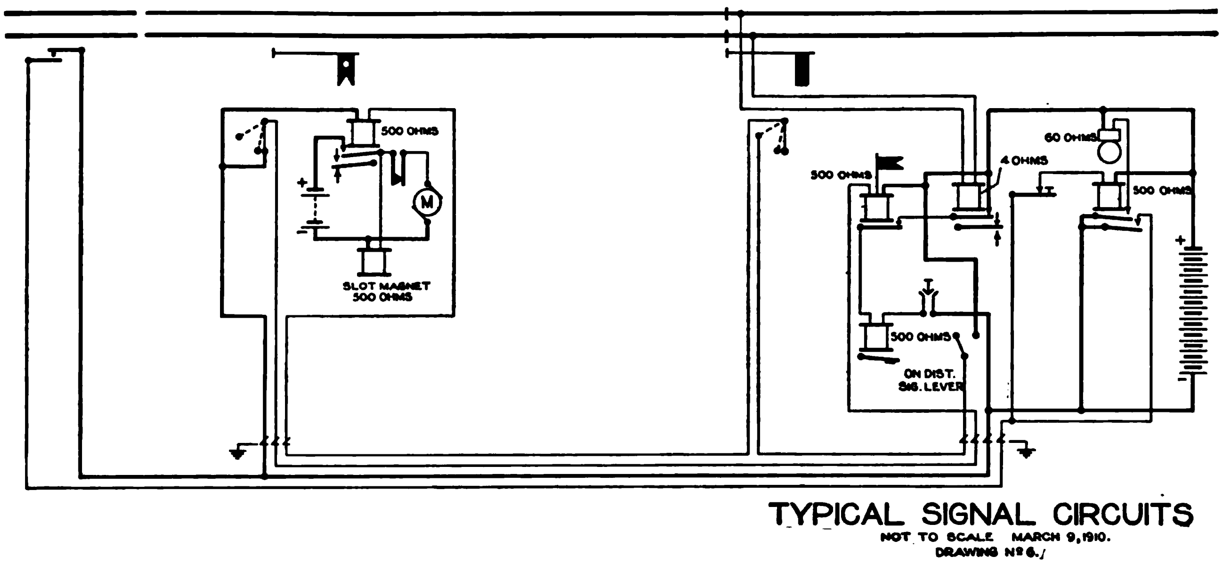
500 OHMS
SLOT MAGNET
500 OHMS
M
+
-
60 OHMS
4 OHMS
500 OHMS
500 OHMS
500 OHMS
ON DIST.
SIG. LEVER
+
-
TYPICAL SIGNAL CIRCUITS
NOT TO SCALE MARCH 9, 1910.
DRAWING Nº 6./

Fig. 68

36. **Sheet 6.** Draw the *circuit plan,* Fig. 68, noting instructions with regard to sheet 5. Some points of difference in the symbols are that No. 17 (Fig. 66) has a blade of the distant signal type and No. 18 is shown de-energized. Finish with ink lines.

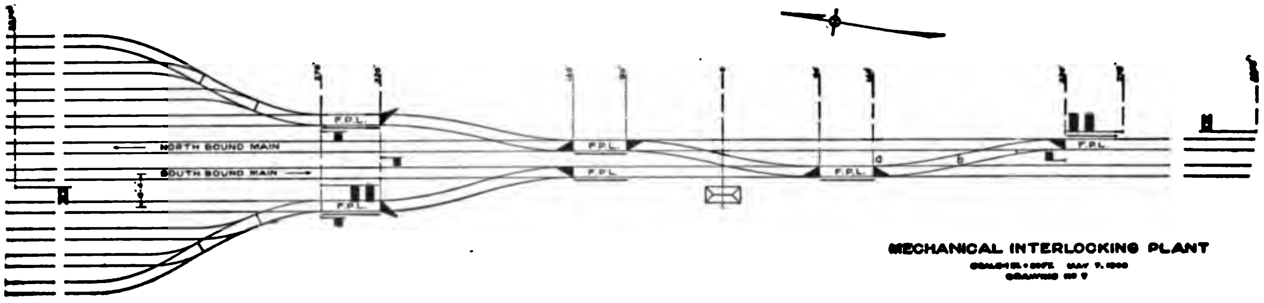
NORTH BOUND MAIN
SOUTH BOUND MAIN
F.P.L.
F.P.L.
F.P.L.
F.P.L.
F.P.L.
F.P.L.
F.P.L.
a
b
MECHANICAL INTERLOCKING PLANT

Fig. 69

37. Sheet 7. Draw the track plan shown in Fig. 69.

To afford practice in the use of the "Engineer's" scale, the signals, switches, etc., should be located in their correct positions according to the dimensions given for each, a scale of 1 in. = 50 ft. being used. The dimensions are laid off from the position marked 0. The distance from center to center of tracks is twelve feet* and the gauge** four feet eight and one-half inches. The measurement from point of switch (*a*) to point of frog (*b*) is 77′ 0″ for a No. 10 frog as used in this layout, with the exception of the turnouts for the yard tracks. Indicate the dimensions plainly after the method illustrated. Finish with ink lines.

---

*When a high signal occurs between tracks the track centers must be spaced at least 15 ft. 4 in.

**4 ft. 8½ in. from inside to inside of rail heads.

GEOMETRICAL PROBLEMS

DEFINITIONS

38. An **object** is generally described by three dimensions, length, breadth, and thickness. The boundary of such an object or solid is a surface or combination of surfaces.

39. A **surface** is generally described by two dimensions, length and breadth. A surface has no thickness. The boundary of a surface is a line or combination of lines.

40. A **line** has only one dimension, length. A line is limited or located by two or more points.

41. A point has position only and no dimension.

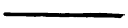

Fig. 70

42. A **straight line** is one which has the same direction throughout its entire length. Fig. 70.

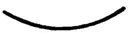

Fig. 71

43. A **curved line** is one no part of which is straight. Fig. 71.

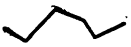

Fig. 72

44. A **broken line** is a series of different successive straight lines. Fig. 72.

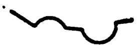

Fig. 73

45. A **mixed line** is one composed of straight and curved lines. Fig. 73.

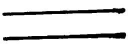

Fig. 74

46. Lines are **parallel** when they are everywhere at the same distance from each other and have the same direction. Fig. 74.

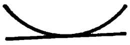

Fig. 75

47. A straight line and a curved line, or two curved lines, are **tangent** when they have one point in common, but do not intersect or cut each other on being prolonged. Fig. 75.

48. Points, lines, or surfaces are said to **coincide** when they occupy exactly the same position.

49. An **angle** is the difference in direction of two straight lines that meet or tend to meet.

The lines are called the **sides** of the angle and the point where they meet, the **vertex.**

When the sides of an angle do not meet within the boundary of a drawing they are sometimes called **inclined lines.**

50. If the circumference of any circle* is divided into 360 equal parts and straight lines drawn to the center through two adjacent points, the small angle between the lines is said to have a magnitude of **one degree.** Thus an angle may be measured by means of an arc of a circle described from the vertex as a center and included between its sides. The degree is subdivided into 60 equal parts called **minutes,** and the minute into 60 equal parts called **seconds.****

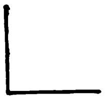

Fig. 76

51. An angle is called a **right angle** or angle of 90 deg. when the included arc is ¼ of the entire circumference. Fig. 76. When this is increased to 180 deg. or ½ the circumference, it is called a **straight angle.**

52. An **oblique angle** is one which is not a right or a straight angle.

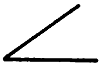

Fig. 77

53. An **acute angle** is an angle less than a right angle. Fig. 77.

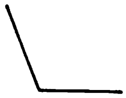

Fig. 78

54. An **obtuse angle** is one greater than a right angle. Fig. 78.

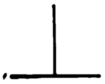

Fig. 79

55. Two straight lines are said to be **perpendicular** to each other when they make an angle of 90 deg. Fig. 79.

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*See Arts. 68-69.

**Degrees are indicated by (°), minutes by (′), seconds by (″). Thus 3 degrees, 15 minutes, and 30 seconds is written 3° 15′ 30″.

56. A **plane** or **plane surface** is one upon which a straight line can be drawn in any direction.

57. A **curved surface** is one no part of which is plane.

58. A line or plane is said to be **horizontal** when it is parallel to a level surface such as the surface of smooth water. On a drawing, lines parallel to the top or bottom of the sheet are considered as horizontal.

59. A line or plane is said to be **vertical** when it is perpendicular to a horizontal line or plane.

60. **Oblique** means neither horizontal nor vertical.

61. **Describe** generally means to draw.

62. **Circumscribe** means to draw around.

63. **Inscribe** means to draw inside.

64. When a figure is described or circumscribed about or inscribed within another, it is generally implied that, as in the case of a polygon (see Art. 96) about a circle, each side shall be tangent to the circle or, in the case of a circle about a polygon, that the circumference of the circle shall pass through all the vertices of the polygon.

65. To **bisect** is to divide into two equal parts.

66. To **construct** generally means to draw by a method of building up starting with certain given quantities and applying known rules.

67. A plane figure is part of a plane surface bounded on all sides by lines. The boundary of a closed plane figure is called its **perimeter**.

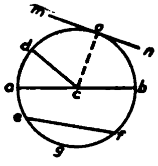


Fig. 80

68. A **circle** is a plane figure bounded by a curved line all points of which are at the same distance from a point within called the **center.** Fig. 80.

69. The boundary line of a circle is called the **circumference.**

70. A **diameter** of a circle is a straight line through the center joining opposite points in the circumference, as (ab), Fig. 80.

71. The **radius** of a circle is the distance from the center to the circumference, as (cd), Fig. 80.

72. A **semicircle** is half a circle, made by dividing a circle into two equal parts with a diameter, as (aboda), Fig. 80.

73. An **arc** of a circle is any part of the circumference, as (ad), Fig. 80.

74. A **chord** is any straight line whose ends are in the circumference of a circle, as (ef), Fig. 80.

75. A **segment** is a part of a circle bounded by an arc and a chord, as (efge), Fig. 80.

76. A **sector** is a part of a circle bounded by two radii and an arc, as (acda), Fig. 80.

77. A **tangent** to a circle is a straight line which meets the circumference but does not cut it on being prolonged, as (mn). The point of meeting (o) is called the **point of tangency.**

78. A **triangle** is a plane figure bounded by three straight lines called the **sides.** The angles they form are called the **angles of the triangle** and the vertices of these angles, the **vertices of the triangle.**

A triangle is called:

79. A **scalene triangle** when no two sides are equal.

80. An **isosceles triangle** when two sides are equal. Fig. 81.

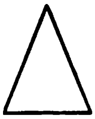


FIG. 81

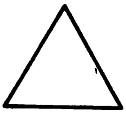


Fig. 82

81. An **equilateral triangle** when the three sides are equal. Fig. 82.

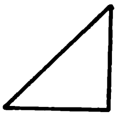


Fig. 83

82. A **right triangle** when one of the angles is a right angle. Fig. 83.

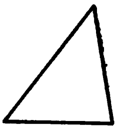


Fig. 84

83. An **acute triangle** when all of the angles are acute. Fig. 84.

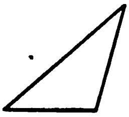


Fig. 85

84. An **obtuse triangle** when one of the angles is obtuse. Fig. 85.

85. The **base** of a triangle is the side on which it is supposed to stand. The equal sides of an isosceles triangle are known as the **legs** and the third side as the base.

In any other triangle any side may be the base.

86. The vertex of the angle opposite the base is called the **vertex** of the triangle.

87. The **altitude** is the perpendicular distance from the vertex to the base.

88. A **quadrilateral** is a plane figure bounded by four straight lines called the sides.

A quadrilateral is called:

89. A **parallelogram** when the opposite sides are parallel.

90. A **trapezium** when no two sides are parallel. Fig. 86.

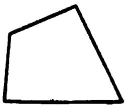


Fig. 86

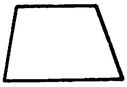


Fig. 87

91. A **trapezoid** when two sides only are parallel. Fig. 87.

Fig. 88

92. A **rectangle** when the angles are right angles. Fig. 88.

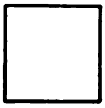


Fig. 89

93. A **square** when the angles are right angles and the sides equal. Fig. 89.

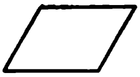


Fig. 90

94. A **rhomboid** when the opposite sides are parallel and the angles oblique. Fig. 90.

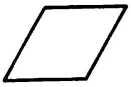


Fig. 91

95. A **rhombus** when the opposite sides are parallel and of equal length and the angles oblique. Fig. 91.

96. A **polygon** is a plane figure bounded by straight lines. When the sides are equal it is called **equilateral** and when the angles are equal, **equiangular.**

97. A **regular polygon** is one which is both equilateral and equiangular.

Polygons are named according to the number of sides as follows:

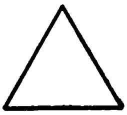


Fig. 92

98. A **triangle,** one with three sides, Fig. 92.

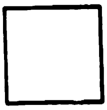
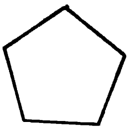


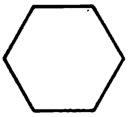
Fig. 93

99. A **quadrilateral,** one with four sides, Fig. 93.



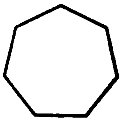
100. A **pentagon,** one with five sides, Fig. 94.

Fig. 94



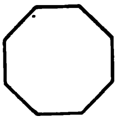
101. A **hexagon,** one with six sides, Fig. 95.

Fig. 95



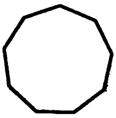
102. A **heptagon,** one with·seven sides, Fig. 96.

Fig. 96



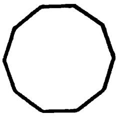
103. An **octagon,** one with eight sides, Fig. 97.

Fig. 97



104. A **nonagon,** one with nine sides, Fig. 98.

Fig. 98



105. A **decagon,** one with ten sides, Fig. 99.

Fig. 99

The figures show regular polygons.

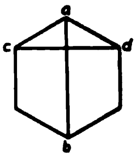


Fig. 100

106. A **diagonal** is a straight line in any polygon which joins vertices not adjacent. In a regular polygon diagonals are called *long* when they pass through the center, as (ab), Fig. 100, and *short* when they extend between parallel sides, as (cd).

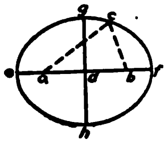


Fig. 101

107. An **ellipse** is a plane figure bounded by a line generated by a point which moves so that the sum of its distances, (ac + bc), Fig. 101, from two fixed points, (a) and (b), is constant. The two fixed points are called the **foci.** The point, (d), midway between the foci is called the **center.** The longest diameter, (ef), that can be drawn in an ellipse is called the **major axis** and passes through the foci. The shortest diameter, (gh), is called the **minor axis** and is at right angles to the major axis.

PROBLEMS OF CONSTRUCTION

108. Sheet 8. Divide the sheet into twelve equal spaces by a horizontal and vertical lines. Reserve the space in the lower right hand corner for the title. The title should be the same as shown in Fig. 64 with the exception that the words "Geometrical Problems" should be substituted for "General Methods."

Draw problems in Arts. 109, 110, 111, 112, 113, 114, 115, 118, 119, 120, 122. (Note that problems in Arts. 116, 117 and 121 are not to be drawn. However, they should be studied carefully as soon as the problems preceding them have been drawn.) Use fine distinct pencil lines. Do not ink this drawing. Make construction lines dotted and omit the reference letters. Draw the problems twice the size shown in the illustrations. Perform the work as accurately as possible.

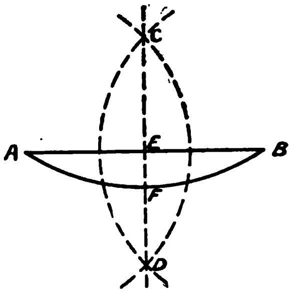


Fig. 102

109. To bisect a straight line AB, or an arc of a circle AFB. Fig. 102.

With the ends A and B as centers and a radius greater than ½-AB, describe arcs intersecting at C and D. Draw a straight line through C and D. The line CD is perpendicular to AB and bisects it at E, and the arc AFB at F.

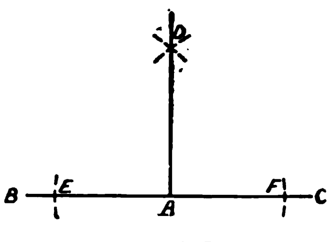


Fig. 103

110. To draw a perpendicular to a straight line BC from a given point A in that line. Fig. 103.

With A as a center and any radius, describe arcs cutting the line at E and F. With E and F as centers and a radius greater than ½-EF, describe arcs intersecting at D. Connect D and A with a straight line which is the required perpendicular.

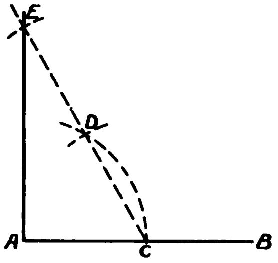


Fig. 104

111. To draw a perpendicular to a straight line AB from a point A at or near its end. Fig. 104.

With A as a center and a radius AC, describe an arc DC. With C as a center and the same radius, describe an arc cutting DC at D. Draw a straight line through C and D prolonging it beyond D. With D as a center and the same radius as above, describe an arc cutting this line at E. The straight line joining E and A is the required perpendicular.

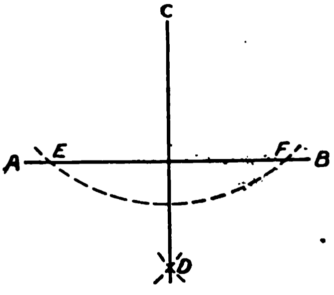


Fig. 105

112. To draw a perpendicular to a straight line AB from any point C outside that line. Fig. 105.

With C as a center and a suitable radius, describe an arc cutting AB at E and F. With E and F as centers and a radius greater than $\frac{1}{2}$-EF, describe arcs intersecting at D. Join C and D with a straight line which is the required perpendicular.

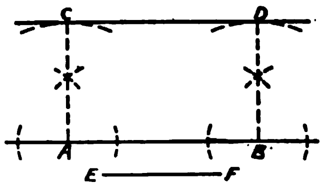


Fig. 106

113. To draw a straight line parallel to a given line AB at a given distance EF from it. Fig. 106.

At any two points on the given line as A and B erect perpendiculars. (Art. 110.) With A and B as centers and a radius equal to EF, describe arcs cutting these perpendiculars at C and D. Connect C and D by a straight line which is the line required.

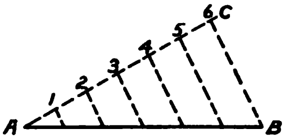


Fig. 107

114. To divide a straight line AB into any number of equal parts, for example six. Fig. 107.

Draw the line AC at any angle to AB. From A on AC set off any distance six times. Connect B with the sixth point by a straight line. Through the other points draw lines parallel to B6. (Art. 113.) These lines divide AB as required.

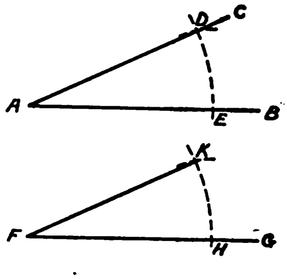


Fig. 108

115. Upon a straight line, FG, to draw an angle equal to a given angle CAB. Fig. 108.

With A as a center and any radius describe an arc cutting the sides of CAB at D and E. With F as a center and the same radius describe an arc HK cutting FG at H. With H as a center and a radius equal to DE, describe an arc cutting HK at K. Connect K and F with a straight line and KFG is the required angle.

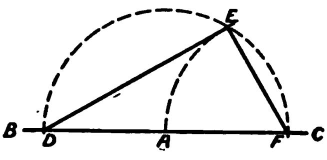


Fig. 109

116. To draw angles of 30 deg. and 60 deg. on the straight line BC. Fig. 109.

With any point A on the line as a center and with any radius, describe a semicircle DEF cutting BC at D and F. With F as a center and a radius equal to AF, describe an arc cutting the semicircle at E. Connect D and E and E and F with straight lines. FDE is the required angle of 30 deg. and DFE the angle of 60 deg.

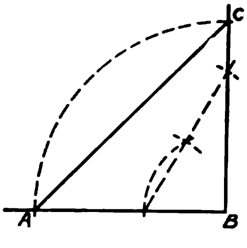


Fig. 110

117. To draw an angle of 45 deg. on the straight line AB. Fig. 110.

At B erect a perpendicular. (Art. 111.) With B as a center and any radius describe an arc cutting AB at A and the perpendicular at C. Join A and C with a straight line. The angle BAC is the required angle of 45 deg.

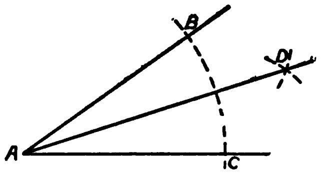


Fig. 111

118. To bisect a given angle BAC. Fig. 111.

With A as a center and any radius, describe an arc cutting the sides of the angle at B and C. With B and C as centers and a suitable radius, describe arcs intersecting at D. Connect A and D with a straight line which divides the angle into two equal parts.

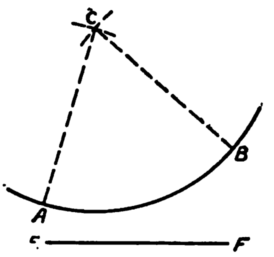


Fig. 112

119. Through two given points, A and B, to describe an arc of a circle with a given radius EF. Fig. 112.

With A and B as centers and a radius equal to EF, describe arcs intersecting at C. With C as a center and the same radius, describe the required arc through A and B.

120. To find the center of a circle or of an arc of a circle ABC. Fig. 113.

Take three points on the circumference or arc well apart as A, B and C. Bisect and draw perpendiculars to the arcs AB and BC. (Art. 109.) Prolong the perpendiculars to intersect at D which is the required center.

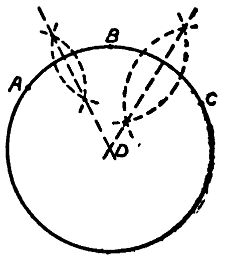


Fig. 113

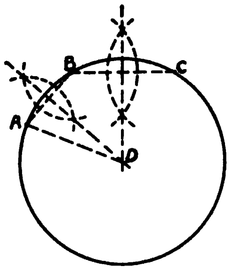


Fig. 114

121. To draw a circle through three given points A, B and C not in the same straight line. Fig. 114.

Connect A to B and B to C with straight lines. Bisect and draw perpendiculars to AB and BC. (Art. 109.) With the point of intersection, D, of the perpendiculars as a center and a radius equal to DA draw the required circle through A, B and C.

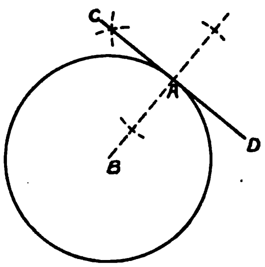


FIG. 115

122. To draw a tangent to a circle at a given point A in the circumference. Fig. 115.

Draw the radius BA and erect a perpendicular CD at A, (Art. 110), which is the required tangent.

123. **Sheet 9.** Draw problems in Arts. 124, 127, 131, 134, 135, 139, 142, 147, 150,151 and 152 following the instructions given for sheet 8. The problem in Art. 152 should be drawn the same size as shown in the illustration in order that the ellipse may be drawn complete.

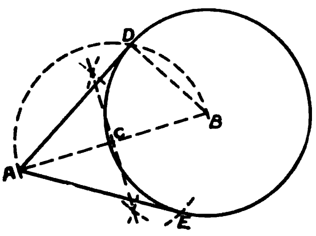


FIG. 116

124. To draw tangents to a circle from a given point, A, outside its circumference. Fig. 116.

Connect A with the center B of the circle by a straight line. Bisect AB at C. (Art. 109.) With C as a center and a radius equal to CB, describe a circle cutting the circumference of the given circle at D and E. Join A to D and A to E with straight lines which are the required tangents.

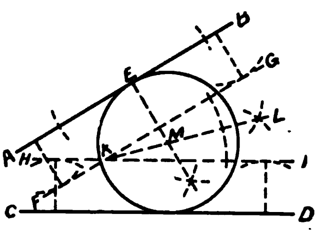


FIG. 117

125. To draw a circle tangent to two given lines AB and CD inclined to each other, one tangential point E being given. Fig. 117.

Draw the straight line FG parallel to AB and at any distance from it, (Art. 113), and HI parallel to and at the same distance from CD. These lines intersect at K. Draw the line KL bisecting the angle GKI. (Art. 118.) At E erect a perpendicular to AB (Art. 110), intersecting KL at M. With M as a center and a radius equal to ME, draw the required circle.

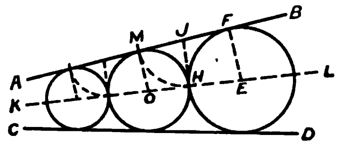


Fig. 118

126. To draw any number of circles tangent to each other and to two given inclined lines AB and CD. Fig. 118.

Bisect the inclination of the lines AB and CD by the line KL as in Art. 125. From point E in this line draw the perpendicular EF to AB. (Art. 112.) With E as a center and a radius equal to EF, draw a circle touching AB and CD and cutting KL at H. At H draw HJ perpendicular to KL. (Art. 110.) With J as a center and a radius JH, describe an arc cutting AB at M. At M erect a perpendicular to AB cutting KL at O, which is the required center for the next circle. Repeat the process as many times as there are circles required.

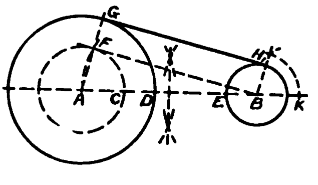


Fig. 119

127. To draw a tangent to two given circles with centers at A and B. Fig. 119.

Connect A and B with a straight line. Set off CD equal to EB. With A as a center and a radius AC, describe a circle and draw BF tangent to it from point B. (Art. 124.) Through the point of tangency, F, draw the straight line AF and prolong it to G. Make the angle HBK equal to GAD. (Art. 115.) Connect H and G with a straight line which is the required tangent.

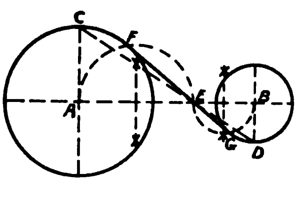


Fig. 120

128. To draw a tangent to two given circles which shall pass between them. Fig. 120.

Connect the centers A and B of the given circles with a straight line. At A and B erect perpendiculars to A B, (Art. 110), cutting the circumferences of the given circles at C and D. Join C and D with a straight line intersecting AB at E. Find F and G, at which points lines drawn from E would be tangent to the given circles. (Art. 124.) Connect points F and G with a straight line which is the required tangent.

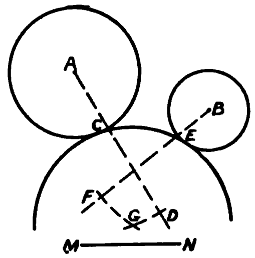


Fig. 121

129. **To draw a circle with a given radius, MN, tangent to two given circles with centers at A and B.** Fig. 121.

Through A and B draw straight lines of indefinite length. Set off CD and EF each equal to MN. With A as a center and a radius AD, describe an arc and with B as a center and radius BF, another arc intersecting the first at G. The point of intersection, G, is the center of the required circle which may be drawn with a radius equal to MN.

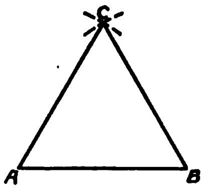


Fig. 122

130. **To construct an equilateral triangle having given a side AB.** Fig. 122.

With the ends A and B as centers and a radius equal to AB, describe arcs intersecting at C. Connect A to C and C to B with straight lines and ABC is the required triangle.

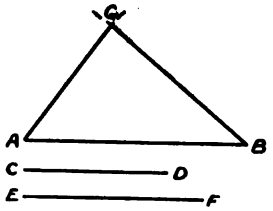


Fig. 123

131. **To construct a triangle of unequal sides, the sides AB, CD and EF being given.** Fig. 123.

With A as a center and a radius equal to CD describe an arc. With B as a center and a radius equal to EF, describe another arc cutting the first at G. Join A to G and B to G with straight lines and ABG is the required triangle.

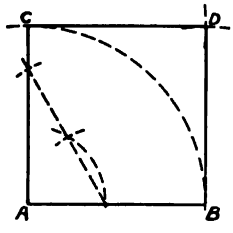


Fig. 124

132. **To construct a square on a given straight line AB.** Fig. 124.

At A erect a perpendicular to AB. (Art. 111.) With A as a center and a radius equal to AB, describe an arc cutting the perpendicular at C. With B and C as centers and the same radius, describe arcs intersecting at D. Connect C to D and D to B with straight lines and ABDC is the square required.

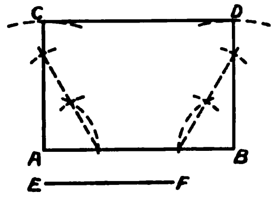


Fig. 125

133. To construct a rectangle with a given base AB and height EF. Fig. 125.

At A and B erect perpendiculars to AB. (Art. 111.) With A and B as centers and a radius equal to EF, describe arcs cutting these perpendiculars at C and D. Connect C and D with a straight line and ABDC is the required rectangle.

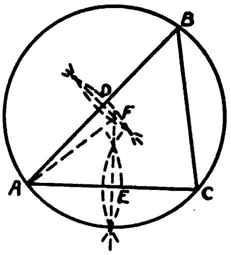


Fig. 126

134. To circumscribe a circle about a given triangle ABC. Fig. 126.

Bisect AB at D and AC at E erecting perpendiculars. (Art. 109.) With the point of intersection, F, of these perpendiculars as a center and a radius FA, draw the required circle.

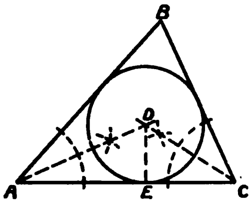


Fig. 127

135. To inscribe a circle within a given triangle ABC. Fig. 127.

Bisect two of the angles as BAC and ACB by the lines AD and CD. (Art. 118.) From the point of intersection, D, of these lines draw the perpendicular DE to AC. (Art. 112.) With D as a center and a radius equal to DE, draw the required circle.

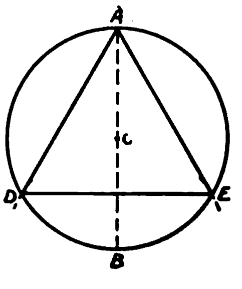


Fig. 128

136. To inscribe an equilateral triangle within a circle whose center is at C. Fig. 128.

Draw the diameter AB. With B as a center and a radius equal to BC, describe arcs cutting the circumference at D and E. Join points A, D and E with straight lines and ADE is the triangle required.

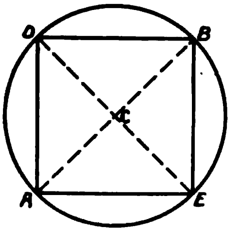


FIG. 129

137. To inscribe a square in a circle whose center is at C. Fig. 129.

Draw the diameter AB and the diameter DE perpendicular to it. (Art. 110.) Join A to D, D to B, B to E, and E to A with straight lines and ADBE is the square required.

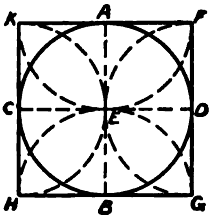


FIG. 130

138. To circumscribe a square about a circle whose center is at E. Fig. 130.

Draw the diameter AB and the diameter CD perpendicular to it. With A, D, B and C as centers and a radius equal to AE, describe arcs intersecting at F, G, H and K. Join F, G, H and K with straight lines and FGHK is the square required.

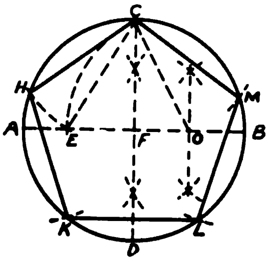


FIG. 131

139. To inscribe a pentagon in a circle whose center is at F. Fig. 131.

Draw the diameter AB and the diameter CD perpendicular to it. Bisect FB at O. (Art. 109.) With O as a center and a radius OC, describe an arc cutting AB at E. With C as a center and radius CE, describe an arc cutting the circumference at H and with the same radius step off K, L and M on the circumference. Connect these points to form the required pentagon CHKLM.

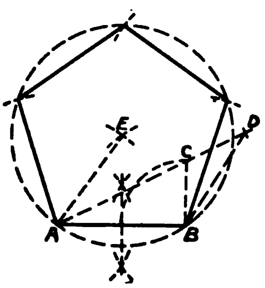


FIG. 132

140. To construct a pentagon on a given straight line AB. Fig. 132.

Erect a perpendicular, BC, at B equal to ½-AB, using Arts. 109 and 111. Join A to C with a straight line prolonged to D, making CD equal to BC. Join B to D. With A and B as centers and a radius equal to BD, describe arcs intersecting at E. With E as a center and a radius EA, describe a circle. Step off the distance AB around the circumference and connect the points thus found to form the required pentagon.

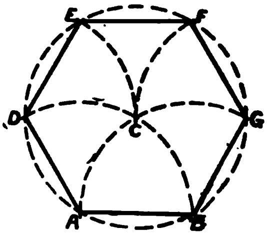


FIG. 133

141. To construct a hexagon on a given straight line AB. Fig. 133.

With A and B as centers and a radius equal to AB, describe arcs intersecting at C. With C as a center and the same radius, describe a circle passing through A and B and intersecting these arcs at D and G. With the same radius describe arcs CE and CF with centers at D and G respectively. Join A to D, D to E, etc., with straight lines to form the required hexagon.

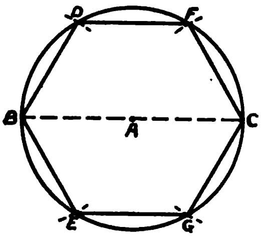


FIG. 134

142. To inscribe a hexagon within a circle whose center is at A. Fig. 134.

Draw the diameter BC. With B and C as centers and a radius equal to BA, describe arcs cutting the circumference at D, E, F and G. Join B to D, D to F, etc., to form the required hexagon.

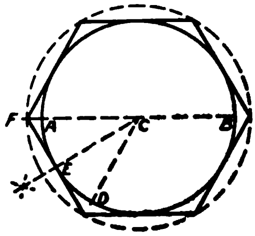


FIG. 135

143. To circumscribe a hexagon about a circle whose center is at C. Fig. 135.

Draw the diameter AB. With A as a center and a radius equal to AC, cut the circumference at D. Bisect the angle ACD, (Art. 118), with the radius CE which intersects the circumference at E. At E draw a perpendicular to CE, (Art. 110), intersecting AB prolonged at F. From the center C with a radius CF, describe a circle. Complete the problem by inscribing a hexagon in this circle according to Art. 142.

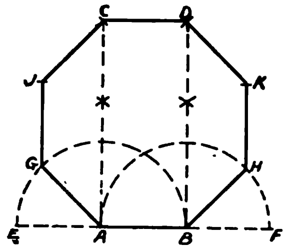


Fig. 136

144. To construct an octagon on a given straight line AB. Fig. 136.

Prolong AB in both directions to E and F. At A and B erect perpendiculars AC and BD, (Art. 110). Bisect the angles EAC and FBD by the lines AG and BH, (Art. 118). Make AG and BH each equal to AB. From G and H draw lines GJ and HK parallel to AC and BD and equal in length to AB. From J and K as centers with a radius equal to AB, describe arcs cutting AC and BD at C and D. Complete the required octagon by joining J to C, C to D, and D to K with straight lines.

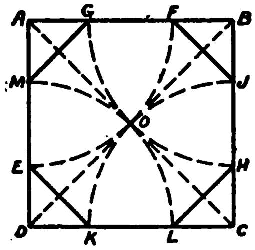


Fig. 137

145. To inscribe an octagon within a given square ABCD. Fig. 137.

Draw the diagonals AC and BD intersecting at O. From A, B, C and D as centers with a radius equal to AO, describe arcs cutting the sides of the square at E, F, G, H, J, K, L and M. Connect M to G, F to J, H to L, and K to E by straight lines to form the required octagon.

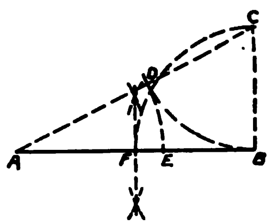


Fig. 138

146. To divide a given line AB in extreme and mean ratio. Fig. 138.

Bisect AB at F, (Art. 109). At B erect a perpendicular BC, (Art. 111), equal to BF. Connect A and C with a straight line. With C as a center and a radius CB, describe an arc cutting AC at D. With A as a center and a radius AD, describe an arc cutting AB at E. Point E divides AB in extreme and mean ratio or AB is to AE as AE is to BE.

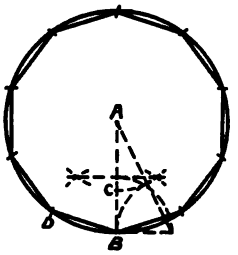


Fig. 139

147. To inscribe a regular decagon in a given circle whose center is at A. Fig. 139.

Draw any radius AB and divide it in extreme and mean ratio, (Art. 146), so that AB is to AC as AC is to BC. With B as a center and a radius equal to AC, describe an arc cutting the circumference at D. BD is one side of the required decagon and by stepping around the circumference with the length BD and joining the points thus found the decagon may be constructed.

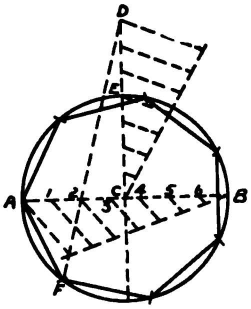


Fig. 140

148. To inscribe a regular polygon of any number of sides, for example seven, within a given circle whose center is at C. Fig. 140.

Draw the diameter AB and at the center C erect the perpendicular CD, (Art. 110), cutting the circumference at E. Divide CE into four equal parts (Art. 114), and lay off a distance equal to three of these parts from E to D. Divide AB into as many equal parts as the polygon is to have sides, i. e. seven, (Art. 114), and through D and the second point of division draw a straight line cutting the circumference at F. Draw AF which is a side of the required polygon. By stepping the length AF around the circumference and connecting the points thus found the polygon may be constructed.

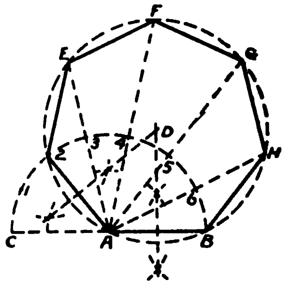


Fig. 141

149. On a given side AB to construct a regular polygon having any number of sides. Fig. 141.

Prolong AB toward C and with A as a center and a radius equal to AB, describe a semicircle and divide it into as many equal parts as the polygon is to have sides. This operation may be performed by the method given in Art. 148, by completing the circle and dividing it into twice as many equal parts as the required polygon is to have sides. Connect A to the second point of division 2. A2 is another side of the required polygon. Find the center of the circumscribing circle by Art. 121. Draw the circle and step the length AB around the circumference to complete the required polygon. If desired lines intersecting the circumference may be drawn through A and the points 3, 4, 5 and 6 and the polygon completed by joining the points 2, E, F, G, H and B.

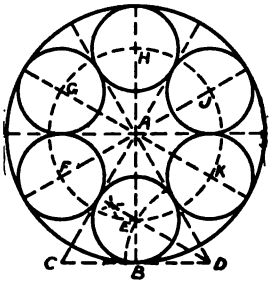


Fig. 142

150. Within a given circle to draw any number of equal circles, for example six, tangent to each other and the given circle whose center is at A. Fig. 142.

Divide the circumference (Art. 148) into twice as many equal parts as there are circles required. Draw radii through the points of division. At the end of any radius as AB draw a tangent CD to the circle. (Art. 122.) Prolong a radius adjacent to AB to intersect CD at D. Bisect the angle ADB by the line DE, (Art. 118), and the point E where this line intersects AB is the center of one of the required circles. With A as a center and a radius equal to AE, describe a circle cutting the radii at F, G, H, J and K, which intersections locate the centers of the other circles which may be drawn with a radius equal to EB.

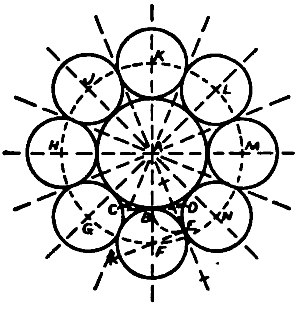


Fig. 143

151. About a given circle to describe any number of circles, for example eight, tangent to each other and the given circle whose center is at A. Fig. 143.

Divide the circumference (Art. 148) into twice as many equal parts as there are circles required. Draw radii through the points of division. At the end of any radius as AB draw a tangent CD to the circle. (Art. 122.) Prolong a radius adjacent to AB to intersect CD at D and make DE equal to DB. At E erect a perpendicular, (Art. 110), cutting AB prolonged at F. F is the center of one of the required circles. With A as a center and a radius AF, describe a circle cutting the radii prolonged at G, H, J, K, L, M and N, which intersections locate the centers of the other circles which may now be drawn with a radius equal to FB.

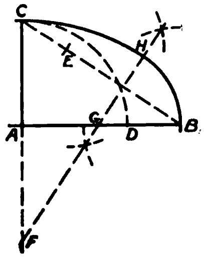


Fig. 144

152. To describe an ellipse approximately by means of circular arcs of two radii, given the major and minor axes. Fig. 144.

Let AB be one-half the major axis and AC one-half the minor axis. With A as a center and a radius AC, describe an arc cutting AB at D. Connect B and C with a straight line and set off CE equal to BD. Bisect BE and erect a perpendicular at the center, (Art. 109), intersecting AC prolonged at F and AB at G. With F as a center and a radius equal to FC, draw the arc CH and with G as a center and a radius equal to GB, draw the arc HB. Complete the ellipse by drawing the three remaining quarters in the same manner.

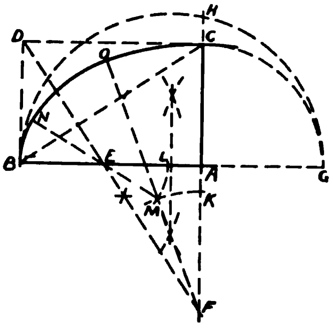


Fig. 145

153. To describe an ellipse approximately by means of circular arcs of three radii, given the major and minor axes. Fig. 145.

Let AB be one-half the major axis and AC one-half the minor axis. At B and C erect perpendiculars intersecting at D. (Art 111.) Connect B and C with a straight line and draw DF perpendicular to BC. (Art. 112.) DF intersects AC prolonged at F. Prolong AB to G and make AG equal to AC. Bisect BG, (Art. 109), and from the central point with a radius equal to ½-BG, describe a semicircle cutting

AC prolonged at H. Set off AK equal to CH. With F as a center and a radius equal to FK, describe an arc KM. With B as a center and a radius equal to AH, describe an arc intersecting AB at L. With E as a center and a radius equal to EL, describe an arc intersecting KM at M. Draw MEN and FMO. With E as a center and a radius equal to EB, draw the arc BN; with M as a center and a radius equal to MN, draw arc NO; and with F as a center and a radius equal to FO, draw arc OC. Complete the other quarters in the same manner.

EXAMINATION QUESTIONS

(1) What is the difference between an object and a point?

(2) State the reason why a curved line cannot be drawn parallel to a straight line.

(3) (*a*) What is an angle of 51 deg. called?
(*b*) What is an angle of 99 deg. called?

(4) State the difference between a vertical line and a perpendicular line.

(5) What is the difference between the diameter and circumference of a circle?

(6) What is the name of a triangle having angles of 41 deg., 41 deg. and 98 deg.?

(7) If the altitude of an isosceles triangle is drawn, what kind of triangles are formed?

(8) What is the name of the figure formed by placing two equal squares so that a side of one coincides with a side of the other?

(9) If a regular hexagon is divided by means of long diagonals joining all the vertices, what are the parts called? Give the complete name.

(10) How is an ellipse different from a circle?

(11) How is a regular polygon different from a circle?

(12) If you were given a straight line to divide into four equal parts state two ways in which you could do it.

(13) If two lines are drawn parallel to each other and a perpendicular drawn to one of them, what angles does it make with the other if prolonged to meet it?

(14) If two circles are each drawn tangent to the same two parallel lines and the circles are some distance apart, how do their diameters compare with each other and with the distance between the parallel lines?

(15) If the long diagonals of a square are drawn; what is the number of degrees in each of the four angles at the center formed by the intersection of the diagonals?

WORKING DRAWINGS

PROJECTION

154. In order to represent an object on paper in such a way as to give its true form and size, it is generally necessary to draw several views as seen from different points of observation. The views are made either the same size as the object or some proportionate size, for example one quarter size. The dimensions of the object are indicated on the views by figures. In addition all information is given regarding material, allowance for finishing, etc., to enable a workman to produce the piece without further question. The views when drawn correctly and in proper relation to each other form what is called a working drawing. The method of making the views is called orthographic projection.

155. If an object, for example a cube, is placed in back of a transparent plane and lines perpendicular to the plane passed through all points of the object, these lines where they meet the plane, trace a full size view of the object which is said to be projected on the plane.

In Fig. 146 the cube ABCDEFG is placed in back of the transparent plane WXYZ, with the face ABCD parallel to the plane. Assume that lines perpendicular to WXYZ are extended from points A, B, C, D, E, F, G and H* till they inter-

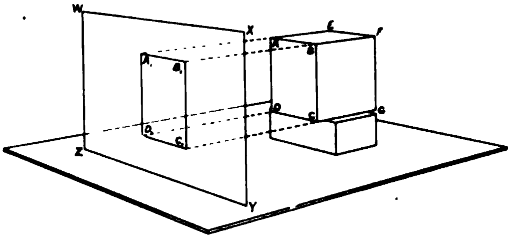


Fig. 146

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*The point H is hidden behind the cube and therefore does not show in the Fig.

sect the transparent plane. The cube is a body whose edges are all at right angles to each other and of equal length, so that the perpendiculars from points E, F, G and H coincide with edges EA, FB, etc. and with the perpendiculars extended from A B, C and D to WXYZ. Therefore the points A_1, B_1, C_1 and D_1, where these perpendiculars intersect the transparent plane, are the projections E, F, G and H as well as A, B, C and D. By joining A_1 to B_1, etc., a square is drawn the same size as the face ABCD. This square is one view or projection of the cube and is the only portion of the cube visible from this point of observation.

156. Planes in different positions may be used similarly to obtain views of the other faces, in order to show details not given by the first view.

Suppose several transparent planes at right angles to each other are hinged together to form a transparent box. Fig. 147 shows such a box hinged at the edges RS, RV, ST and VT and arranged to part at the other edges. Place a point A anywhere in the interior of the box and through it pass lines perpendicular to and intersecting the walls of the box. The points of intersection will be at A_1 on plane RSXW, A_2 on plane RSTV, A_3 on plane SXYT, A_4 on plane RWZV, A_5 on plane VTYZ and A_6 on plane WXYZ. Each of these points of intersection is a view or projection of point A.

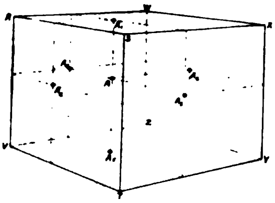

Fig. 147

Now open the transparent box bending back the walls at the hinged edges and lay it down on a plane surface, such as the top of a drawing board. Fig. 148 gives its appearance and shows the positions that the projections of point A take.* It will be noted that A_1, A_2 and A_5 lie on a line which is perpendicular to RS and VT and A_4, A_2 and A_3 on a line perpendicular to RV and ST and therefore to the line joining A_1A_5. Also that A_1, A_3, A_5 and A_4 are all at the same distance from the respective hinged edges RS, ST, etc. Therefore if the projection A_6 were missing it could be determined as follows.

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*To avoid confusion the points W, X, Y and Z are marked W', X', Y' and Z' on the end views.

Draw a perpendicular to RV through A_2 and a line parallel to RS through A_1 to intersect RW at B. With R as a center

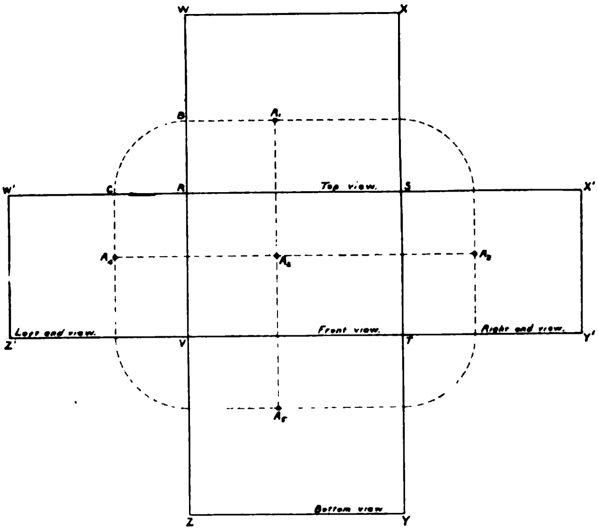

Fig. 148

and a radius RB describe an arc cutting RW′ at C. Through C draw a parallel to RV intersecting the perpendicular through A_2 at A_4. A_4 is the required projection on plane RVZ′W′, since it lies on a line through A_2 perpendicular to RV and is at the same distance (by construction) from RV, as A_1 is from RS. The projection on any of the other planes may be found similarly.

The different projections are named according to their position, top view or plan (A_1), front view or elevation (A_2), right end view (A_3), left end view (A_4), and bottom view (A_5). (A_5) as well as the back view (A_6), which is not shown in Fig. 148, are seldom used.

In order to make it convenient to refer to the various planes, WXSR is called the top horizontal plane, RSTV the front vertical plane, SX′Y′T the right end vertical plane, and RVZ′W′ the left end vertical plane.

157. Instead of one point, a series of points may be projected in the same manner on the walls of the transparent box and these points may be the limits of lines that determine the boundaries of an object. For example, the same cube used in Fig. 146 may be placed in the center of the space enclosed by the box and its views projected on the walls as described above and illustrated by Fig. 149. Fig. 150 shows the appearance of the projected views when the box is opened at the hinged edges. This set of views if properly dimensioned and lettered would be a complete working drawing of the cube.* However, for simple objects such as this, a top and front view are usually sufficient and it is seldom that more than a top, front and end view are required for any object. Thus, Fig. 151 is a working drawing of a regular hexagonal

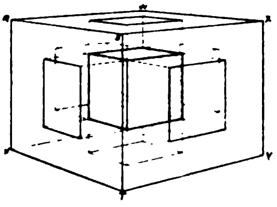

Fig. 149

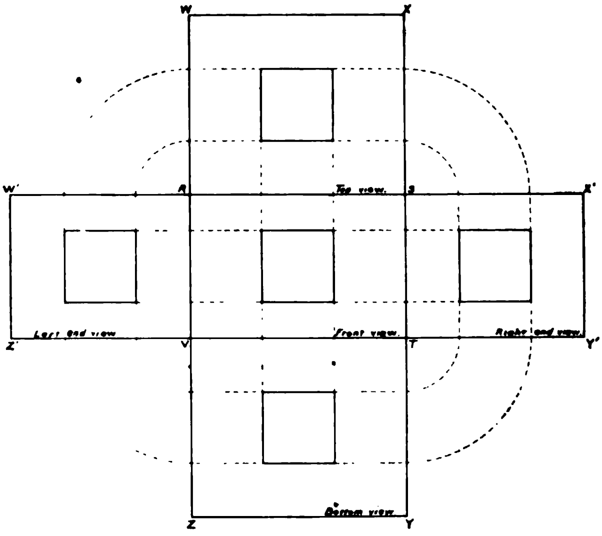

Fig. 150

pyramid.* The pyramid is placed with its base resting on a horizontal plane and one edge of the base, 5-6, parallel to the front vertical plane. Its dimensions are 2″ on a short diagonal of the base by 4″ high. The points are numbered so as to show their location on the different views. The method of making the drawing is as follows.

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*In finishing a working drawing it is usual to omit all construction lines.

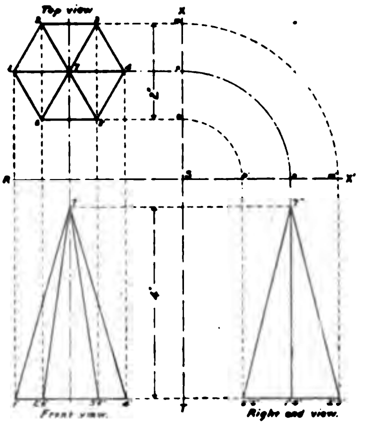

Fig. 151

Estimate the space required by the different views and draw the vertical line XST and the horizontal RSX′. In this case the front and end views each require approximately the same space while the top view needs only about half the amount taken by the front view. The line RSX′ should therefore be drawn nearer the top of the sheet than the bottom. XST and RSX′ correspond to XST and RSX′ in Figs. 148 and 150.

First, draw the view about which most information is given, in this example the top view. This view is a hexagon with the side 5-6 parallel to RSX′ and the vertices 1, 2, 3, 4, 5 and 6 joined to the center 7 by straight lines representing the edges between the sides. The hexagon may be constructed by describing a circle 2″ in diameter, at a convenient distance above RSX′ and to the left of XST, and drawing the sides tangent to its circumference with the T square and 30° × 60° triangle. At some convenient distance, greater than 4″, below RSX′ draw a horizontal line 1′-4′. From the points 1, 2, 3, 4, 5 and 6 of the top view draw vertical lines downward to intersect this line at 1′, 2′, 3′, 4′, 5′ and 6′ noting that points 2′, 6′ and 3′, 5′ coincide. Draw another vertical downward from point 7 and on it lay off point 7′, 4″ above 1′-4′. Join this point to 1′, 2′, 3′, 4′, 5′ and 6′ with straight lines to complete the front view.

Now to determine the right end view draw two horizontals to the right from the front view, one through points 1′ to 4′, and the other through 7′. From points 2 and 3 of the top view draw a parallel to RSX′ to intersect XST at m. With S as a center and a radius Sm describe an arc cutting RSX′ at m′. Through m′ draw a vertical downward to intersect the horizontal through points 1′ to 4′ at 2″3″, thus locating these points on the right end view, since they lie on a line through 2′, 3′, which is perpendicular to XST, and are at the same distance from XST as 2 and 3 are from RSX′. It will be noted that these points coincide. In the same manner the other points 1″, 4″ which coincide, 5″, 6″ which coincide, and 7″ may be found and by

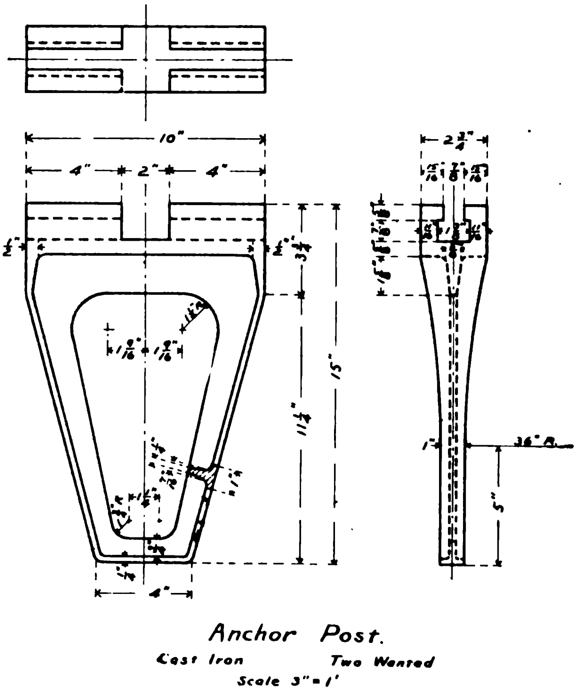

Fig. 152

connecting 7″ to 5″6″, 1″4″, and 2″3″ with straight lines the required right end view is produced.

As another illustration, Fig. 152 is a working drawing of the anchor post shown in Fig. 153. Three views are shown corresponding to the views in Fig. 151, but the construction lines are omitted. Attention is called to the method of dimensioning where an arc of a circle is used to join two lines at an angle to each other. If the position of one of the lines is known, as in this case, the center of the arc is located in a direction parallel to this line and the radius indicated by a dimension. If the positions of both lines are unknown the center should be located exactly by two dimensions.

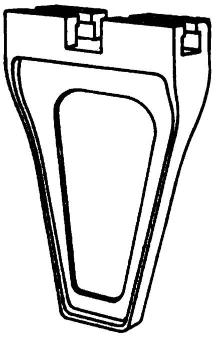

Fig. 153

158. When it is difficult to determine the projection of a line directly, it should be remembered that every line is determined by two or more points. The projections of the points may be found as described in Art. 156 and if a line is drawn through them it will be the required projection of the given line.

159. In some cases in order to get the true form and size of a portion or face of an object which is not parallel to any of the planes described above, it is necessary to project it on an auxiliary plane parallel to the face, and then swing this plane around till it lies in the same plane as the rest of the drawing. For example, suppose it is desired to find the true shape of one face, ABC, of a square pyramid. Fig. 154 illustrates the manner in which the glass box would be arranged and the method of projecting to accomplish this result. Note that plane STYX is parallel to the face in question. Fig. 155 shows the box opened. The construction is as follows.

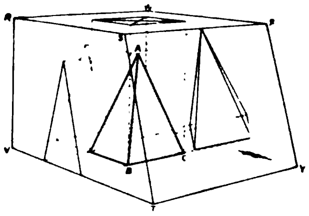

Fig. 154

Take any point as B, on the front view, and from it draw a perpendicular to ST. Through the corresponding point on the top view, draw a line parallel to RS to intersect SX. With S as a center swing the point of intersection around on to SX′ and through this point draw a parallel to ST. The intersection between this parallel and the perpendicular first drawn is the required point on the auxiliary plane. The other points are determined in the same manner. In this case not only the required face, but also some of the faces beyond it are visible and therefore show in this view.

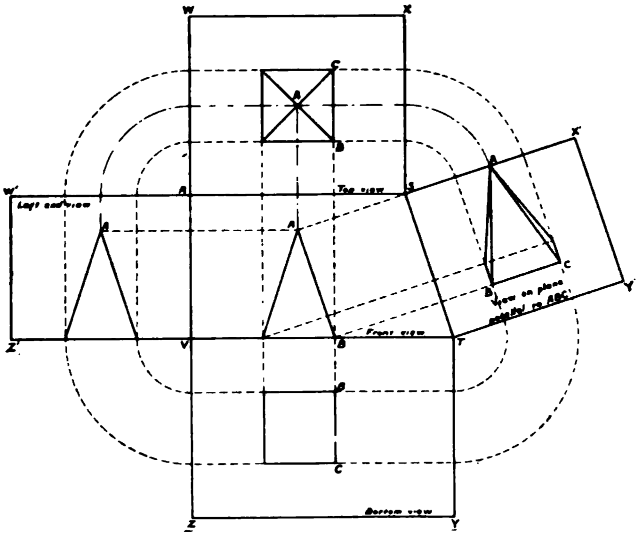

Fig. 155

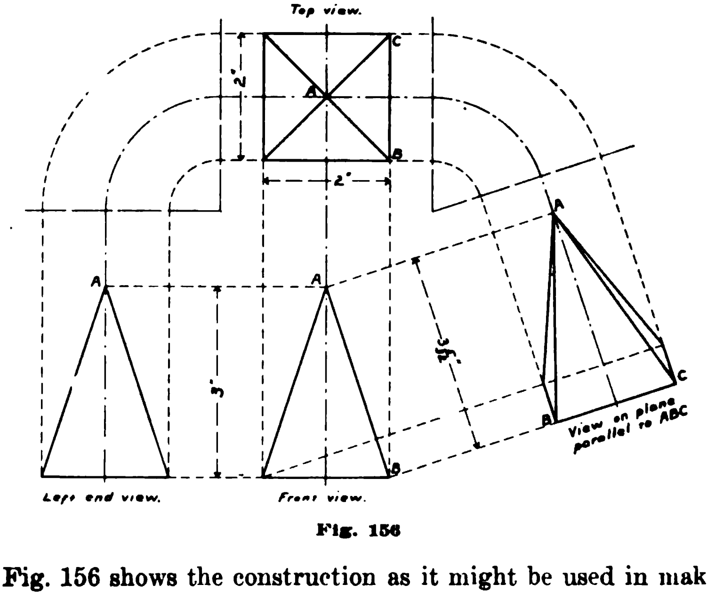

Fig. 156

Fig. 156 shows the construction as it might be used in making a working drawing.

160. Fig. 157 indicates the method of projecting curved lines on auxiliary planes. In this case it is desired to find the true shape of the section made by cutting off the top of a right circular cone with an oblique plane AB. In the top view, the circumference of the base is divided into any number of equal parts 1, 2, 3, etc. and the points of division projected down on the front view. These points are connected to the apex 0 of the cone in both views by straight lines called elements.* Assume the plane CD parallel to AB. To follow out the construction take any element as 03 on the front view, which AB cuts at 3′. The corresponding point on the top view is found as follows. Point 3′ lies on the line or element 03 in the front view and therefore on the same line in the top view. Also since the projections of a point on the front and top views lie in the same vertical line, a vertical line through 3′ intersects 03 in the top view at 3″, the required point. To find the projection on the auxiliary plane pass a line perpendicular to CD through 3′. Through 3″ draw a horizontal intersecting the vertical line CE at m and, with C as a center and a radius Cm, describe an arc intersecting CF, which is perpendicular to CD, at m′. Through the latter point draw a parallel to CD cutting the perpendicular through 3′ at 3‴ which is the required point on this plane. All the other points are determined in the same manner both for the section and the base, and joined by curved lines to complete the view on the auxiliary plane.

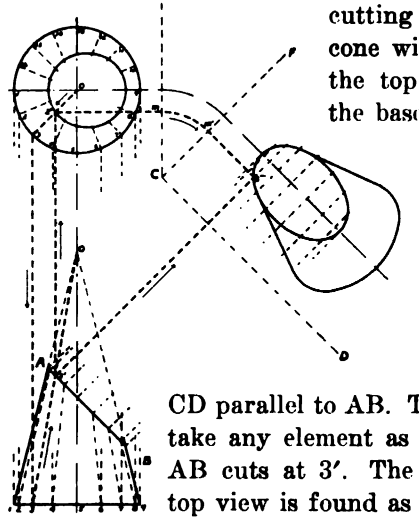

Fig. 157

161. Sections, see Art. 23, aid greatly in making plain the form and size of interior portions and others difficult to show by the regular views. The section is made by cutting the object at the desired place with an imaginary plane, removing the part toward the observer, and projecting the section and parts beyond on a plane parallel to the cutting plane. Thus in Fig. 46 the outside of the piece is shown on one side of the vertical center line, while on the other side the piece is cut at the center and the part toward the observer removed to show the interior construction. The drawing of the pipe carrier roller in Fig. 65 further illustrates the use of sections for showing clearly details of construction.

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*To make the cut clearer these lines are not actually drawn through 0, but only the parts necessary for construction are indicated.

162. In practice the use of planes as described above is purely imaginary, the draftsman carrying on a similar process in his mind. It is necessary, however, to project from one view to another, but, as before noted, the construction lines do not appear on the finished drawing. Also the construction is much simplified by using the dividers for transferring distances. Thus in Fig. 151, if the center lines are drawn first, the projection lines may be omitted by carrying the distances from one view to another with the dividers.

The object should always be placed so as to make the views as simple and few in number as possible. Generally if an object is intended to stand in a certain position, the drawing should show it in the same position. The space required by each view should be carefully estimated and the drawing arranged symmetrically in its allotted space. It is not necessary to finish one view before starting another. In fact, it is often desirable to carry along the work on the different views at the same time. If the points are numbered it will be less difficult to determine their position in the different views.

163. In addition to the instructions in Arts. 25, 26, 27 and 154, note that in general a dimension given on one view should not be repeated on another. Machine finished surfaces are marked with a small (f), or similar character, and the allowance for finishing specified either at the surface or in a note. The pattern material (if a pattern is required), number of pieces of each kind required by the device of which they are a part, material from which pieces are to be made, etc., are other items of information that should be given.

EXAMINATION QUESTIONS

(1) Under what conditions would you use three views in making a working drawing?

(2) In what position should an object be placed with reference to the imaginary transparent planes, so as to make it as easy as possible to draw the different views?

(3) If the front view of a point, placed in a transparent box similar to the one shown in Fig. 147, is 2½″ from RS and 3″ from ST, and the top view is 3¼″ from RS, what is the position of the right end view? Show this by a sketch.

(4) If the projection of a point is located on the left end vertical plane at a distance of 5″ from RV, and on the front vertical plane at a distance of 1½″ from RS and 4″ from RV, what is the location on the top horizontal plane? Show this by a sketch.

(5) Suppose that several points lie in a straight line perpendicular to the top horizontal plane, what is their projection on that plane?

(6) On what planes are the projections of a point definitely located if the point is 3¾″ back from the front vertical plane and 2½″ down from the top horizontal plane?

(7) When would you use an auxiliary plane in making a working drawing?

(8) If you were making a drawing of an object, under what conditions would you consider it unnecessary to make use of sections?

(9) State the relation between the projections of the same point on the top horizontal and front vertical planes.

(10) State the relation between the projections of the same point on the top horizontal and right end vertical planes.

EXERCISES

164. Sheet 10: Divide this sheet into three equal parts by vertical lines. Reserve a space 3″ long by 1″ wide in the lower right hand corner for the title. The title is to be the same as shown in Fig. 64, but with the word "Projection" substituted for "General Methods." Dimensions and shade lines are not required on this sheet. Finish the work including construction lines with ink. Make the drawings full size.

In the first space draw three views of a square prism standing on end on a horizontal plane with one of its faces parallel to the front vertical plane, as shown in Fig. 158. The dimensions of the prism are 1½″ square by 3″ long. The method of making the drawing, see Fig. 159, is as follows.

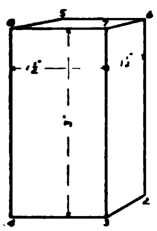

Fig. 158

First, draw the top view. This is a square measuring 1½″ on each side. Since the prism stands with one of its sides parallel to the front vertical plane, two of the lines of the top view will be horizontal and should be drawn with the T square. The other two lines will be at right angles to these and should be drawn by using one of the triangles on the T square blade.

Second, draw the front view which is a rectangle 1½″ wide by 3″ long. Use the T square and triangles as just described, obtaining the width by projecting down from the top view.

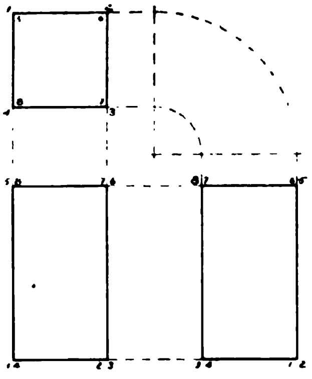

Fig. 159

Third, draw the right end view by projecting across from the front view and around from the top view, locating the points as described in Art. 157. This view is another rectangle with the same dimensions as the front view.

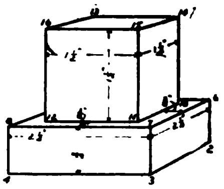

Fig. 160

In the second space draw a top, front and right end view of a cube placed centrally on top of a square plinth, as illustrated in Fig. 160. The cube measures 1½″ on each edge and the plinth is 2¼″ square by ¾″ high. The group rests on a horizontal plane with the faces toward the observer parallel to the front vertical plane. Fig. 161 shows the drawing completed. Draw the top view first, then the front and finally the end view, following the same method as described in connection with the previous problem.

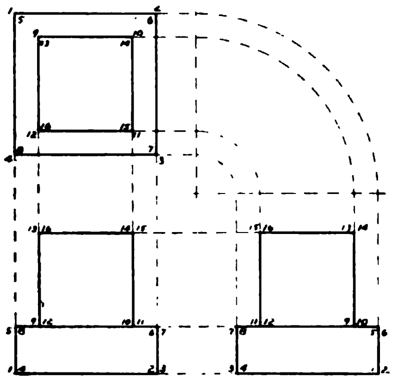

Fig. 161

In the third space draw three views of the group shown in Fig. 162. This consists of a circular plinth 2¼″ in diameter by ¾″ high placed centrally on the side of a hexagonal prism. The prism measures 1½″ on a short diagonal of the end face by 3″ long. It rests on a horizontal plane and its long edges are parallel to the front vertical plane. The method of proceeding in this case, see Fig. 163, is as follows.

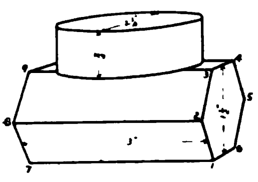

Fig. 162

First, draw the end view of the hexagonal prism by describing a circle 1½″ in diameter and circumscribing a hexagon around it with the T square and 30° × 60° triangle.

Second, determine the horizontal center line of the top view by projecting around from the center of the end face of the hexagonal prism. Take a point on this line as a center and draw the top view of the circular plinth, which is a circle 2¼″ in diameter. Draw a vertical center line through the center of this circle and 1½″ on each side of it draw lines which correspond to the end faces of the hexagonal prism. Complete this view by projecting around from the end view to obtain the edges between the sides. Now draw the end view of the circular plinth which is a rectangle 2¼″ long by ¾″ high placed on top of the end face of the hexagonal prism. The 2¼″ dimension is obtained by projecting around from the top view. Third, draw the front view obtaining the points by projection from the top and end views.

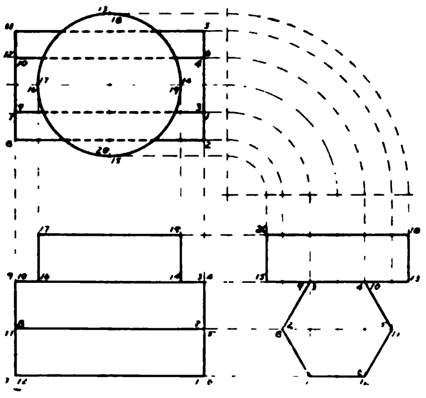

Fig. 163

165. **Sheet 11**: In laying out this sheet and making the drawings observe the instructions with regard to sheet 10, with the exception that the drawings should be made half size.

In the first space draw three views of the triangular prism shown in Fig. 164. Each of the three rectangular faces of the prism is 1½″ wide by 3″ long. The prism is placed with one side resting on a horizontal plane and its long edges making angles of 45° with the front vertical plane. The drawing, see Fig. 165, is made in the following manner.

First, draw the top view which is a rectangle 1½″ wide by 3″ long, with its edges making angles of 45° with the horizontal. This operation is performed with the aid of the T square and 45° triangle. Complete the view by drawing another 45° line through the center of the rectangle to represent the edge between the upper inclined faces.

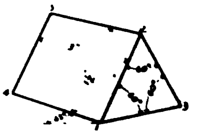

Fig. 164

Second, draw a horizontal line 4-3 to represent the lower edges of the faces showing in the front view and project points 4, 6, 1 and 3 from the top view on to this line. The height of this view corresponds to the altitude of an end face of the

prism and is found as follows. Extend the line 3-2-1 and a short **distance below the top view draw** a horizontal line AB, inter**secting it at D. With D as a center,** swing points 1 and 3 around on to AB and then project them vertically downward on to the line 4-3 prolonged. The points thus obtained are numbered 1′ and 3′ and the distance 1′-3′ is equal to the width of the bottom face. Since all the rectangular faces are of equal width, with points 1′ and 3′ as centers and a radius equal to 1′-3′, describe arcs intersecting at 2′. This operation is equivalent to projecting the end face on a plane parallel to itself and then swinging the parallel plane around so as to lie in the plane of the drawing, thus giving the true form and size of this face, see Art. 159. Through 2′ draw a horizontal line and project points 5 and 2 on it from the top view. Join 5 to 4 and 6, and 2 to 1 and 3 to complete the front view.

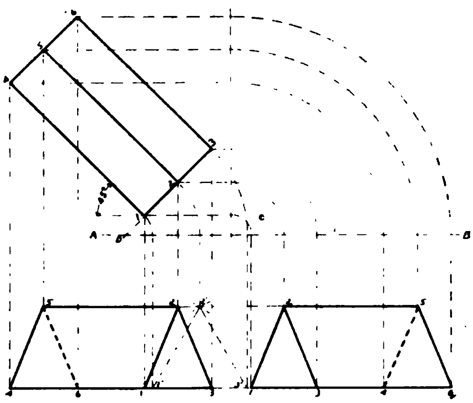

Fig. 165

Third, draw the end view, projecting across from the front view and around from the top view to locate the points.

In the second space draw three views of the group shown in Fig. 166. The dimensions of the square prism are the same as in Fig. 158. Each of the two smaller rectangular faces of the right triangular prism measures 1½″ by 3″. The group rests on a horizontal plane and is placed so that the horizontal edges make angles of 30° and 60° with the front vertical plane.

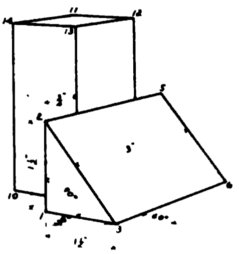

Fig. 166

Fig. 167 shows the method of making the drawing. The top view should be drawn first, then the front and end views.

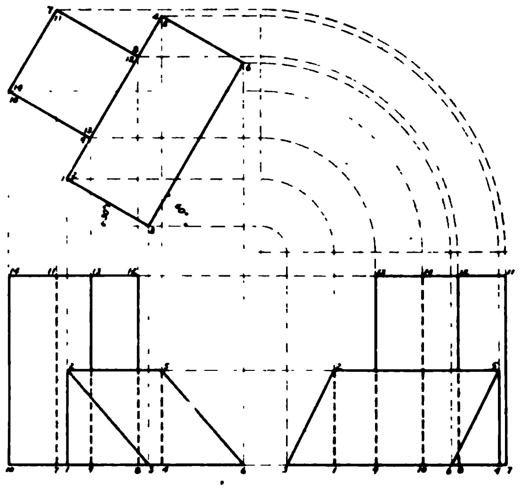

Fig. 167

For the third space three views of the group shown in Fig. 168 are required. The square and hexagonal prisms have the same dimensions as in the preceding problems. The hexagonal prism is placed centrally on top of the square prism which rests on a horizontal plane. The horizontal edges of the square prism make angles of 45° with the front vertical plane and the long edges of the hexagonal prism angles of 60°.

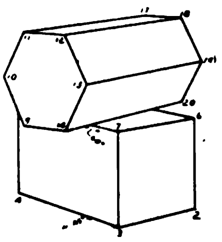

Fig. 168

First, draw the three views of the square prism, see Fig. 169, allowing space in the front and end views so that the hexagonal prism may be placed on top of it.

Second, draw a center line hj through the top view of the square prism, and through its middle point k, draw another line mn making an angle of 60° with the horizontal. On this line lay off points m and n, at distances of 1½″ both sides of the point of intersection k, and through m and n draw perpendiculars, 16-19 and 10-13 to the line mn. These perpendiculars correspond to the end faces of the hexagonal prism. A short distance below the top view draw a horizontal line ED, intersecting one of these perpendiculars at D, and with D as a center describe an arc

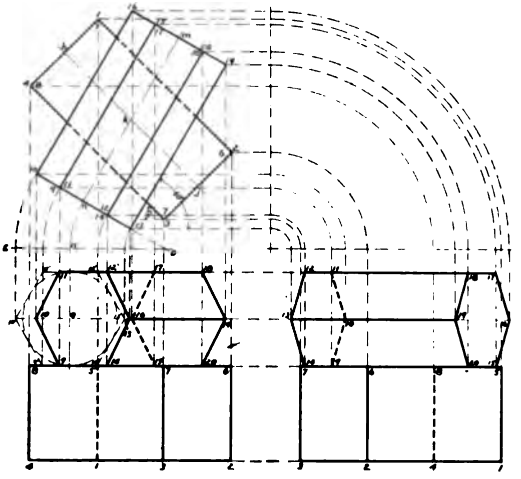

Fig. 169

through n, to intersect ED at n′. Extend a vertical line downward from n′ and on it lay off a distance of ¾″ up from the top of the square prism in the front view. With this point 0 as a center, describe a circle 1¼″ in diameter and circumscribe a hexagon 9′ 10′ 11′ 12′ 13′ 14′ around it. As in the case of the triangular prism in the first space, this operation gives the true form and size of an end face of the hexagonal prism. Project points 9′, 10′, 11′, 12′, 13′ and 14′ on to ED and swing them around D as a center on to D-10, giving points 9, 10, 11, 12, 13 and 14. Note that points 9, 11 and 12, 14 coincide. Complete the top view of the hexagonal prism by drawing lines parallel to the center line mn through these points, to represent the edges between the sides.

Third, finish the front view by projecting down from the top view of the hexagonal prism and across from the construction of the end face 9′ 10′ 11′ 12′ 13′ 14′.

Fourth, complete the end view by projecting across from the front view and around from the top view.

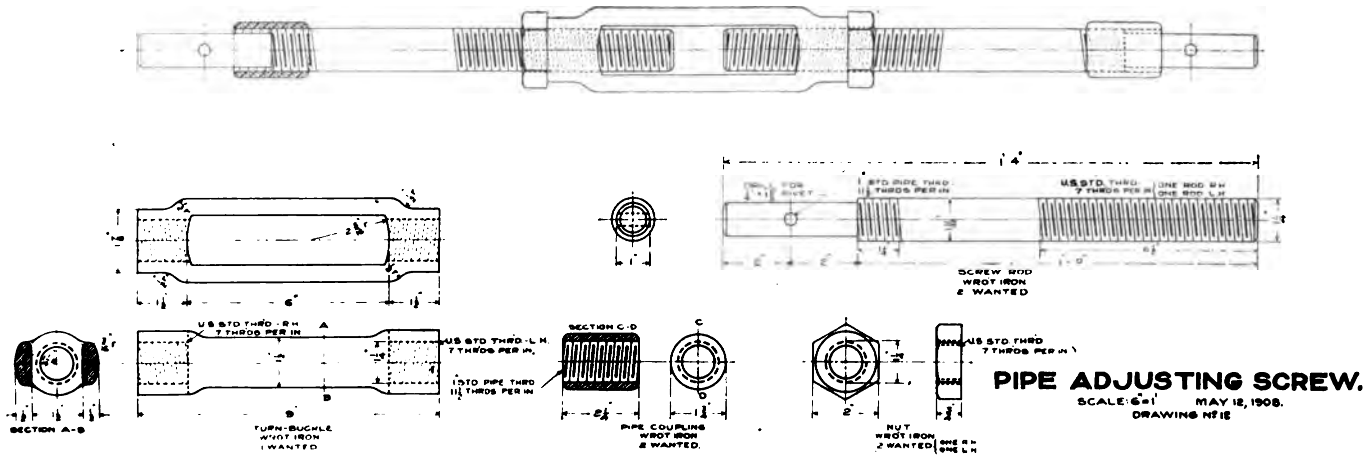
1' 4"
FOR RIVET
STD PIPE THRD
11½ THRDS PER IN
U.S.STD. THRD
7 THRDS PER IN
ONE ROD R.H.
ONE ROD L.H.
1¼
6½
1' 0"
SCREW ROD
WROT IRON
2 WANTED
6"
1½
SECTION A-B
U.S STD THRD - R.H.
7 THRDS PER IN
A
B
U.S STD THRD - L.H.
7 THRDS PER IN.
1 STD PIPE THRD
11½ THRDS PER IN
9"
TURN-BUCKLE
WROT IRON
1 WANTED
SECTION C-D
C
D
2¼
1¾
PIPE COUPLING
WROT IRON
2 WANTED.
2"
U.S STD THRD
7 THRDS PER IN
NUT
WROT IRON
2 WANTED
ONE R.H.
ONE L.H.
PIPE ADJUSTING SCREW.
SCALE: 6"=1'
MAY 12, 1908.
DRAWING No. 12

Fig. 170

166. Sheet 12: Fig. 170 shows the assemblage and details of a pipe adjusting screw. The assemblage, in this case, placed at the top of the sheet, shows the different pieces of the adjusting screw put together in their proper positions. The details are the drawings of each separate piece giving all the dimensions and information necessary to enable a workman to make it. Ordinarily no dimensions are placed on an assemblage, although in some cases it is desirable to do so in order to show the relation of one part to another, or to save the time necessary to draw a detail.

When starting a view decide where it is to be placed on the sheet, then draw the center lines and construct the view around them. Draw the assemblage first, taking the dimensions from the details in the illustration. Then draw the details, arranging them so that they will not crowd each other, giving all the views, dimensions and information shown in the illustration. Print the title in its proper place, giving the date the sheet is finished instead of the one shown. For the large size printing in the title use letters $\frac{1}{4}''$ high, and for the small printing and other notes use letters $\frac{3}{32}''$ high. Finish with ink lines and finally check the drawing carefully.

These instructions apply to all the following sheets except where noted otherwise.

Make this drawing to a scale of $6'' = 1'$.

In that branch of Signal Work called Mechanical Interlocking, pipe is used to transmit motion from one piece of apparatus to another. Under certain conditions it is necessary to change the length of the pipe in order to adjust the apparatus so that it will operate properly. The adjusting screw is used for this purpose.

The assemblage requires only one view as the device is simple and the details show all the features of the separate pieces.

A front and end view serve to make the construction of the screw rod plain. This is a straight round bar with one end, called a tang end, reduced in size so that it will fit inside 1″ pipe. Just beyond the tang end, a pipe thread 1¼″ long is cut and on the other end, the bar is threaded with a 1¼″ U. S. Standard thread, 6½″ long.

The pipe coupling requires only a section and end view. The section is taken through the center, at CD, to show the thread. The operation of cutting an inside thread is usually done with a tool know as a tap and is called tapping. These couplings are tapped with either tapered or straight threads.

A top and front view and a section are required to show the turn-buckle. The section is taken at AB to show the form of the sides.

Two views are sufficient to show the nut.

Attention is called to the following facts:

First with regard to pipe and pipe threads:—Pipe is always specified by its approximate inside diameter, thus 1″ pipe is 1.048″ inside diameter* and 1.315″, or slightly over $1\frac{5}{16}''$, outside diameter. Standard pipe threads are always cut in a certain way. For example, a 1″ standard pipe thread always has 11½ threads per inch of length and when properly cut, the outside diameter of the thread at the end of the pipe, or bar, is 1.283″ and the thread is tapered, growing larger from this point.

Second with regard to U. S. Standard threads:—When the diameter of a thread is specified, it is understood to mean the diameter of the piece on the outside of which the thread is to be cut or the outside diameter of the tap in the case of an inside thread. Thus a 1¼″ outside thread would be cut on a piece 1¼″ in diameter and, if an inside thread, would be cut with an 1¼″ tap. Threads with the same diameter always have the same number of threads per inch of length. For example, $1\frac{1}{4}''$ U. S. Standard threads always have 7 threads per inch of length.

---

*This dimension is 0.951″ for extra strong pipe and 0.587″ for double extra strong.

The following abbreviations are used on this sheet, r = radius, St'd. = standard, Thr'd. = thread, R. H. = right hand, L. H. = left hand, Wro't. = wrought, in. = inch.

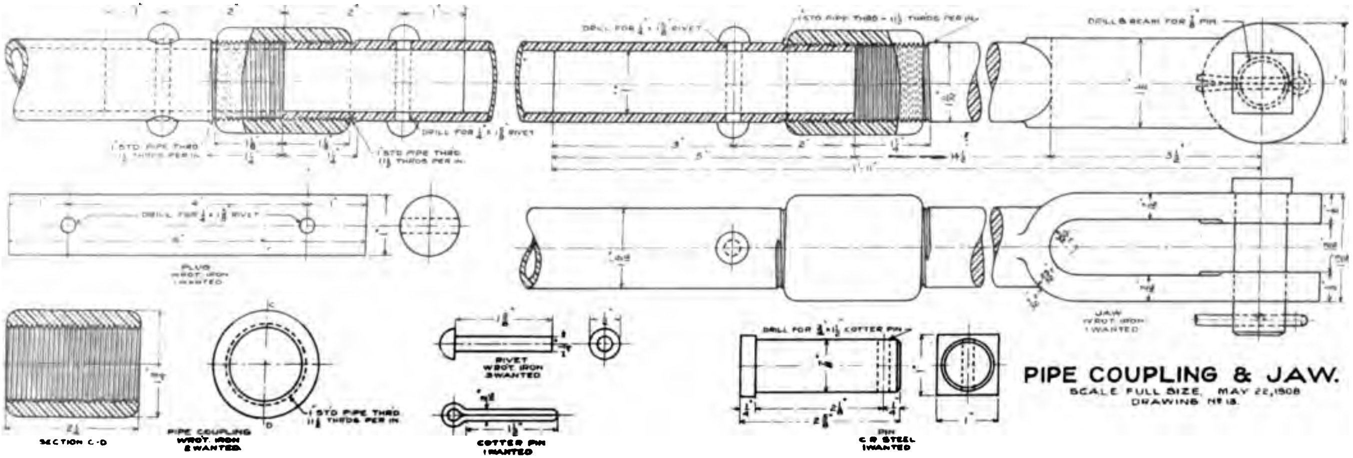
DRILL FOR 1/4" × 1 3/8" RIVET
1" STD PIPE THRD 11 1/2 THRDS PER IN.
1" STD PIPE THRD - 11 1/2 THRDS PER IN.
DRILL & REAM FOR 5/8" PIN
3"
5"
2"
1 1/4"
1'-11"
3 1/2"
PLUG
WROT IRON
1 WANTED
JAW
WROT IRON
1 WANTED
SECTION C-D
PIPE COUPLING
WROT. IRON
2 WANTED
1" STD PIPE THRD
11 1/2 THRDS PER IN.
RIVET
WROT IRON
3 WANTED
COTTER PIN
1 WANTED
DRILL FOR 1/8" × 1 1/2" COTTER PIN
PIN
C R STEEL
1 WANTED
PIPE COUPLING & JAW.
SCALE FULL SIZE. MAY 22,1908
DRAWING Nº 13.

Fig. 171

167. Sheet 13: On this sheet draw the pipe coupling and jaw shown in Fig. 171, noting the general instructions in Art. 166. Make the drawing full size. These pieces, as in the case of the adjusting screw, are portions of the apparatus used in Interlocking.

In this case, the assemblage gives two views and the dimensions of the jaw as well as other information, so that no detail of this piece is required. Attention is called to the method of showing the long shank of the jaw, by breaking out the center portion, and to the method of representing breaks in pipe.

Pipe and U. S. Standard threads have sides making angles of 60° with each other and are drawn with the aid of the 30° × 60° triangle.

The plug, rivet and pin each require only a front and end view.

The pipe coupling requires a section and end view.

One view is sufficient to show the cotter pin, since it is a well known piece.

The abbreviation C. R. steel is used to denote cold rolled steel.

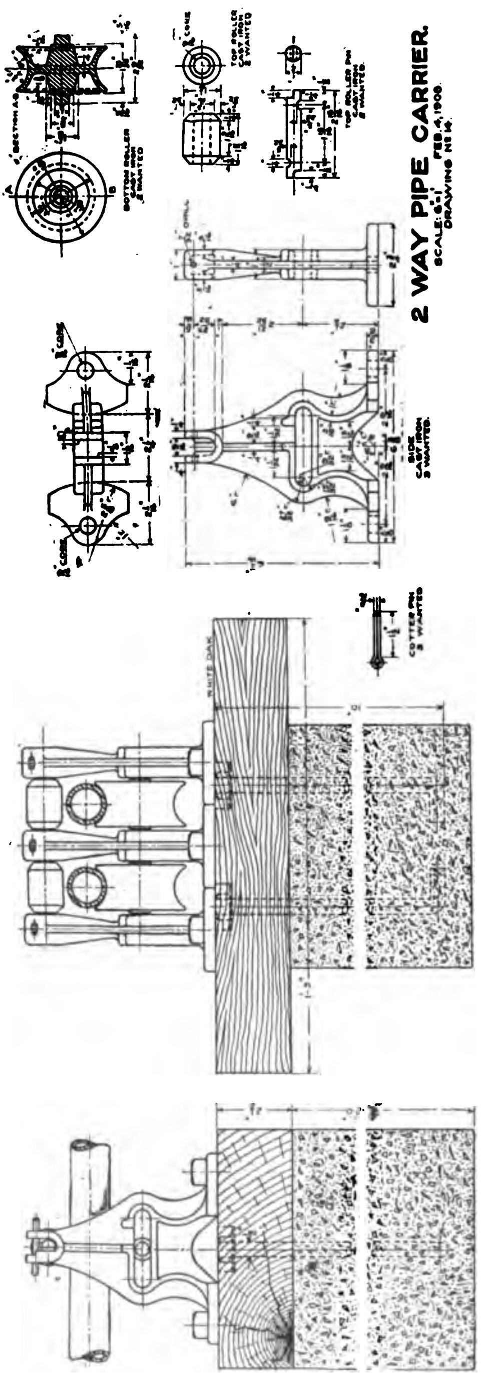
2 WAY PIPE CARRIER.
SCALE 6"=1'
FEB. 4, 1906
DRAWING No 14.
BOTTOM ROLLER CAST IRON 2 WANTED
SECTION A-B
TOP ROLLER CAST IRON 2 WANTED
TOP ROLLER PIN CAST IRON 2 WANTED
SIDE CAST IRON 3 WANTED
CORE
DRILL
COTTER PIN 3 WANTED
WHITE OAK

Fig. 172

168. **Sheet 14**: The 2 way pipe carrier, shown in Fig. 172, is to be drawn on this sheet to a scale of 6″ = 1′. This device is used to guide and support the pipes connecting Mechanical Interlocking apparatus and is designed to allow the pipe to move with little friction.

On account of the rather complex nature of the device the assemblage requires a front and end view which are placed at the left end of the sheet. The assemblage shows the method of mounting the carrier on its foundation, which is composed of a concrete column with an oak plank (8″ × 15″)* bolted on top by means of two hook bolts. The sides of the carrier are fastened to the plank with lag screws. Attention is called to the method of designating wood and concrete by surface lining, in a similar manner to the way in which sections of the same material would be shown.

The details of the side of the carrier requires a top, front and end view to show all its features.

The other details are shown by two views each. In the case of the bottom roller, a section through the center, at AB, is shown.

Note the method of indicating the operation of the carrier by showing the pipe in position. As the pipe is of indefinite length, it is shown with broken ends in the front view and in section in the end view.

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*The length of the plank varies according to the number of ways.

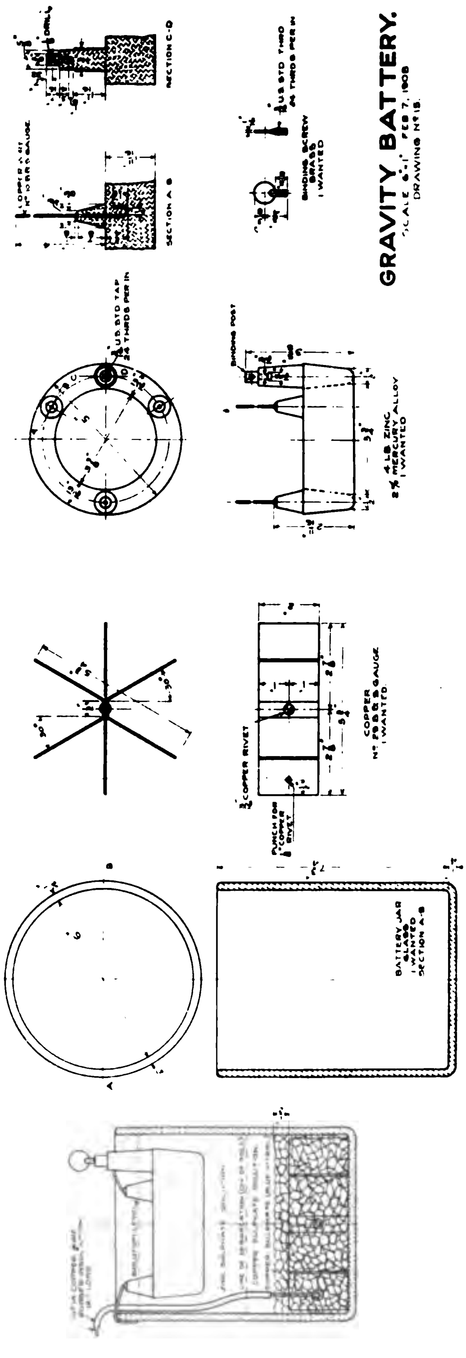
GRAVITY BATTERY.
SCALE 6"=1' FEB 7, 1908
DRAWING No 15.
SECTION A-B
SECTION C-D
COPPER No 20 B&S GAUGE
DRILL
US STD TAP 24 THRDS PER IN
BINDING POST
4 LB ZINC
2% MERCURY ALLOY
1 WANTED
BINDING SCREW
BRASS
1 WANTED
US STD THRD 24 THRDS PER IN
COPPER RIVET
PUNCH FOR COPPER RIVET
COPPER
No 20 B&S GAUGE
1 WANTED
BATTERY JAR
GLASS
1 WANTED
SECTION A-B

Fig. 173

169. Sheet 15: On this sheet make the drawing shown in Fig. 173 using a scale of 6″ = 1′. The drawing illustrates the construction of one cell of a gravity battery, a type extensively used in signal work.

As the device is very simple the assemblage requires only one view. In addition to the information that would ordinarily be given on a working drawing, this assemblage shows the quantity of copper sulphate that should be placed in the cell*, the line of demarcation between the copper sulphate and the zinc sulphate solutions and the level at which the surface of the solution should be kept. Attention is called to the type of sectioning used on the assemblage and on the detail of the jar to indicate glass.

Top and sectional views are sufficient to show the details of the jar. The copper requires a top and front view. As the drawing of the zinc shows the supporting wires and binding post in position, two partial sections are required, in addition to the top and front views, to give some further details of these pieces and show the method of construction. Note the type of sectioning used to indicate zinc.

The details of the binding screw requires a front and end view.

In two places on this sheet gauge numbers are used instead of actual dimensions. Thus No. 14 copper wire indicates that the wire is No. 14 Brown & Sharpe gauge which has a diameter of .064″ and the No. 29 B. & S. gauge specified in the case of the copper indicates that the copper is .011″ thick.

On account of limited space on the drawing the abbreviations div. and sols. are used in place of division and solutions.

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*Practice varies on different roads regarding the quantity of copper sulphate to be used.

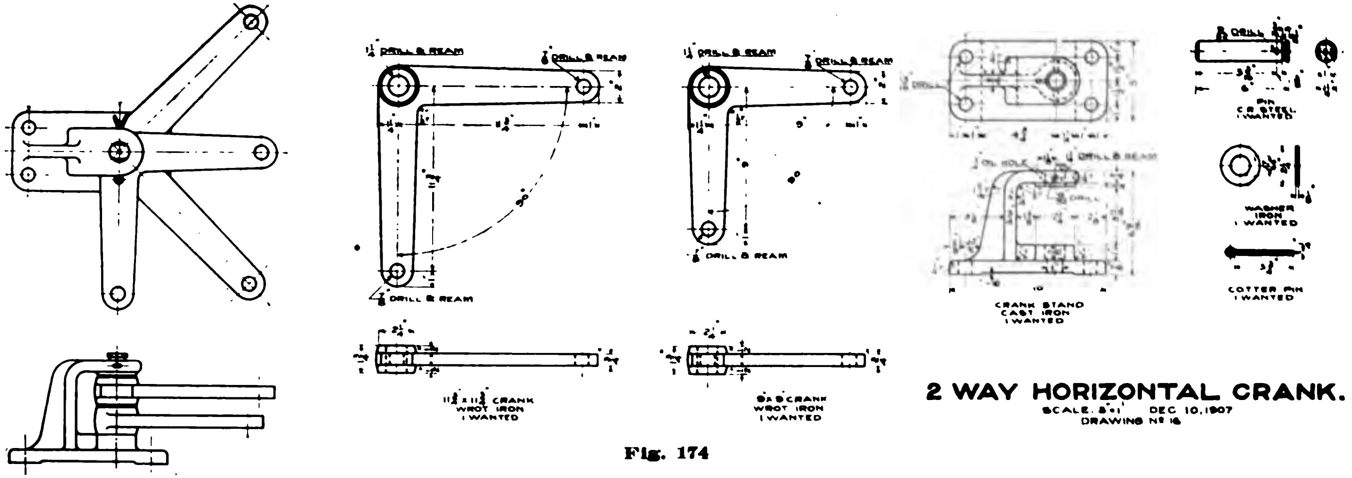


Fig. 174

170. Sheet 16: Fig 174 shows the details of the parts and method of assembling a two way horizontal crank. This is another piece of apparatus used mainly in connection with Mechanical Interlocking. Its purpose is for changing motion from one direction to another, in this case at 90° to the first.

The assemblage and details of this piece of apparatus are of such a nature that it is sufficient to draw two views for each. Use a scale of $3'' = 1'$.

Instead of inking directly on the paper make a tracing from this drawing. (See Arts. 16 and 28, Part I.)

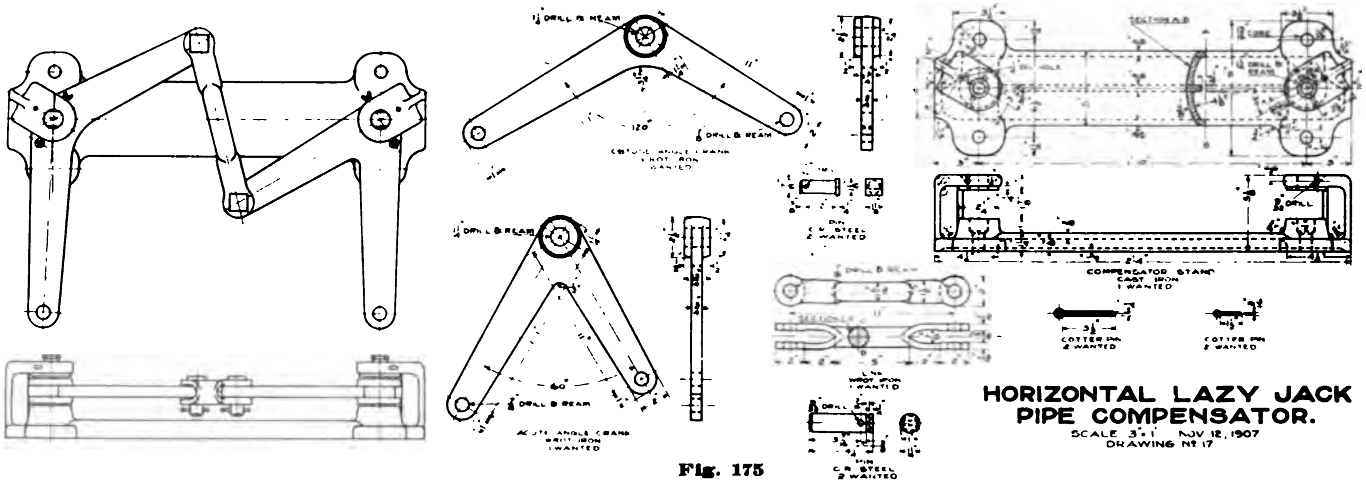


Fig. 175

171. Sheet 17: The horizontal lazy jack pipe compensator, shown in Fig. 175, is a piece of Mechanical Interlocking apparatus used to compensate for the expansion and contraction in lines of pipe, due to changes of temperature.

Draw the assemblage and details on this sheet to a scale of 3" = 1'.

Attention is called to the method of sectioning used in drawing the top view of the compensator stand. This is a convenient way of showing the form of a piece at some particular place. The same method is used to show the form of the center portion of the link.

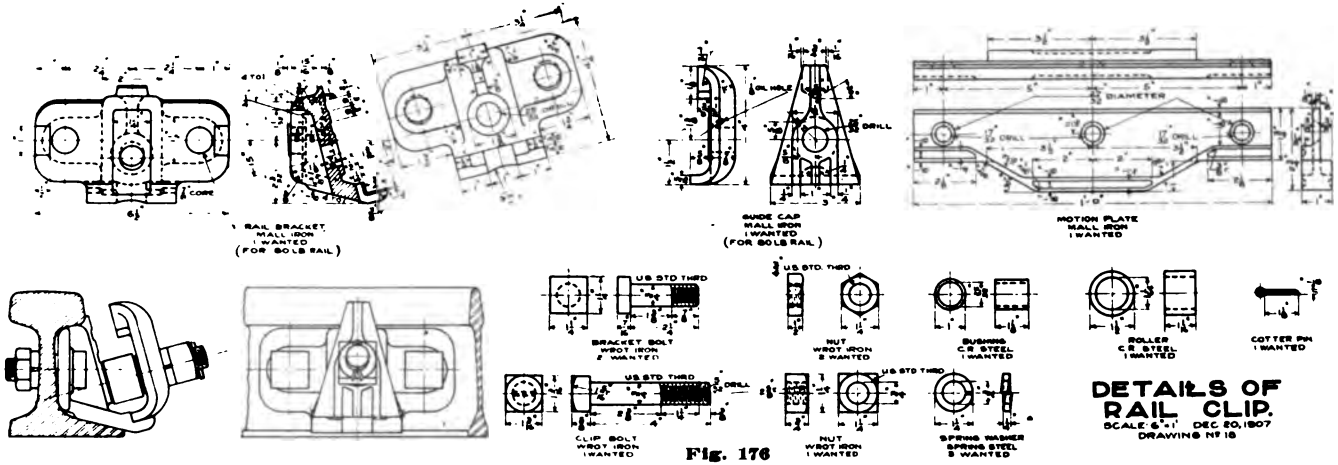


Fig. 176

172. Sheet 18: Fig. 176 shows the details of a rail clip for an 80 lb. rail and also the detail of a motion plate, which devices are used to support and guide a detector bar. This bar is located at switches on the outside of the head of the rail, $\frac{1}{8}''$ below the top. Its motion at the start of its operation is upward and before a switch can be unlocked this bar has to be operated. If the wheels of a train are standing on a portion of track where one of these bars is located, the bar cannot be operated and therefore the switch cannot be unlocked. The type of motion plate shown is used for Power Interlocking. Make the drawing to a scale of $6'' = 1'$. Note that a U. S. Standard thread $\frac{3}{4}''$ in diameter has 10 threads per inch of length. The detail of the rail bracket is a good example of the use of the auxiliary plane. (See Art. 159.)

Note the use of the abbreviation mall. for malleable.

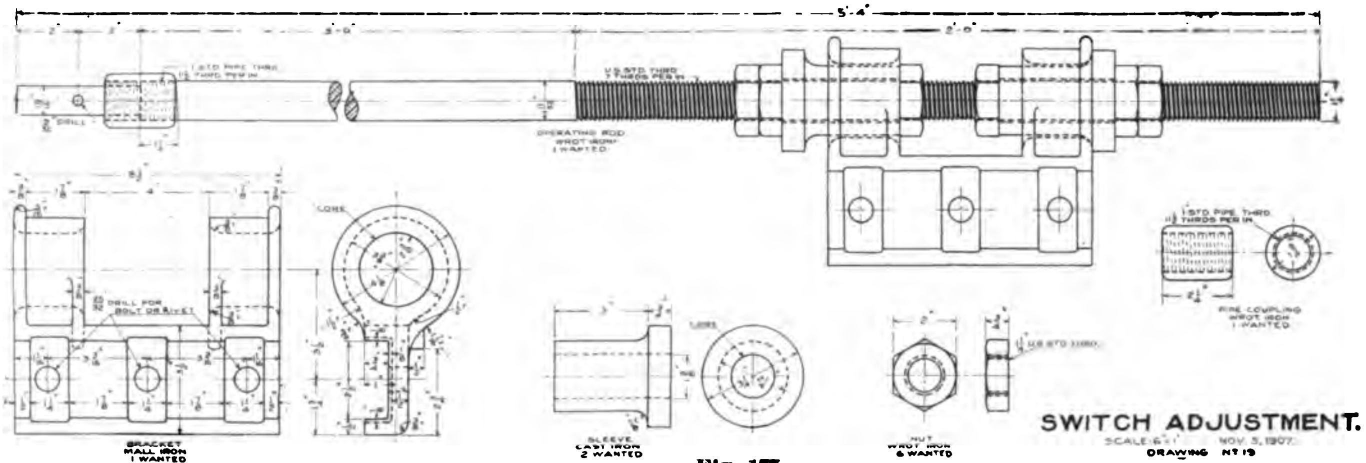


Fig. 177

173. Sheet 19: Make a drawing of the switch adjustment shown in Fig. 177, using a scale of 6" = 1'.

The switch adjustment is a device which in use is riveted or bolted to a switch rod (a bar, joining the points of the switch, known as the head rod). It connects the switch to the interlocking apparatus which operates it.

Its purpose is to provide a certain amount of lost motion, which can be varied according to requirement. This variation or adjustment becomes necessary when the stroke of the operating rod changes. It further provides for adjustment in case the position of the track changes in relation to the operating connections.

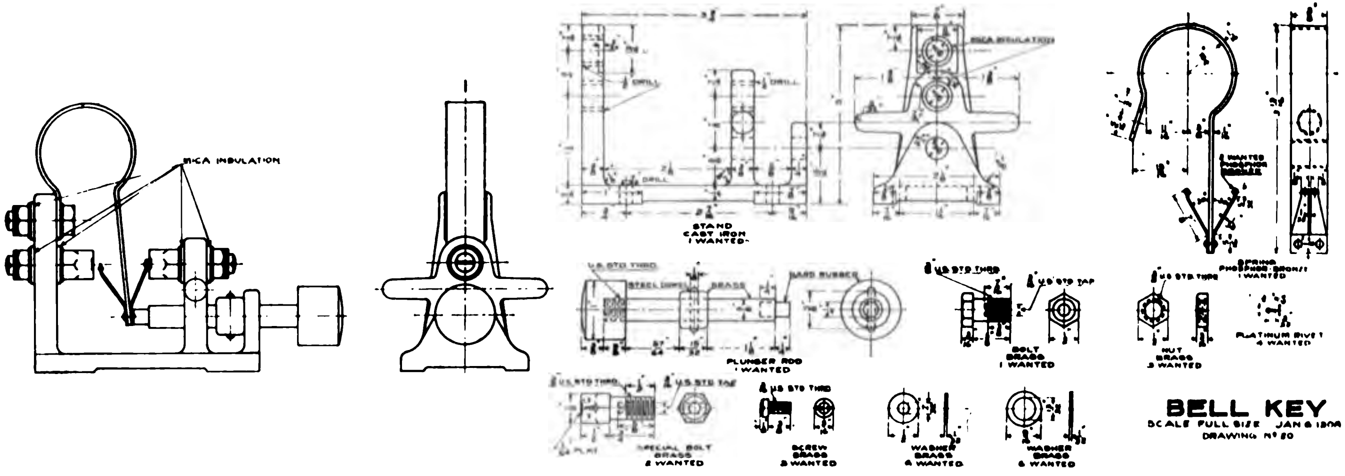


Fig. 178

174. Sheet 20: Draw the bell key and details shown in Fig. 178, making the drawing full size. This device is used in signal towers to operate an electric bell at a distant point, such as another signal tower and is sometimes used in connection with telephone installations.

U. S. Standard threads $\frac{3}{8}''$ and $\frac{3}{16}''$ in diameter have 16 and 24 threads per inch of length respectively. Ink this sheet on tracing cloth, as in the case of No. 16, instead of inking directly on the paper.

APPENDIX

MENSURATION

The sum of the three angles of a triangle equals 180°.

The **hypothenuse** of a **right triangle** is the side opposite the right angle. Its length equals the square root of the sum of the squares of the other two sides.

The **area** of a **triangle** equals the base multiplied by one half the altitude.

The **diagonal** of a **rectangle** equals the square root of the sum of the squares of two adjacent sides.

The **area** of any **parallelogram** equals the base multiplied by the altitude.

To find the **area** of any **quadrilateral,** divide it into two triangles and the sum of the areas of the triangles is the area required.

The **area** of a **regular polygon** may be found as follows. Multiply the length of a side by the perpendicular distance to the center. Multiply this product by the number of sides and divide by 2.

The **circumference** of a **circle** equals the diameter multiplied by 3.1416. The number 3.1416 is called (Pi) and is denoted by the symbol π.

The **area** of a **circle** equals the square of the diameter multiplied by 0.7854.

The total **surface** of a **right prism** or **cylinder** equals the perimeter of the base multiplied by the altitude, plus the areas of the two ends.

The **volume** of a **prism** or **cylinder** is equal to the area of the base multiplied by the altitude.

The total **surface** of a **regular pyramid** or **cone** equals the perimeter of the base multiplied by one half the slant height plus the area of the base.

The **volume** of a **pyramid** or **cone** equals the area of the base multiplied by one third the altitude.

To find the total **surface** of the **frustrum** of a **pyramid** or **cone** add together the perimeters of the upper and lower bases and multiply the sum by one half the slant height. The product plus the areas of the bases is the surface required.

To find the **volume** of the **frustrum** of a **pyramid** or **cone** add to the sum of the areas of the two ends the square root of their product and multiply the result by one third the altitude.

The **surface** of a **sphere** is equal to the square of the diameter multiplied by 3.1416.

The **volume** of a **sphere** is equal to the cube of the diameter multiplied by 0.5236.

Table Giving the Approximate Radius Corresponding to the Degree of Curve For Track Plans Made to Different Scales

DEGREE OF CURVE

Degs.	Min.	SCALE 1" = 100'	1" = 80'	1" = 60'	1" = 50'	1" = 40'	1" = 30'	1" = 20'	1" = 10'
0	15	229	286	382	458	573	764	1146	2292
	30	115	143	191	229	286	382	573	1146
	45	76	95	127	153	191	255	382	764
1	00	57	72	96	115	143	191	287	573
	15	46	57	76	92	115	153	229	458
	30	38	48	64	76	96	127	191	382
	45	33	41	55	65	82	109	164	327
2	00	29	36	48	57	72	96	143	287
	15	25	32	42	51	64	85	127	255
	30	23	29	38	46	57	76	115	229
	45	21	26	35	42	52	69	104	208
3	00	19	24	32	38	48	64	96	191
	15	18	22	29	35	44	59	88	176
	30	16	20	27	33	41	55	82	164
	45	15	19	25	31	38	51	76	153
4	00	14	18	24	29	36	48	72	143
	15	13	17	22	27	34	45	67	135
	30	13	16	21	25	32	42	64	127
	45	12	15	20	24	30	40	60	121
5	00	11	14	19	23	29	38	57	115
	15	11	14	18	22	27	36	55	109
	30	10	13	17	21	26	35	52	104
	45	10	12	17	20	25	33	50	100
6	00	10	12	16	19	24	32	48	96
	15	9	11	15	18	23	31	46	92
	30	9	11	15	18	22	29	44	88
	45	8	11	14	17	21	28	42	85
7	00	8	10	14	16	20	27	41	82
	15	8	10	13	16	20	26	40	79
	30	8	10	13	15	19	25	38	76
	45	7	9	12	15	19	25	37	74
8	00	7	9	12	14	18	24	36	72
	15	7	9	12	14	17	23	35	70
	30	7	8	11	14	17	23	34	68
	45	7	8	11	13	16	22	33	66
9	00	6	8	11	13	16	21	32	64
	15	6	8	10	12	16	21	31	62
	30	6	8	10	12	15	20	30	60
	45	6	7	10	12	15	20	29	59
10	00	6	7	10	11	14	19	29	57
	30	5	7	9	11	14	18	27	55
11	00	5	7	9	10	13	17	26	52
	30	5	6	8	10	12	17	25	50
12	00	5	6	8	10	12	16	24	48
	30	5	6	8	9	11	15	23	46
13	00	4	6	7	9	11	15	22	44
	30	4	5	7	9	11	14	21	43
14	00	4	5	7	8	10	14	21	41
	30	4	5	7	8	10	13	20	40
15	00	4	5	6	8	10	13	19	38
16	00	4	4	6	7	9	12	18	36
17	00	3	4	6	7	8	11	17	34
18	00	3	4	5	6	8	11	16	32
19	00	3	4	5	6	8	10	15	30
20	00	3	4	5	6	7	10	14	29

METHOD OF USING THE TABLE.

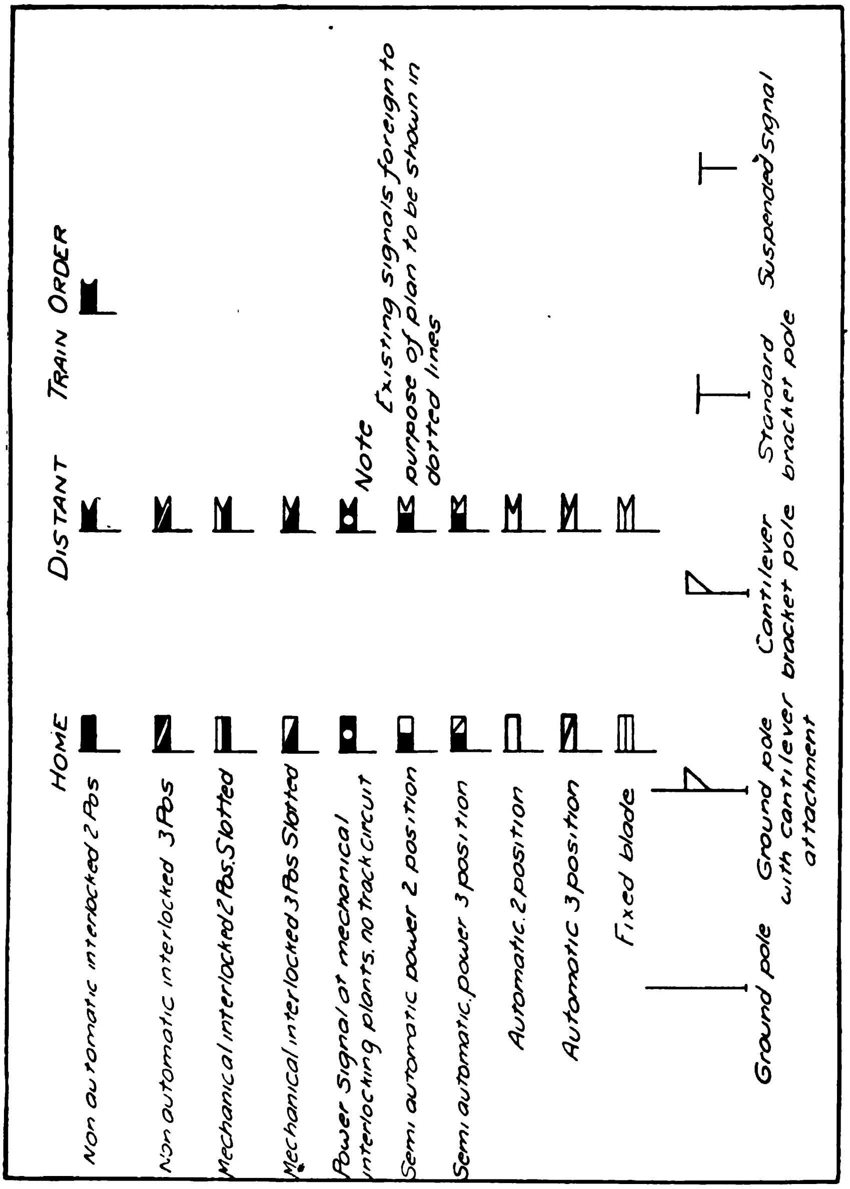
Suppose that it is desired to find the proper radius to draw a curve of 3° - 15' on a plan made to a scale of 1" = 40'. Find 3° - 15' in the left hand column. Opposite this, in the column under 1" = 40', the figure 44 is found denoting that the approximate radius for the given degree and scale is 44". If a set of curves is available, but contains curves of 42" and 45" radius instead of 44", select the one with the nearest radius, in this case 45".

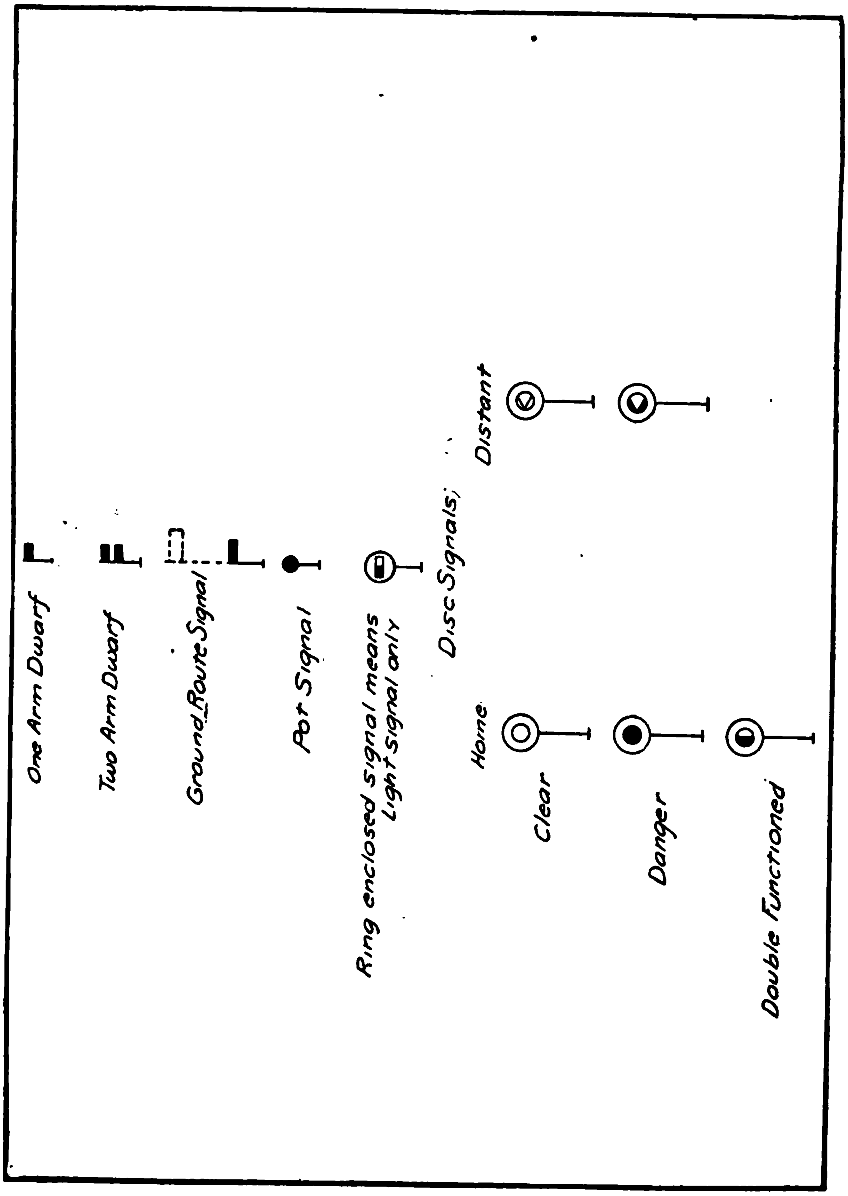
STANDARD SYMBOLS FOR RAILWAY SIGNAL PLANS

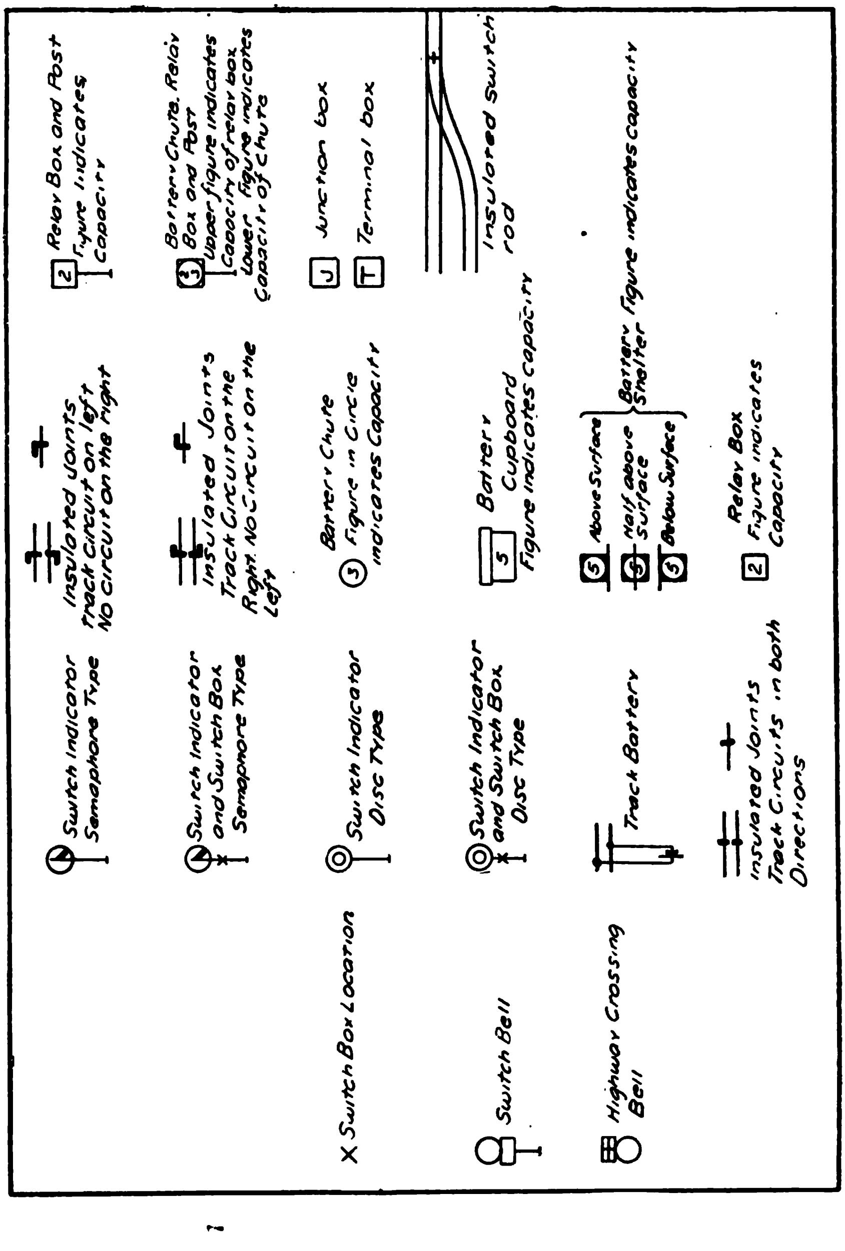
On account of the variety of symbols used in signal work and to overcome the difficulties arising from different Railroads and Signal Companies using different methods of representing the same piece of apparatus, The Union Switch and Signal Co., The Hall Signal Co., The General Railway Signal Co. and The Federal Signal Co. compiled and adopted a set of symbols as a common standard. These were published in pamphlet form and have been widely distributed.

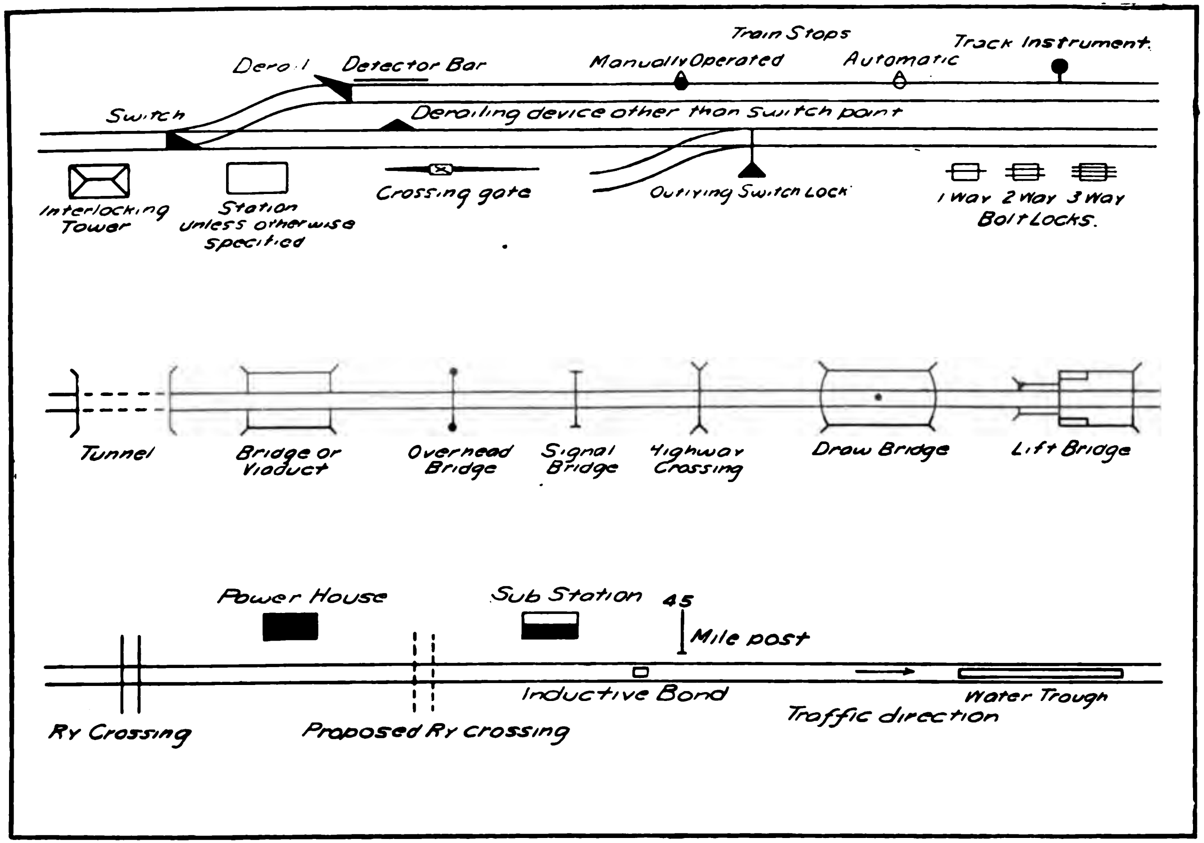
These symbols, together with a few that have been added recently, are shown on the following pages and are used throughout the course.

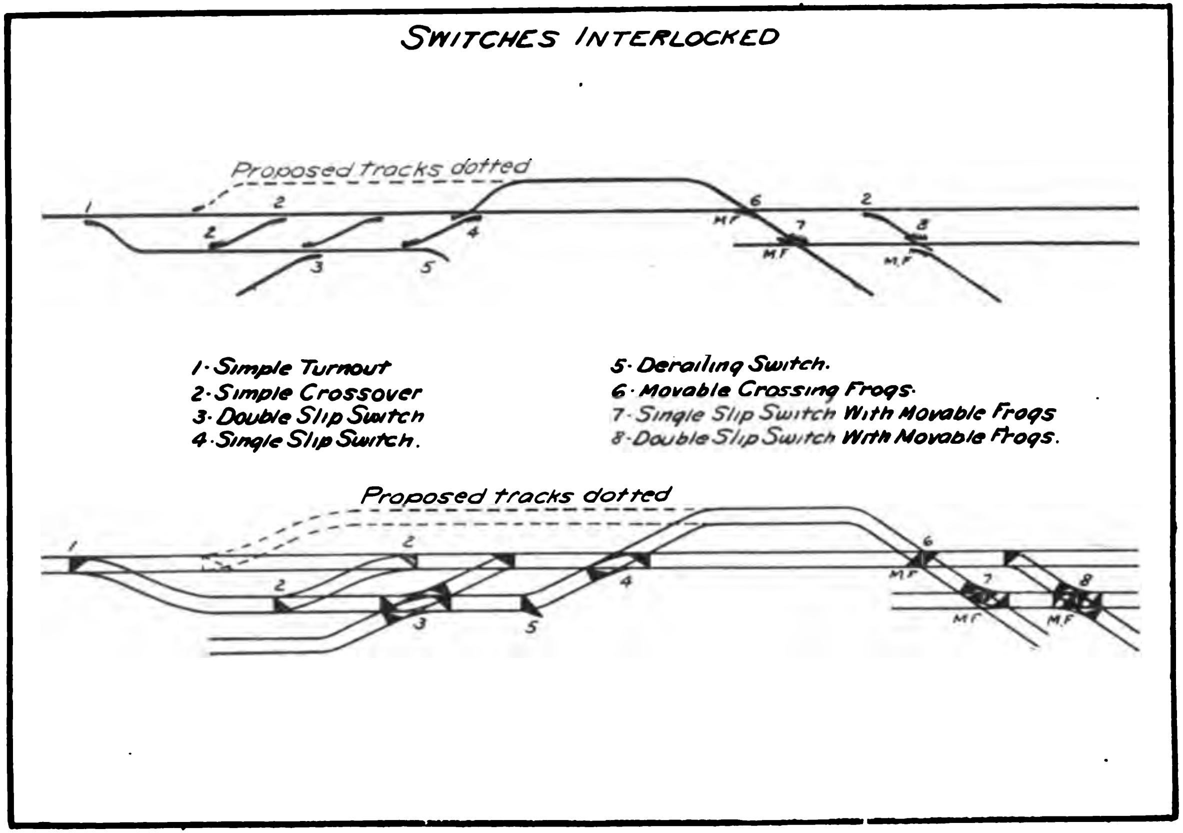
Explanations of same are given only at this point.


HOME
DISTANT
TRAIN ORDER
Non automatic interlocked 2 Pos
Non automatic interlocked 3 Pos
Mechanical interlocked 2 Pos. Slotted
Mechanical interlocked 3 Pos Slotted
Power Signal at mechanical interlocking plants, no track circuit
Semi automatic power 2 position
Semi automatic, power 3 position
Automatic. 2 position
Automatic 3 position
Fixed blade
Note Existing signals foreign to purpose of plan to be shown in dotted lines
Ground pole
Ground pole with cantilever attachment
Cantilever bracket pole
Standard bracket pole
Suspended signal

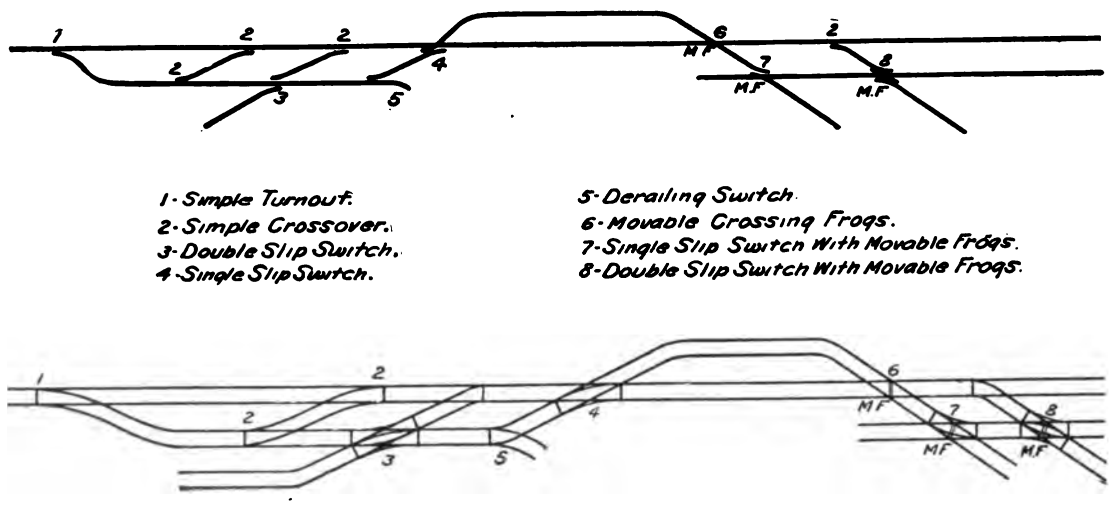

One Arm Dwarf
Two Arm Dwarf
Ground Route Signal
Pot Signal
Ring enclosed signal means
Light Signal only
Disc Signals;
Home
Distant
Clear
Danger
Double Functioned


X Switch Box Location
Switch Bell
Highway Crossing Bell
Switch Indicator Semaphore Type
Switch Indicator and Switch Box Semaphore Type
Switch Indicator Disc Type
Switch Indicator and Switch Box Disc Type
Track Battery
Insulated Joints Track Circuits in both Directions
Insulated Joints track circuit on left No circuit on the right
Insulated Joints Track Circuit on the Right No Circuit on the Left
3
Battery Chute Figure in Circle indicates Capacity
5
Battery Cupboard Figure indicates capacity
5
Above Surface
5
Half above Surface
5
Below Surface
Battery Shelter Figure indicates capacity
2
Relay Box Figure indicates Capacity
2
Relay Box and Post Figure indicates Capacity
3
3
Battery Chute, Relay Box and Post Upper figure indicates Capacity of relay box Lower figure indicates capacity of chute
J
Junction box
T
Terminal box
Insulated Switch rod

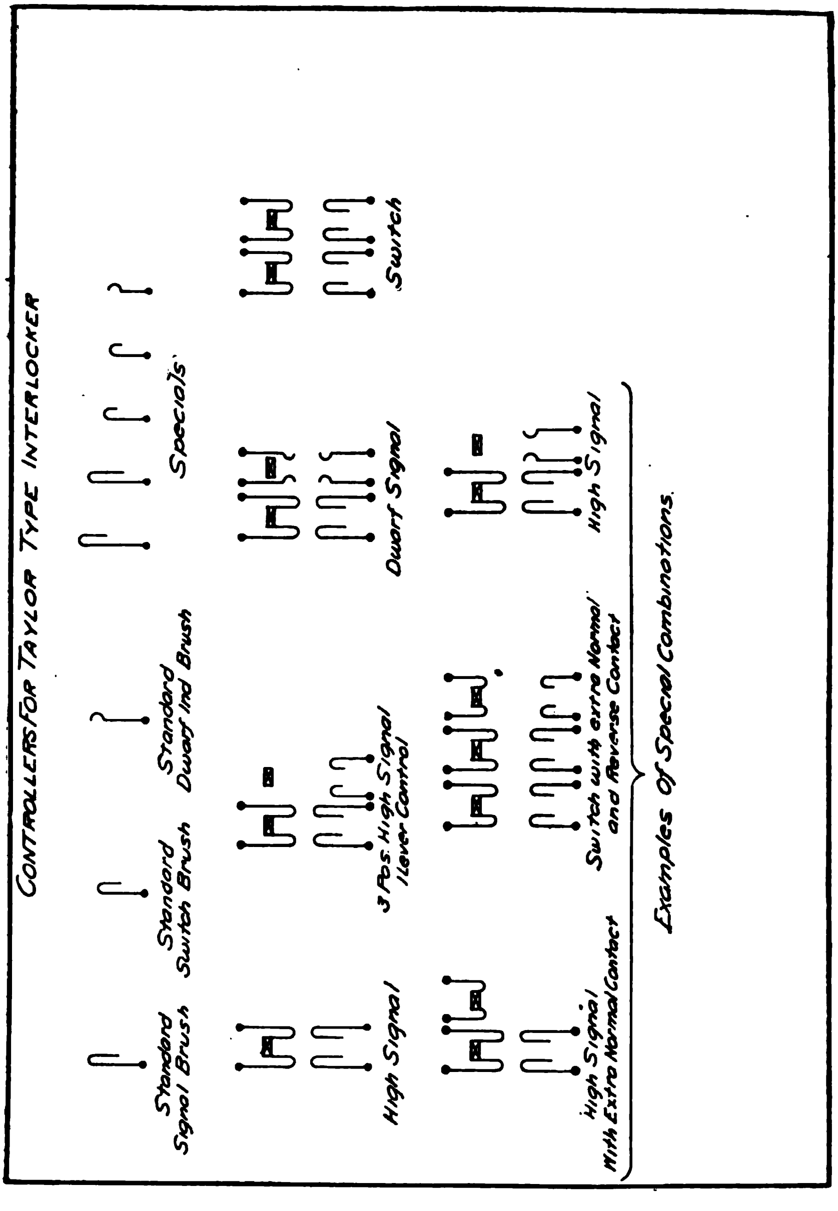

Train Stops
Track Instrument.
Derail
Detector Bar
Manually Operated
Automatic
Switch
Derailing device other than Switch point
Interlocking Tower
Station unless otherwise specified
Crossing gate
Outlying Switch Lock
1 Way 2 Way 3 Way
Bolt Locks.
Tunnel
Bridge or Viaduct
Overhead Bridge
Signal Bridge
Highway Crossing
Draw Bridge
Lift Bridge
Power House
Sub Station
45
Mile post
Inductive Bond
Water Trough
Traffic direction
Ry Crossing
Proposed Ry crossing

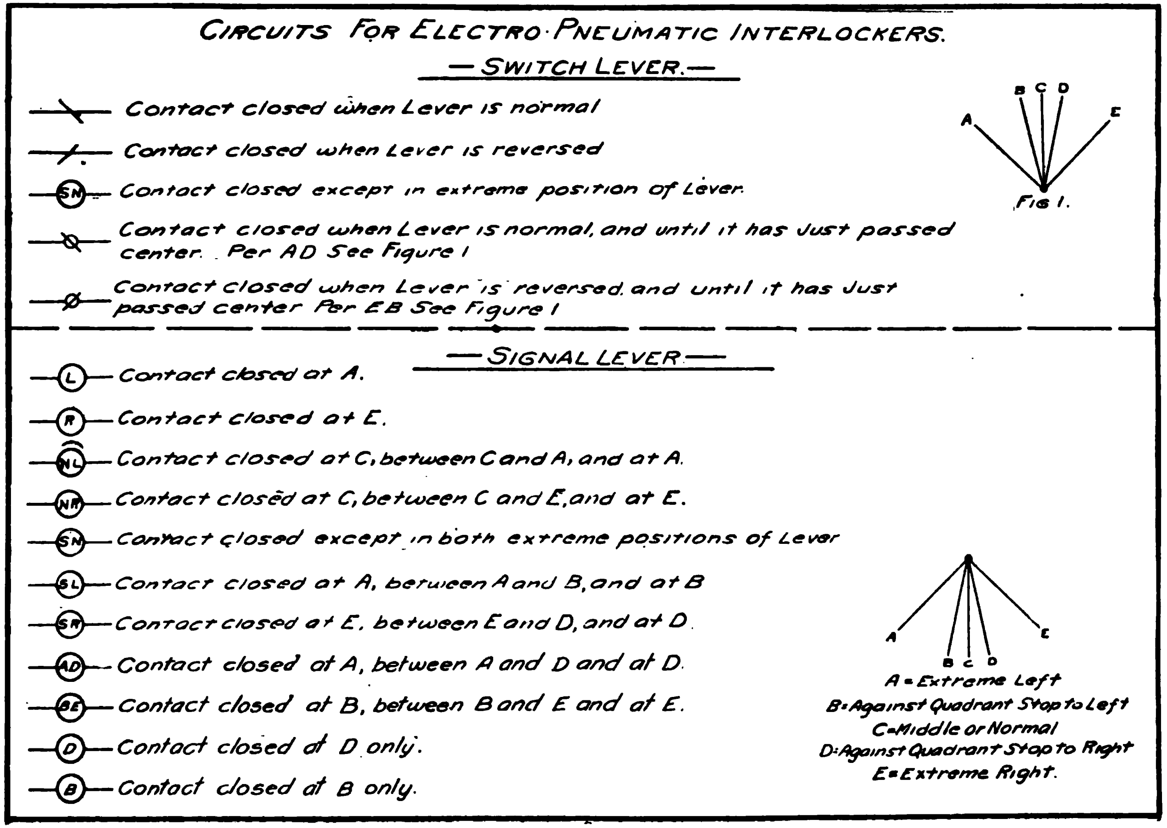

SWITCHES INTERLOCKED
Proposed tracks dotted
1
2
2
3
4
5
6
M.F
7
M.F
2
8
M.F
1·Simple Turnout
2·Simple Crossover
3·Double Slip Switch
4·Single Slip Switch.
5·Derailing Switch.
6·Movable Crossing Frogs.
7·Single Slip Switch With Movable Frogs
8·Double Slip Switch With Movable Frogs.
Proposed tracks dotted
1
2
2
3
4
5
6
M.F
7
M.F
8
M.F

SWITCHES-NON INTERLOCKED



1-Simple Turnout.
2-Simple Crossover.
3-Double Slip Switch.
4-Single Slip Switch.
5-Derailing Switch.
6-Movable Crossing Frogs.
7-Single Slip Switch With Movable Frogs.
8-Double Slip Switch With Movable Frogs.


CONTROLLERS FOR TAYLOR TYPE INTERLOCKER
Standard Signal Brush
Standard Switch Brush
Standard Dwarf Ind Brush
Specials
High Signal
3 Pos. High Signal 1 Lever Control
Dwarf Signal
Switch
High Signal With Extra Normal Contact
Switch with extra Normal and Reverse Contact
High Signal
Examples Of Special Combinations.



CIRCUITS FOR ELECTRO-PNEUMATIC INTERLOCKERS.

— SWITCH LEVER. —

Contact closed when Lever is normal

Contact closed when Lever is reversed

SN — Contact closed except in extreme position of Lever.

Contact closed when Lever is normal, and until it has just passed center. Per AD See Figure 1

Contact closed when Lever is reversed, and until it has just passed center Per EB See Figure 1

— SIGNAL LEVER —

L — Contact closed at A.

R — Contact closed at E.

NL — Contact closed at C, between C and A, and at A.

NR — Contact closed at C, between C and E, and at E.

SN — Contact closed except in both extreme positions of Lever

SL — Contact closed at A, between A and B, and at B

SR — Contact closed at E, between E and D, and at D.

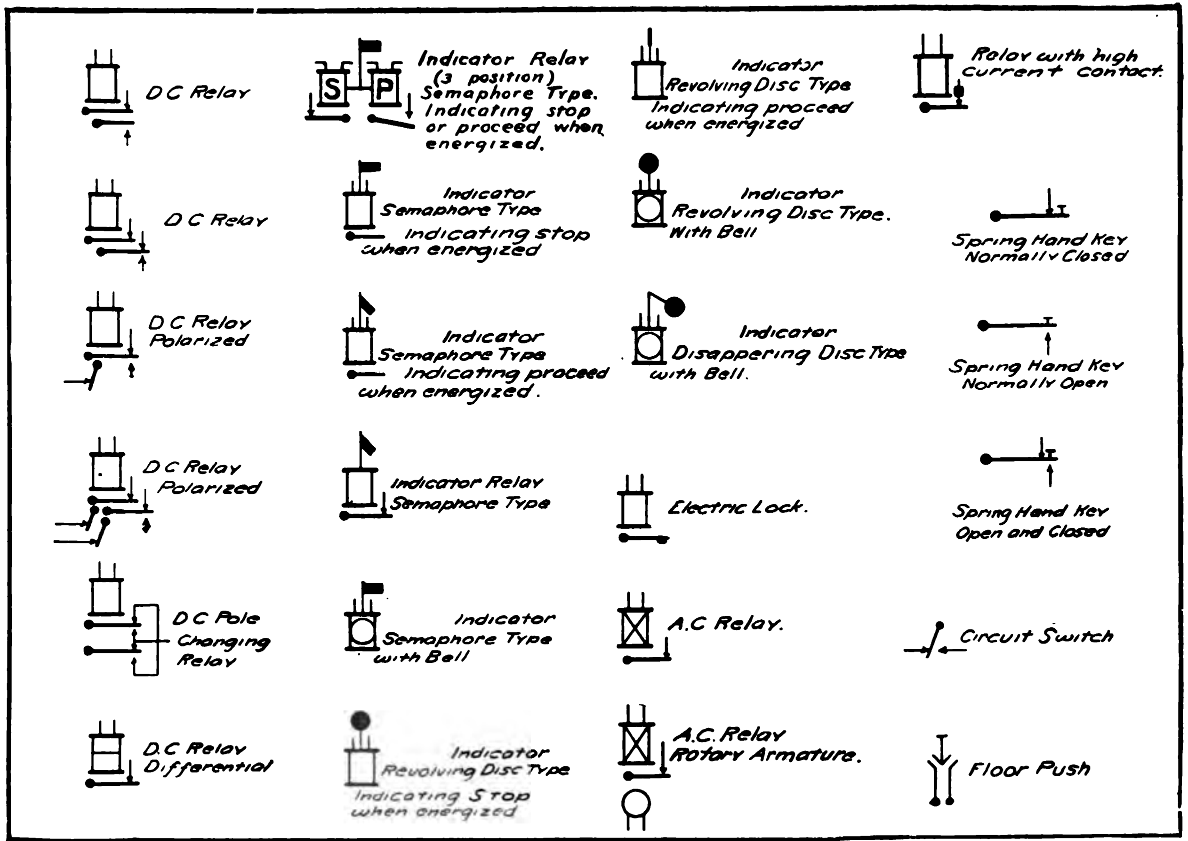
AD — Contact closed at A, between A and D and at D.

BE — Contact closed at B, between B and E and at E.

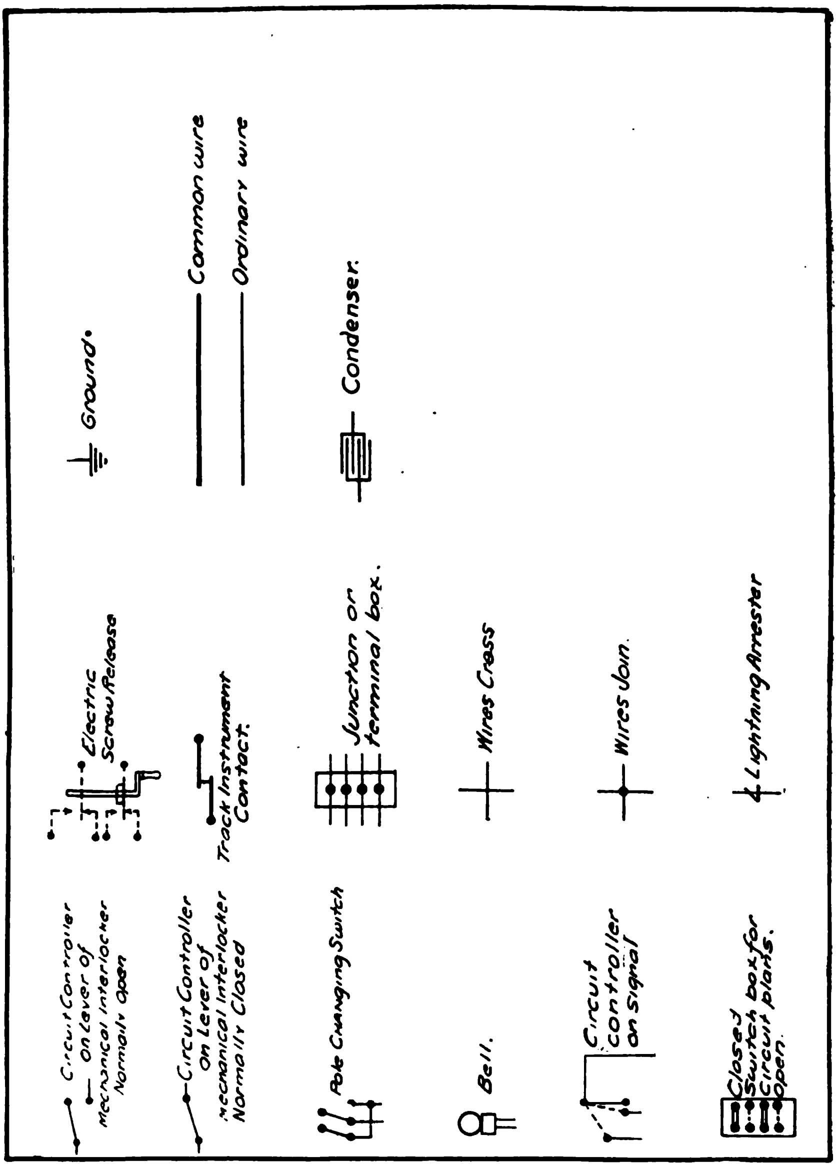
D — Contact closed at D only.

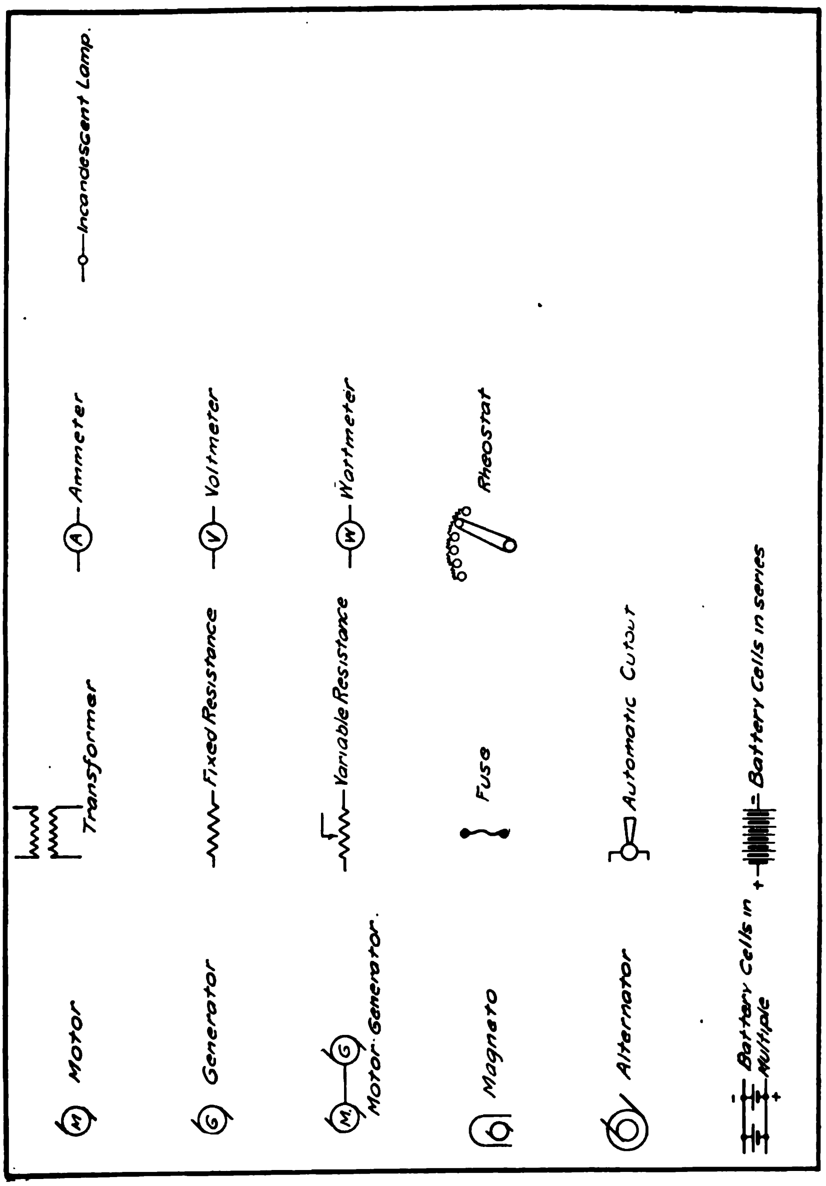
B — Contact closed at B only.

A = Extreme Left
B = Against Quadrant Stop to Left
C = Middle or Normal
D = Against Quadrant Stop to Right
E = Extreme Right.


D C Relay
Indicator Relay (3 position) Semaphore Type. Indicating stop or proceed when energized.
Indicator Revolving Disc Type Indicating proceed when energized
Relay with high current contact.
D C Relay
Indicator Semaphore Type Indicating stop when energized
Indicator Revolving Disc Type. With Bell
Spring Hand Key Normally Closed
D C Relay Polarized
Indicator Semaphore Type Indicating proceed when energized.
Indicator Disappering Disc Type with Bell.
Spring Hand Key Normally Open
D C Relay Polarized
Indicator Relay Semaphore Type
Electric Lock.
Spring Hand Key Open and Closed
D C Pole Changing Relay
Indicator Semaphore Type with Bell
A.C Relay.
Circuit Switch
D.C Relay Differential
Indicator Revolving Disc Type Indicating Stop when energized
A.C. Relay Rotary Armature.
Floor Push
S
P

Note.- When an indicator is used as a distant signal repeater it indicates "caution" or "clear" instead of "stop" or "proceed".


Ground.
Common wire
Ordinary wire
Condenser.
Electric Screw Release
Track Instrument Contact.
Junction or terminal box.
Wires Cross
Wires Join.
Lightning Arrester
Circuit Controller on Lever of mechanical interlocker Normally open
Circuit Controller on Lever of mechanical interlocker Normally Closed
Pole Changing Switch
Bell.
Circuit controller on signal
Closed Switch box for Circuit plants. Open.


M
Motor
Transformer
A
Ammeter
Incandescent Lamp.
G
Generator
Fixed Resistance
V
Voltmeter
M.
G
Motor-Generator.
Variable Resistance
W
Wattmeter
Magneto
Fuse
Rheostat
Alternator
Automatic Cutout
-
+
Battery Cells in Multiple
+
-
Battery Cells in Series

ELEMENTS OF MECHANICS

MATTER

1. **Mechanics** is the science that treats of the actions of bodies upon each other and of their relative motions.

2. Anything that occupies space is **matter.** There are many different kinds of matter or substances, such as iron, wood, oil, etc. Distinct portions of matter, such as a rail, a tie, or an insulator are called **bodies.**

3. It is assumed that matter is composed of exceedingly small particles called **molecules** which in turn are groups or systems of indivisible particles called **atoms**; that the atoms forming a molecule and in turn the molecules of a body are held together by certain attractive forces; and that for a given kind of matter each one of the molecules is just like every other molecule both as to the number and arrangement of atoms.

4. Bodies are divided into three classes called **solid, liquid** and **gaseous*** bodies. The term fluids includes both liquids and gases.

5. **Solid bodies** are bodies whose molecules are held together so closely that the bodies tend to keep a given shape and to resist, to a certain degree, efforts to change it. Zinc, wood, and rubber are examples of this class.

6. **Liquid bodies** are bodies whose molecules, while tending to cling together, move easily with relation to one another, as

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*Aeriform.

for example water or alcohol. Bodies of this class retain no definite form except that determined by the containing vessel. The free surfaces of such bodies always assume a horizontal or level position.

7. **Gaseous bodies** differ from liquid bodies in that the molecules tend to withdraw from each other. This class is subdivided into two groups called **permanent gases** and **vapors.** A **permanent gas,** such as carbon dioxide,* is one that remains in the gaseous condition at ordinary temperatures and pressures while under the same conditions a **vapor,** such as steam, changes to a liquid or solid.

8. By changing the conditions it is known to be possible in most cases to cause matter to take any of the forms mentioned. As a familiar illustration of this, water may be changed to steam (a vapor) by heating, or to ice (a solid) by cooling.

9. Some other effects due to changes of conditions are called **solution, diffusion,** and **crystallization.**

10. When a solid or gaseous body is changed to the liquid form by means of a liquid known as a **solvent,** the transformation is called **solution.** Thus salt may be dissolved in water thereby changing from the solid to the liquid form, or carbon dioxide may be similarly dissolved changing from the gaseous to the liquid form. The term **saturated solution** signifies that the solvent has dissolved as large a quantity of the given solid or gas as it can. The quantity of a solid in a saturated solution in almost all cases varies with the temperature, and of a gas, with the temperature and pressure.

11. When two liquids or gases placed in contact with each other gradually mix of their own accord the action is called **diffusion.** The mixing of alcohol and water is an illustration of this. All liquids will not mix with each other as in the case of oil and water.

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*Carbonic acid gas.

12. Many bodies when changing slowly from the gaseous or liquid to the solid condition take regular forms termed crystals, this action being called **crystallization.** The crystals of a given substance always take the same general form and arrangement under the same conditions. Fig. 1 shows crystals of salt and alum.

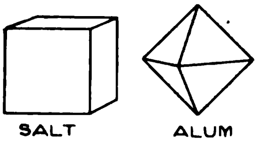

FIG. 1

CHARACTERISTICS OF MATTER

13. Matter has certain characteristics or properties of which the most general are defined in the following paragraphs.

14. The **mass** of a body is the quantity of matter which it contains and is constant for a given body. For example, regardless of where we put or what we do with a body, such as a porcelain insulator, it contains the same quantity of matter as long as matter is not taken from or added to it.

15. All bodies are attracted toward the earth. The measure of this attraction is called **weight.** Thus the insulator in Art. 14 is attracted to the earth by a definite pull which may be measured by a spring balance.

16. **Inertia** is that property of matter which is indicated by the tendency of a body at rest to remain at rest and of a body in motion to move at a uniform rate in a straight line. An example of inertia is found in the effort required to start and stop a train.

17. **Impenetrability** may be defined as a characteristic of matter on account of which two bodies cannot occupy the same space at the same time. Thus a hole must be drilled or punched through a steel rail before a bolt can be put through the rail.

18. **Porosity** signifies that the molecules of a body do not fit closely together, but that there are spaces between them. This fact is illustrated by the action of alcohol and water when mixed. If a measured volume of alcohol is poured into a measured volume of water, the volume of the mixture will be less than the sum of the original volumes. The bodies do not occupy the same space at the same time but the molecules of one wedge themselves between the molecules of the other.

As a result of porosity bodies may be compressed.

19. **Elasticity** is a property of matter which makes it possible for bodies when they have been distorted by an external force to regain their original form or volume when the force is removed. As regards form a steel spring illustrates the characteristics of a highly elastic body. Fluids are perfectly elastic as regards volume, but have no elasticity of form.

20. The term **conservation of matter** expresses the fact that matter cannot be destroyed. When wood or coal is burned apparently all but a small portion is destroyed, while the fact is that the quantity of matter is the same as before, only in a different form; ie, water, vapor, gas, etc.

21. **Hardness** is the tendency of a substance with this quality to resist any effort to penetrate between its particles. Thus steel is harder than lead and resists any attempt to scratch it with the lead.

22. **Brittleness** is the inability of a substance with this quality to resist shock, although it is capable of withstanding considerable pull or pressure when applied steadily. Glass is a familiar example.

23. **Tenacity** is the ability of a substance with this quality to resist a force tending to pull it apart. A rod of manganese bronze shows a high degree of tenacity while a similar rod of lead shows little.

24. **Malleability** signifies that a substance with this quality may be rolled or hammered into sheets. Thus copper is said to be very malleable.

25. **Ductility** signifies that a substance with this quality can be drawn into wire. For example iron is considered to be ductile.

MOTION AND FORCE

MOTION

26. When a body changes its position it is said to be in **motion**; for example, a moving train.

27. The **path** of a body in motion is the line traversed by some point about which all the matter composing the body may be balanced.

28. Newton's Laws of Motion:—

(1) *All bodies continue in a state of rest or of uniform motion in a straight line unless compelled to change that state by an external force.*

(2) *A given force has the same effect in changing motion, whether the body on which it acts is in motion or at rest; whether it is acted upon by that force alone, or by others at the same time.*

(3) *Action and reaction are equal and opposite in direction.*

29. The first law is a description of the behavior of a body when no force is acting. Thus if a body is moving and no force acts to retard or increase its rate of motion the body will move on at the same speed indefinitely. This law cannot be verified by experiment since there are always forces acting, such as the resistance of air, that tend to bring a moving body to rest. If a body is at rest and no forces act to set it in motion it is evident that it will remain at rest.

30. The second law is a description of the behavior of a body subject to the action of a force. Thus a body when acted upon by a force changes its rate of motion in the direction of the force and at a rate which is proportional to the force and inversely proportional to the mass of the body. The effect of a force may be considered as the result produced when it acts alone. Its tendency to produce this result is the same when other forces are acting.

31. The third law sets forth the fact that a force is simply the action of one body upon another and that the second body opposes the action of the first with an equal and opposite force. Thus a pail hung on a spring will stretch the spring till it pulls with a force that just balances the weight of the pail; then if the pail is filled with water the spring is stretched to another point where it just balances the weight of both pail and water.

32. In the preceding text the *rate of motion* has been referred to. *Rate of motion is expressed in terms of the distance traveled in a given period of time and is called* **velocity.** Thus air flowing through a pipe may be said to have a velocity of so many feet per second, as 20 ft. per sec.

33. The relation of *velocity* to *time* and *distance* is expressed by the following formulas. If $d =$ distance in feet, $t =$ time in seconds and $v =$ velocity in feet per second; then

$$v = \frac{d}{t}; \qquad (1)$$

$$d = tv; \qquad (2)$$

$$t = \frac{d}{v}. \qquad (3)$$

34. PROBLEM.—A train moving with uniform velocity passes over a distance of 1,500 ft. in 30 sec. (*a*) What is the velocity in feet per second? (*b*) How many miles would the train travel in 1 hr.? (*c*) How long would it take the train to run 90 mi.?

SOLUTION.—(*a*) The problem states that the distance in feet or $d = 1{,}500$ and that the time in seconds or $t = 30$. Substituting these values in formula (1), we have

$$v = \frac{1500}{30} = 50 \text{ ft. per sec., } \textit{Ans.}$$

(*b*) Since 1 hr. equals 3,600 sec. $= t$ and, from (*a*), $v = 50$ ft. per sec., substituting in formula (2), we have

$$d = 3600 \times 50 = 180{,}000 \text{ ft.}$$

This may be reduced to miles by dividing by 5,280, the number of feet per mile; thus

$$\frac{180000}{5280} = 34.09 \text{ mi., } \textit{Ans.}$$

(*c*) Since 1 mi. equals 5,280 ft.,

$$d = 5280 \times 90 = 475{,}200 \text{ ft.}$$

Substituting in formula (3),

$$t = \frac{475200}{50} = 9{,}504 \text{ sec.}$$

Dividing by 60 to reduce to minutes,

$$\frac{9504}{60} = 158 \text{ min., } 24 \text{ sec.}$$

Dividing 158 by 60 to reduce to hours,

$$\frac{158}{60} = 2 \text{ hr., } 38 \text{ min.}$$

Therefore the required time is 2 hr., 38 min., 24 sec., *Ans.*

35. PROBLEMS.—(1) Two signals governing movements in the same direction on a single track are located 4,000 ft. apart. Train A is traveling at a uniform speed of 30 mi. per hr. and is just passing the first signal. Train B is 5,500 ft. behind train A and is traveling at a uniform speed of 36 mi. per hr.* (*a*) Will train A pass the second signal before train B reaches the first signal? (*b*) State how far apart the trains will be when A reaches the second signal.

(2) Two trains A and B are running in the same direction on parallel tracks and A is traveling at a uniform speed of 25 mi. per hr. If they both pass a given point at the same time and if when A has reached a signal 3,000 ft. beyond the given point, B is 1,800 ft. ahead of A, what is the velocity of B in miles per hour?

(3) If signals on a single track are located at equal distances from each other and a train moving at a uniform velocity of 40 mi. per hr. passes one signal at 2:15 P. M., and the fifteenth signal beyond this at 2:35 P. M., what is the distance between signals?

(4) If the outside diameter of the armature of an electric motor is 3 in. and it makes 1,800 r. p. m.;† (*a*) What is the velocity of the surface in feet per second? (*b*) How long in minutes and seconds would it take this surface to travel 5 mi.? (Use 3.14 as the value of π.)‡

(5) If two signal cabins are 2 mi. apart, at what rate in feet per second would a repairman have to walk in order to get from one to the other in 35 min.?

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*To reduce miles per hour to feet per second multiply by the fraction $\frac{22}{15}$.

If answers obtained are very near, but do not exactly agree with the answers given it is probable that a different number of decimal places was used in the solution.

In some instances where the unit is very small, as in the case of seconds, the decimal places have been dropped from the final result.

†Revolutions per minute.

‡See Mensuration in Drawing Appendix.

Answers.—(1) (*a*) Yes, (*b*) 4,700.48 ft. (2) 40 mi. per hr. (3) 4,693.33 ft. (4) (*a*) 23.55 ft. per sec., (*b*) 18 min., 41 sec. (5) 5 ft. per sec.

36. *Velocity may be uniform or variable.* Thus when a locomotive is starting its velocity is constantly changing. *Velocity may change either at a uniform or variable rate. If a body moving with a velocity that varies uniformly passes over a certain distance in a certain time, the uniform velocity that would carry it over the same distance in the same time is called the* **average velocity** *for that period.*

37. The relation of *average velocity* to *initial* and *final velocities* is expressed by the following formulas.

If v_1 = initial, v_2 = final and v_0 = average velocity[*]; then

$$v_0 = \frac{v_1 + v_2}{2}. \qquad (4)$$

If the body starts from a *state of rest* the initial velocity $v_1 = 0$ and formula (4) reduces to

$$v_0 = \frac{v_2}{2}. \qquad (5)$$

38. Problem.—A train passing out of a station has a velocity of 5 mi. per hr. and when passing a signal 3,000 ft. beyond the station has attained a velocity of 35 mi. per hr., the rate of increase being uniform. (*a*) What is the average velocity that would have carried it over the same distance in the same time? (*b*) If the train starts from a state of rest, 500 ft. before passing out of the station, what is the average velocity? (*c*) How long did it take to travel the 3,000 ft. in the first case?

Solution.—(*a*) From the statement of the problem the initial velocity $v_1 = 5$ mi. per hr. and the final velocity $v_2 = 35$ mi. per hr. Substituting these values in formula (4), we have

$$v_0 = \frac{5 + 35}{2} = 20 \text{ mi. per hr., } \textit{Ans.}$$

(*b*) In this case $v_1 = 0$ since the train starts from rest or a state of no velocity, therefore substituting in formula (5),

$$v_0 = \frac{35}{2} = 17\tfrac{1}{2} \text{ mi. per hr., } \textit{Ans.}$$

(*c*) Substituting the value of v_0, obtained in (*a*), reduced to feet per second for v in formula (3),

$$t = \frac{3000}{20 \times \frac{22}{15}} = 1 \text{ min., } 42 \text{ sec., } \textit{Ans.}$$

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[*] When these symbols are used in formulas combined with symbols such as t – times in seconds or d – distance in feet they will have to be expressed in feet per second.

39. Problems.—(1) A train starts from rest and is brought up to speed in 2 min. (*a*) What is the final velocity if the average velocity is 30 mi. per hr.? (*b*) What is the distance traversed?

(2) If the signals on a railroad are 2,500 ft. apart and a train starts from rest exactly half way between two signals, how many will it have passed when at the end of 4 min. it has attained a speed of 60 mi. per hr.?

Answers.—(1) (*a*) 60 mi. per hr., (*b*) 1 mi. (2) 4 signals.

40. *When velocity varies uniformly the rate of change expressed in units of distance and time is called* **uniform acceleration** *or* **uniform retardation** *according as the velocity is increased or decreased.* Thus a train in starting may be accelerated at a rate of ½ ft. per sec. per sec.*

41. For bodies considered to start from a state of rest, or $v_1 = 0$, the relation of *uniform acceleration* to *time* and *distance* is expressed by the following formulas. If $a =$ acceleration in feet per second per second, $v_2 =$ final velocity in feet per second,† $d =$ distance in feet, $d_1 =$ distance in feet traversed in any particular unit of time, and $t =$ time in seconds; then

$$v_2 = at; \qquad (6) \qquad\qquad d_1 = \frac{a(2t-1)}{2}; \qquad (10)$$

$$v_2 = \sqrt{2ad}; \qquad (7) \qquad\qquad t = \sqrt{\frac{2d}{a}}; \qquad (11)$$

$$d = \frac{at^2}{2}; \qquad (8) \qquad\qquad a = \frac{2d}{t^2}; \qquad (12)$$

$$d = \frac{v_2 t}{2}; \qquad (9) \qquad\qquad a = \frac{v^2_2}{2d}. \qquad (13)$$

42. Problem.—A train starting from a state of rest is brought up to a speed of 60 mi. per hr. in 2 min. (*a*) Compute the acceleration. (*b*) Compute the distance traveled. (*c*) Compute the distance traveled during the 70th sec. after starting. (*d*) Compute the time required to travel 3,000 ft. (*e*) If the train had traveled 4,000 ft. in 1 min. 40 sec. compute the acceleration.

Solution.—(*a*) The problem states that the final velocity is 60 mi. per hr. Reducing this to feet per second, $v_2 = 88$. Similarly the value of t is given as 2 min. or 120 sec. Substituting the values of v_2 and t in formula (6) and transposing, we have

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*This means that a change of velocity of ½ ft. per sec. takes place in 1 sec.

†In cases of retarded motion this should be considered the initial velocity.

$$a = \frac{88}{120} = 0.73 \text{ ft. per sec. per sec., } \textit{Ans.}$$

(*b*) Substituting their values for v_2 and t in formula (9).

$$d = \frac{88 \times 120}{2} = 5{,}280 \text{ ft. } = 1 \text{ mi., } \textit{Ans.}$$

(*c*) Substituting their values for a and t, which in this case equals 70, in formula (10),

$$d_1 = \frac{0.73\,(2 \times 70 - 1)}{2} = 50.73 \text{ ft., } \textit{Ans.}$$

(*d*) This is obtained from formula (11), by substituting 3,000 for d and in place of a its value obtained in (*a*); thus,

$$t = \sqrt{\frac{2 \times 3000}{0.73}} = 90 \text{ sec., } \textit{Ans.}$$

(*e*) The required result is obtained by substituting in formula (12); giving

$$a = \frac{2 \times 4000}{100^2} = 0.8 \text{ ft. per sec. per sec., } \textit{Ans.}$$

43. PROBLEMS.—(1) If two trains start from the same point at the same time and train A is accelerated at a rate of 0.5 ft. per sec. per sec. and train B at a rate such that it will travel 50 ft. during the 80th sec.; (*a*) How far apart will they be at the end of the 80th sec.?* (*b*) In what time will each attain a speed of 75 mi. per hr.? (*c*) How far will each travel before reaching a speed of 30 mi. per hr.?

(2) If the speed of a motor starting from rest is uniformly accelerated so as to bring it to 1,800 r. p. m. in 1½ sec.; what is the acceleration at the surface of the armature if it is 3 in. in diameter?

(3) If a train is uniformly accelerated at a rate of 0.5 ft. per sec. per sec.; (*a*) Compute and make a table of the distances passed over in each second from 1 to 6. (*b*) Compute and make a table of the velocities at the end of each second from 1 to 6. (*c*) State the relation that exists between the consecutive figures in each case.

(4) If a train starts at one signal and has just reached speed when passing a signal 4,000 ft. beyond the first and it has taken 3 min. to travel the distance between signals; (*a*) What is the acceleration? (*b*) What is the final velocity?

(5) If a train leaving a station was accelerated at a rate of 0.6 ft. per sec. per sec. and after passing a signal 3,000 ft. from the start-

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*Use formula (10) to find the acceleration of train B. Then use formula (8).

ing point power was shut off and brakes applied bringing it to a stop in 600 ft.; (*a*) Compute the retardation.* (*b*) How much time was taken to stop the train? (*c*) What was the velocity 1 sec. before stopping?

ANSWERS.—(1) (*a*) 412 57 ft., (*b*) Train A in 3 min., 40 sec.; Train B in 2 min., 54.9 sec., (*c*) Train A, 1,936 ft.; Train B, 1,539.12 ft. (2) 15.7 ft. per sec. per sec. (3) (*a* and *b*)

Seconds	Distances	Velocities
1	0.25	0.5
2	0.75	1.0
3	1.25	1.5
4	1.75	2.0
5	2.25	2.5
6	2.75	3.0

(*c*) The distances increase in the same ratio as the numbers 1-3-5-7-9-11 and the velocities in the same ratio as the time. (4) (*a*) 0.246 ft. per sec. per sec., (*b*) 30.19 mi. per hr. (5) (*a*) 3 ft. per sec. per sec., (*b*) 20 sec., (*c*) 3 ft. per sec.

FORCE

44. The first law of motion conveys an idea of force which may be stated as follows:—*A* **force** *is the cause that produces or tends to produce a change in the condition of a body as regards motion or rest.* For example, if a semaphore blade stands in the "stop" position (see Fig. 21), it is necessary to apply a force to move it to the "proceed" position.

45. The measurement or comparison of forces requires knowledge concerning three facts as follows:

The **point of application,** *which is the point where a force acts on a body.*

The **direction,** *which is the straight line along which a force tends to move the point of application.*

The **magnitude,** *which is the value of a force compared with a standard.*

46. The unit which is ordinarily used as a standard by which to measure force is the weight or measure of the earth's attraction for a standard unit of mass and is called the **pound**

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*Use formulas (7) and (13).

(avoirdupois). Thus it may be said that a force of 100 lbs. is required to throw a switch under certain conditions.

47. For the purpose of comparing forces and determining the result of the action of two or more forces on a body it is convenient to represent forces by lines. The *length* of the line is made to correspond to the *magnitude* of a given force and its *direction* to the *direction* of the force. For example the arrow in Fig. 2 is a complete graphic representation of a force. Its length is made to correspond to the magnitude of the force, thus 10 in. may represent 10 lbs. Its direction is the direction of the force and the point of application is at one end or the other according as the force is a push or pull.

Fig. 2

48. *When two or more forces act on a body and produce motion it is called* **resultant motion** *and the single force that would produce a like effect is called the* **resultant.**

49. When the points of application, directions and magnitudes of the several forces acting are known, the direction and magnitude of the resultant may be found by combining the forces. This is known as the **composition of forces.**

50. Some of the more general cases that come under the composition of forces are stated in the following rules:

(1) *The resultant of two forces acting in the same direction along the same line is equal in magnitude to the sum of the given forces, Fig. 3.*

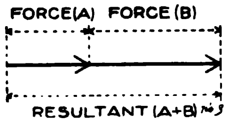

Fig. 3

(2) *The resultant of two forces with different points of application acting in the same direction along parallel lines is equal in magnitude to the sum of the given forces. Its point of application is on a straight line joining the points of application of the given forces and the distance from the greater force to this point, is to the distance from the lesser force, as the lesser force is to the greater, Fig. 4.*

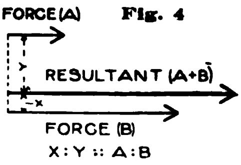

Fig. 4

(3) *The resultant of two forces acting in opposite directions along the same line is equal in magnitude to the difference between the given forces, Fig. 5.*

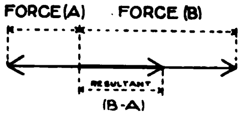

Fig. 5

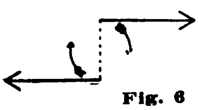

Fig. 6

(4) *Two equal parallel forces with different points of application acting on a body in opposite directions form a couple which produces rotary motion and can have no resultant, Fig. 6.*

(5) *The resultant of two forces that have a common point of application but act in different directions may be found graphically by a construction called the* **parallelogram of forces.**

51. The parallelogram of forces may be described as follows. In Fig. 7 let OA and OB represent the magnitudes and directions of two forces having a common point of application O. Draw AC parallel to OB and BC parallel to OA, forming the parallelogram AOBC. Then the diagonal OC will represent the resultant of OA and OB both as to magnitude and direction.

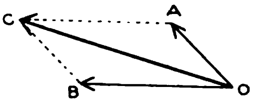

Fig. 7

52. Problems.—(1) Two men are steadying a telegraph pole with pike poles. They push with forces of 20 and 30 lbs. respectively and the pike poles are at an angle of 30 deg. to each other, Fig. 8. Determine graphically by the method of Art. 51 the magnitude of the resultant and its direction relative to the pike poles.

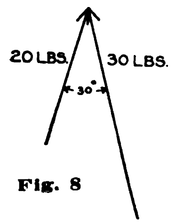

Fig. 8

(2) At the terminal of an electric road it is necessary to take up the pull of the trolley wire by two guy wires run to the sides of the street. These wires are at an angle of 90 deg. to each other and the tension on one of them is 50 lbs. and on the other 200·lbs., Fig. 9. Determine by diagram the pull on the trolley wire and its direction relative to the guy wires.

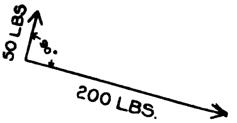

Fig. 9

(3) If two men are pulling in the same direction on a rope fastened to a beam, one with a force of 30 lbs. and the other with a force of 35 lbs., what is the total pull on the beam?

(4) If two men are trying to straighten a wire by pulling at opposite ends and one pulls with a force of 30 lbs. and the other with a force of 40 lbs., determine the resultant both as to magnitude and direction.

(5) Two men, A and B, 10 ft. apart, are carrying a heavy casting by means of a pole slipped through it and supported on their shoulders. A supports a weight of 75 lbs. and B only 25 lbs. Disregarding weight of pole, how much does the casting weigh and how far is it from B?

ANSWERS.—(1) About 48⅓ lbs. at an angle of 12 deg. to the 30 lb. force. (2) About 206 lbs. at an angle of 14 deg. to the 200 lb. force. (3) 65 lbs. (4) 10 lbs., toward the 40 lb. pull. (5) 100 lbs., 2½ ft. from A.

53. When it is desired to find the resultant of more than two forces, the resultant of two of them may be obtained first and then combined with a third and so on; the last diagonal will be the required resultant of all the forces. Thus in Fig. 10, OA, OB and OC represent three forces having a common point of application O. To find the resultant of all three, first combine OA with OB which gives the force OD; then combine OD with OC to obtain the desired resultant OE.

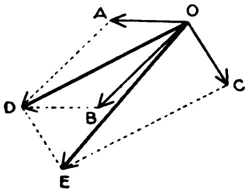

Fig. 10

54. *When several forces acting on a body just balance each other so that there is no tendency to produce motion, the forces are said to be in* **equilibrium** *and their resultant is zero.* Thus a body hung on a spring stretches the spring until it exerts a force that balances the weight of the body and the body comes to rest and remains stationary at this point.

55. By means of the parallelogram of forces a single force may be resolved into components in a manner just the reverse of that used for finding the resultant of several forces. As an illustration, take the case of a yard engine on one track pushing a car on a parallel track by means of a stake. If the stake makes an angle of 15 deg. with the direction of

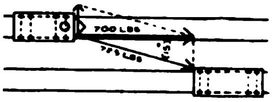

Fig. 11

the track, a force of about 700 lbs. exerted by the locomotive in a direction parallel to the track may be resolved into a force of about 188 lbs. at right angles, exerted by the locomotive wheel flanges against the rails, and a force of about 725 lbs. in the direction of the stake, as in Fig. 11.

GRAVITATION

56. All the particles of matter in the universe are acted upon by an attraction or force tending to pull them together. This force is known as **gravitation.** *It varies directly as the product of the masses of the particles and inversely as the square of the distance between their centers of mass.*

57. The **force of gravity** which is the attraction between the earth and bodies on or near its surface is the most common example of gravitation. The measure of this force is **weight.**

The mass of the earth is a constant quantity so that *doubling the mass of a body doubles the product of the masses, and therefore doubles the weight.* The weight of a given mass varies slightly for different locations on the earth's surface partly on account of the fact that the density and shape of the earth are not uniform.

For changes in position above the earth's surface there is nothing to interfere with the working of the law of gravitation. However, on going below the surface, the particles of the earth between a body and the surface exert an attraction which partially counterbalances that of the rest of the earth.

Thus bodies weigh most at the surface of the earth. Below the surface the weight decreases in the same ratio that the distance to the center of the earth is decreased. Above the surface the weight decreases inversely as the square of the distance from the center.

58. The force of gravity may be considered as a group of parallel forces, each force acting on a particle of a body. The resultant of these forces acts at the same point whatever the position of a body. This point is called the **center of gravity.**

59. If a thin flat object with parallel faces, such as a card, can be balanced on a point, Fig. 12, the center of gravity will lie vertically above the point half way between the faces.

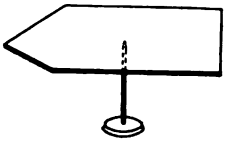

Fig. 12

60. If any solid body is suspended so that it moves freely about the point of suspension (m), view A, Fig. 13, a vertical line through this point contains the center of gravity. Similarily, if suspended from another point, (n), view B, a second vertical line containing the center of gravity is found and the required center is at the intersection, C, of the two lines.

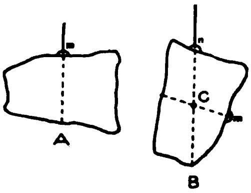

Fig. 13

61. *A body is in equilibrium if a vertical line through the point of support contains the center of gravity,* since in this case the resultant of the forces acting is zero.

A body is said to be in **stable equilibrium** when it tends to return to its original position if slightly displaced. A locomotive bell is a good example.

It is said to be in **unstable equilibrium** if it tends to fall farther away from its original position when displaced, as in the case of a broom balanced on the end of its handle.

If it neither tends to return to its original position nor to fall further away, when displaced, as in the case of a billiard ball, it is said to be in **neutral equilibrium.**

62. The **base** is the side on which a body rests. If a vertical line through the center of gravity falls inside the base, the body is in stable equilibrium and stands.

ACTION OF THE FORCE OF GRAVITY

63. The force of gravity for small distances above the earth's surface may be considered to be constant and therefore to produce uniform acceleration. In the latitude of New York the acceleration is 32.16 ft. per sec. per sec., which is the velocity that a body falling without resistance would gain during the first second of its fall.

64. *The motion of a body falling without resistance is uniformly accelerated.* Therefore the formulas for uniform acceleration hold true and a known quantity, *g*, equal to the acceleration due to gravity may be substituted for the indefinite constant *a*; thus,

$$v_2 = gt; \qquad (14) \qquad\qquad d_1 = \frac{g(2t - 1)}{2}; \qquad (17)$$

$$v_2 = \sqrt{2gd}; \qquad (15)$$

$$d = \frac{gt^2}{2}; \qquad (16) \qquad\qquad t = \sqrt{\frac{2d}{g}}. \qquad (18)$$

65. Problem.—A tie dropped from a railroad bridge reaches the bottom of the gorge in 3 sec. (*a*) How deep is the gorge? (*b*) How long would it have taken the tie to fall half the distance? (*c*) Compute the velocity at the moment in which it reached the bottom.*

Solution.—(*a*) The depth of the gorge is found from formula (16); thus,

$$d = \frac{32.16 \times 3^2}{2} = 144.72 \text{ ft., } \textit{Ans.}$$

(*b*) Half the distance is $\frac{144.72}{2} = 72.36$ ft. and the time is found from formula (18); ie.,

$$t = \sqrt{\frac{144.72}{32.16}} = 2.1 \text{ sec., } \textit{Ans.}$$

(*c*) Substituting in formula (14),

$$v_2 = gt = 32.16 \times 3 = 96.48 \text{ ft. per sec., } \textit{Ans.}$$

66. Problems.—(1) If a bolt dropped from the roof of a tall building reached the sidewalk in 4 sec. what was its velocity just before striking the walk?

(2) Compute the height of the building in the preceding problem.

(3) How far did the bolt fall in the second second?

(4) What was its velocity after falling a distance of 100 ft.?

(5) If the sill of a fifth story window is 45 ft. from the sidewalk, what time was required for the bolt to drop from the roof to this point?

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*In this and following problems take $g = 32.16$ ft. per sec. per sec. and consider the bodies as falling without resistance.

Answers.—(1) 128.64 ft. per sec. (2) 257.28 ft. (3) 48.24 ft. (4) 80.19 ft. per sec. (5) 3.6 sec.

67. If a body is given a certain horizontal velocity and is free to fall at the same time it is called a **projectile.** If two bodies are arranged so that one is projected with horizontal velocity, represented by line AB, Fig. 14, and the other dropped from a state of rest at the same moment, both will reach the surface of the earth at the same instant, since each force acts independently of every other force and a force has the same effect whether the body is in a state of motion or rest. The path of the projectile is represented by line AD and the path of the falling body by AC. The range of the projectile is the horizontal distance, CD, that it travels before reaching the ground.

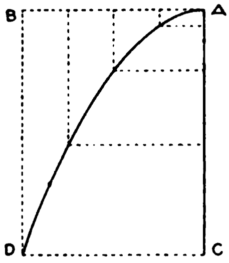

Fig. 14

68. Problem.—If a ball is dropped from a height of 257.28 ft. and at the same instant and place a projectile is given an initial horizontal velocity of 2,000 ft. per sec., assuming that the projectile is retarded uniformly and has a velocity of 1,000 ft. per sec. when it strikes the ground, what is the distance between the two at that time?

Solution.—Both will reach the ground at the same instant and the time required for the ball to fall is obtained by substituting in formula (18); ie.,

$$t = \sqrt{\frac{2 \times 257.28}{32.16}} = 4 \text{ sec.}$$

The average velocity of the projectile is obtained from formula (4) as follows,

$$v_0 = \frac{1000 + 2000}{2} = 1{,}500 \text{ ft. per sec.}$$

The distance traveled in 4 sec. is obtained from formula (2); ie.,

$$d = 1500 \times 4 = 6{,}000 \text{ ft., } \textit{Ans.}$$

This is the required distance between the projectile and ball when they reach the ground.

69. While the mass of a body is a constant, as stated in Art. 14, *the weight varies according to the force of gravity.* The relation between *weight*, *mass*, and the ***acceleration due to gravity*** is expressed by the following formulas. If $m =$ mass, $w =$ weight in pounds and $g =$ acceleration due to gravity in feet per second per second; then

$$w = mg; \qquad (19)$$

$$\text{or, } m = \frac{w}{g}. \qquad (20)$$

70. **Problem.—What is the mass of a body that weights 64.32 lbs.?**

Solution.—Substituting in formula (19),

$$m = \frac{64.32}{32.16} = 2,\text{[*]} \; \textit{Ans.}$$

71. **Problems.—(1) What is the mass of a spectacle casting that weights 68 lbs.?**

(2) What is the weight of a locomotive whose mass is 4,700?

Answers.—(1) 2.11. (2) 151,152 lbs.

72. The second law of motion makes it possible to express *force* in terms of *mass* and *acceleration*. If $f =$ force in pounds, $m =$ mass and $a =$ acceleration in feet per second per second; then

$$f = ma; \qquad (21)$$

$$\text{or, } f = \frac{w}{g}a. \qquad (22)$$

73. **Problem.—What force would be required to accelerate a locomotive weighing 62 tons at a rate of 0.75 ft. per sec. per sec.?**

Solution.—The weight $w = 62$ tons or 124,000 lbs. and the acceleration $a = 0.75$ ft. per sec. per sec. Therefore substituting in formula (22),

$$f = \frac{124000}{32.16} \times 0.75 = 2{,}891 \text{ lbs.}, \; \textit{Ans.}$$

74. **Problems.—(1) What force would be required to accelerate a mass equal to 30 at a rate of 50 ft. per sec. per sec.?**

(2) What acceleration would a force of 10,000 lbs. give to a projectile weighing 1,000 lbs.?

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[*] No name is given to the unit of mass since, with the exception of problems illustrating the formulas, it is always expressed by the fraction $\frac{w}{g}$.

(3) A train weighing 200 tons is required to be accelerated at a rate of 0.5 ft. per sec. per sec. What force is required?

ANSWERS.—(1) 1,500 lbs. (2) 321.6 ft. per sec. per sec. (3) 6,218.9 lbs.

75. *The product of the mass of a body by its velocity, mv, is called* **momentum.**

76. In order that a body in motion may move in a curved path, it is necessary to restrain it to that path by a force. This force is known as the **centripetal force.** The reaction of the body against this centripetal force is commonly called **centrifugal force.**

77. If the body moves in a circular path the value of *centrifugal force* in terms of *mass, velocity* and the *radius of curvature* is expressed by the following formulas. If F = centrifugal force in pounds, R = radius of curvature in feet, m = mass, v = linear velocity in feet per second, w = weight in pounds, g = acceleration due to gravity in feet per second per second, and n = revolutions per minute; then

$$F = \frac{mv^2}{R}; \qquad (23)$$

$$\text{or, } F = \frac{wv^2}{gR}; \qquad (24)$$

$$v = \frac{2\pi Rn}{60}. \qquad (25)$$

78. PROBLEM.—If a body weighing 32.16 lbs. revolves in a circle 2 ft. in diameter at a rate of 200 r. p. m., determine the force that would just hold the body to its path.

SOLUTION.—The velocity of the body is found by means of formula (25) in which $R = 1$ ft. and $n = 200$ r. p. m.; therefore,

$$v = \frac{2 \times 3.14 \times 1 \times 200}{60} = 20.93 \text{ ft. per sec.}$$

The required force is determined by means of formula (24) in which $w = 32.16$ lbs.; therefore,

$$F = \frac{32.16 \times 20.93^2}{32.16 \times 1} = 438.06 \text{ lbs., } \textit{Ans.}$$

79. Problems.—(1) In a centrifugal switch, Fig. 15, what force must the spring exert to just keep the contacts together at a speed of 1,608 r. p. m., if the diameter of the path in which the governing weight moves is 2 in. and the weight weighs 3 oz.?

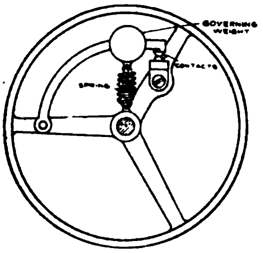

Fig. 15

(2) If one wire in the armature winding of a motor weighs .008 lbs. and revolves in a circle 4 in. in diameter at a speed of 2,000 r. p. m., what force is required to hold it in place?

Answers.—(1) 13.75 lbs. (2) 1.81 lbs.

WORK AND ENERGY

80. *If a force acts against a resistance and as a result produces motion over a certain distance it may be stated that* **work** *has been done.* This does not involve any statement of time required. In other words, *if the same force overcomes the same resistance over the same space, a like amount of work is done whether it takes a day or year to accomplish the operation.*

In order to compare and measure work it is necessary to have some unit to refer to. The one in common use is called the **foot-pound.** *It is the amount of work required to raise one pound one foot vertically.*

81. If W = work in foot-pounds, f = force in pounds, and d = distance in feet, the relation of *work* to *force* and *distance* may be expressed by the formula,

$$W = fd. \qquad (26)$$

82. Problem.—If a force of 200 lbs. is required to move a car on a level track, how much work is done in pushing it a distance of 250 ft.?

Solution.—Substituting in formula (26) the values, $f = 200$ lbs. and $d = 250$ ft., we obtain

$$W = 200 \times 250 = 50{,}000 \text{ ft. lbs.}, \textit{ Ans.}$$

83. Problems.—(1) Compute the amount of work required to raise a spectacle casting weighing 55 lbs. to its position on a pole of 25 ft. above the ground.

(2) If each of two men were required to move a pile of broken stone the same distance and one finished the job in a day while the other took two days was there any difference between the amounts of work done?

(3) How much work is required to move a switch with a 4 in. throw if it is necessary to apply an average force of 100 lbs.

(4) If the operator of an interlocking machine finds that he has to pull with a force of 50 lbs. to operate one lever and 35 lbs. to operate another and his hands travel 2½ ft. in each case, what is the difference in the amounts of work done?

ANSWERS.—(1) 1,375 ft. lbs. (2) No. (3) 33.33 ft. lbs. (4) 37.5 ft. lbs.

84. It is very essential that in measuring work performed we know how long it takes to do it. In other words we wish to know *the rate at which work is done, or the* **power.** The unit ordinarily used for this purpose is called the **horse power.** *Its value is the amount of work required to raise 550 lbs. 1 ft. vertically in 1 sec.*

85. If $H.P.=$ horse-power, $f=$ force in pounds, $d=$ distance in feet and $t=$ time in seconds, the relation between these terms is expressed by the formula,

$$H.P.=\frac{fd}{550\,t}. \qquad (27)$$

86. PROBLEM.—What power would be required to raise a weight of 1,100 lbs. 200 ft. in 5 sec.?

SOLUTION.—Substituting in formula (27) the values, $f=1{,}100$ lbs., $d=200$ ft. and $t=5$ sec., we obtain,

$$H.P.=\frac{1100\times 200}{550\times 5}=80 \text{ H. P., } \textit{Ans.}$$

87. PROBLEMS.—(1) If the locomotive of a train running up grade at a rate of 30 mi. per hr.* exerts a force of 5,500 lbs., what power is it developing?

(2) If a man, operating a track drilling machine with a crank 8 in. long finds it necessary to apply an average force of 11 lbs. to turn the handle 90 r. p. m., what power is he exerting?

(3) If an electric motor is required to throw a switch with a movement of 5 in. from one position to the other in 2 sec., what power is required if the average resistance to be overcome is 75 lbs.?

(4) Two men are operating a diaphragm pump. They find it necessary to push down with a combined force of 140 lbs. for 1 sec. and to pull up with a force of 20 lbs. for 1 sec. The average movement of the handle is 2 ft. What average power are they producing?

ANSWERS.—(1) 440 H.P. (2) 0.125 H.P. (3) .028 H.P. (4) 0.29 H.P.

88. Energy may be defined as the *ability to do work.* A body possesses energy on account of the fact that work has been done upon it. To set a body in motion, to place it in an elevated position, or to distort an elastic body, it is necessary to do work on it, or store energy, and the body in returning to its former state or position is capable of doing an equal amount of work. The stored energy of a body in motion is called **kinetic energy** and that of a distorted elastic body, or body in an elevated position is called **potential energy**. A moving car on a level track is an example of a body whose energy is entirely kinetic, while either a coiled spring at rest under tension, or an object suspended above the ground is an example of a body whose energy is entirely potential.

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*44 ft. per sec.

89. A body falling from a height of ten feet has, at the start, an amount of potential energy equal to the work necessary to raise it ten feet from the ground. If allowed to fall without interference, after passing over the first foot of its path it has an amount of kinetic energy equal to the energy it would give up at this point if stopped, and an amount of potential energy equal to the work necessary to raise it nine feet from the ground. After passing over the next foot it has twice the kinetic energy and its potential energy is equal to the work necessary to raise it eight feet from the ground. Similarly over the other spaces, as it increases its kinetic energy it reduces its potential energy an equal amount until, just as it reaches the ground, its energy is entirely kinetic.

The reverse of this is true for a body that is given an initial velocity upward from the ground. When it starts its energy is entirely kinetic. After passing over a given distance a proportionate part of its kinetic energy is transformed into potential. When, on account of the action of the force of gravity, it comes to rest at an elevation above the ground its energy is entirely potential. If allowed to fall back it will have the same amount of kinetic energy at the moment it reaches the ground, as when it started upward.

Therefore it may be stated that the kinetic energy of a body may be converted into potential energy and inversely the potential energy into kinetic and, as long as no energy is transferred to other bodies, the total amount of energy that a body possesses is constant and is equal to the sum of its kinetic and potential energies.

90. Energy is measured by the same unit as work; ie, the **foot-pound.**

91. *The kinetic energy of a body is equal to the product of its weight by the vertical height to which its velocity would carry it.* Thus if $K.E.=$ kinetic energy in foot-pounds, $w=$ weight in pounds, $v_1=$ initial velocity in feet per second, $h=$ the vertical height in feet that the body would have to fall to acquire this velocity, and $g=$ acceleration in feet per second per second due to gravity; then

$$K.E.=wh; \qquad (28)$$

$$\text{or, } K.E.=\frac{wv^2_1}{2g}. \qquad (29)$$

92. **Problem.**—A gasoline inspection car weighing 500 lbs. is traveling with a velocity of 30 ft. per sec. Compute its kinetic energy.

Solution.—Substituting in formula (29) the values, $w=500$ lbs. and $v_1=30$ ft. per sec., we have

$$K.E.=\frac{500\times 900}{2\times 32.16}=6{,}996 \text{ ft. lbs., } \textit{Ans.}$$

93. **Problems.**—(1) A train is traveling at a speed of 60 mi. per hr. and its weight is 210 tons. What is its kinetic energy?

(2) A hammer weighing 2½ lbs. is dropped from the roof of a signal tower. What is its kinetic energy 16 ft. below the roof?

(3) A brakeman is walking at the rate of 4 mi. per hr. along the roofs of a train of freight cars moving at 26 mi. per hr. If he weighs 150 lbs. and is going in the same direction as the train, compute the energy due to his velocity.

Answers.—(1) 50,567,164 ft. lbs. (2) 40 ft. lbs. (3) 4,514.9 ft. lbs.

94. *The potential energy of a body is equal to the work that it would do if released.* Thus if $P.E.=$ the potential energy in foot-pounds, $w=$ weight in pounds and $h=$ vertical height in feet through which the body would fall if released; then

$$P.E.=wh.^* \qquad (30)$$

95. **Problem.**—Compute the potential energy of a spectacle casting weighing 68 lbs. pivoted on a pole 31 ft. above the ground.

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*Formula (26) will also apply.

Solution.—Substituting in formula (30) the values, $w = 68$ lbs. and $h = 31$ ft., it becomes

$$P.E. = 68 \times 31 = 2{,}108 \text{ ft. lbs., } Ans.$$

96. Problems.—(1) A locomotive weighing 90 tons is standing on a bridge 100 ft. above the bottom of a gorge. Compute its potential energy due to elevation.

(2) A projectile weighing 10 lbs. is discharged vertically with an initial velocity of 1,800 ft. per sec. What is its potential energy at the moment it is brought to rest by the force of gravity?

Note.—If a body is given an initial velocity directly upward, it will rise to the same height that it would have to fall from, to acquire a velocity equal to the initial velocity. Therefore the formulas in Art. 64 may be used by considering v_2 to equal the initial velocity. In this case use formula (15); ie,

$$v_2 = \sqrt{2gd}.$$

Squaring both sides,

$$v^2_2 = 2gd.$$

Transposing,

$$d = \frac{v^2_2}{2g}.$$

(3) A man working on a telephone pole is holding a cross arm in position 25 ft. above the ground. If the cross arm weighs 20 lbs. what is its potential energy?

Answers.—(1) 18,000,000 ft. lbs. (2) 503,731.3 ft. lbs. (3) 500 ft. lbs.

97. *Energy like matter is indestructible.* Although it may be transformed from one form to another the total quantity existing remains unchanged. One of the most frequent transformations that is encountered is the change from mechanical energy to heat energy. Thus the energy that is used in stirring a liquid in a vessel, although it apparently disappears, is changed to heat energy and the temperature of the liquid raised slightly.

THE PENDULUM

98. A single particle of solid matter suspended from a fixed point by a fiber without weight and free to vibrate is a **simple pendulum.** The fixed point is called the **center of suspension.**

A small heavy body suspended by a very fine fiber closely resembles a simple pendulum. The center of gravity of such a pendulum may be considered to be at the center of the small body.

99. When a simple pendulum is in the vertical position, OA, Fig. 16, which is the position of rest, the heavy body may be considered to have no energy either potential or kinetic. If the body is drawn aside from this position to OB, the center of gravity is raised and energy is stored in the body. As long as the body is held in this position its energy is potential and the force of gravity, BC, acting directly downward may be resolved into two components, one, BD, in line with the fibre tending to break it, and another, BE, at right angles to the fibre, tending to return the body to the vertical position. If the body is released it travels toward the vertical position with increasing velocity, transforming its potential energy into kinetic energy until, at the vertical position, the energy is entirely kinetic. This kinetic energy carries the body beyond the vertical position, after passing which point the energy is gradually transformed to potential energy till the body comes to rest at OF and has simply potential energy. The operation is now repeated in the reverse direction and if it were not for friction and other resistances would go on indefinitely.

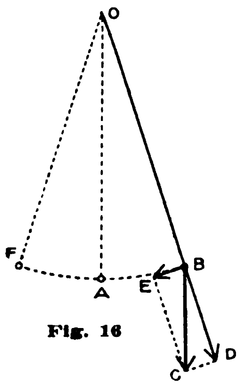

Fig. 16

100. When the pendulum is in motion it is said to vibrate. As long as the angle of vibration does not exceed 4 or 5 deg.

each vibration for a given pendulum takes a like period of time and the vibrations are said to be isochronous.

101. The relation between the *period of vibration,* or time in seconds required to swing from one extreme position to the opposite extreme, denoted by T, the *length of the pendulum* in feet, l, and the *acceleration* in feet per second per second, *due to gravity,* g, is expressed by the formulas,

$$T = \pi\sqrt{\frac{l}{g}}; \qquad (31)$$

$$l = g\left(\frac{T}{\pi}\right)^2. \qquad (32)$$

102. Problem.—*If $g = 32.16$ ft. per sec. per sec., what is the period of vibration of a pendulum 3.2585 ft. long?

Solution.—Substituting in formula (31) the values $l = 3.2585$ ft. and $g = 32.16$ ft. per sec. per sec. the formula becomes

$$T = \pi\sqrt{\frac{3.2585}{32.16}} = 3.1416 \times .3183 = 1 \text{ sec., } \textit{Ans.}$$

103. Problems.—*(1) If the length of a pendulum is 0.8146 ft. what is its period of vibration taking $g = 32.16$ ft. per sec. per sec.?

(2) Compute the length of a pendulum to vibrate in periods of ¼ sec.

Answers.—(1) ½ sec. (2) 0.2036 ft.

104. At New York a pendulum 39.1017 in. long makes one vibration in one second and is therefore called a *seconds pendulum.*

105. An ordinary pendulum as used in a clock or other device has a supporting rod of some size and weight and a fairly large end or bob in place of the small heavy body. Such a pendulum is known as a **compound pendulum.** It may be considered to be composed of an infinite number of simple pendulums, of all lengths within its limits and it may be assumed that one of these simple pendulums is of such a length that it vibrates the same as the given compound pendulum. If the compound pendulum is considered to have the same length as this simple pendulum it can be treated as a simple pendulum.

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*In these problems the calculations should be carried to at least four decimal places.

106. The point in a compound pendulum that vibrates the same as the particle at the end of the simple pendulum described in the preceding article is called the **center of oscillation** or **percussion.** The motion would be the same if the whole mass were collected at this point. The points of suspension and oscillation are interchangeable so that if the pendulum is suspended from the point of oscillation it will vibrate the same as before.

SIMPLE MACHINES

107. A **machine** is a device used to *transfer* energy. Its specific object may be either to change the direction or character of motion or to change the direction or magnitude of force. It does not make or increase energy, but only transfers it.

108. There are two forces that must be considered in connection with the action of a machine, the **applied force** and the **resistance.** *The applied force is the force acting upon a machine; the resistance is the force which acts against a machine.*

GENERAL LAW OF MACHINES

109. In a perfect machine *the applied force multiplied by the distance through which it acts just equals the resistance multiplied by the distance through which it acts.* If $f' =$ applied force, $d' =$ distance through which it acts, $r =$ resistance and $s =$ distance through which it acts, then for a perfect machine this law is expressed by the formula*

$$f'd' = rs. \qquad (33)$$

From formula (33) the proportion

$$f' : r :: s : d',$$

may be obtained. From this it will be observed that if f' decreases, d' correspondingly increases and if f' increases, d' decreases or, f' varies inversely as d'. Similarly r varies inversely as s. Thus to overcome a resistance of 10 lbs. through a distance of 10 ft. either a force of 100 lbs. may act through a distance of 1 ft. or a force of 1 lb. through a distance of 100 ft. or any other combination as long as the product of $f'd' = 100$.

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*Care should be taken when using this and following formulas to express the values of the symbols throughout the same formula in the same units. Thus, if force is expressed in pounds, resistance in the same formula must also be in pounds.

110. A perfect machine is not a possibility since in the operation of every machine a certain amount of energy is wasted due to resistances in the machine itself. *The* **efficiency** *of a machine is the ratio of the energy delivered to the energy supplied.* If this ratio is represented by the letter k;* formula (33) becomes

$$kf'd' = rs. \qquad (34)$$

111. The most important loss of energy in a machine is due to **friction.** This is the force that acts on two bodies, at the surfaces in contact, resisting a change of position of one relative to the other. It may be either sliding or rolling.

112. Friction depends on the material of which the bodies are composed, the force pressing the contact surfaces together, the character of these surfaces, their condition as regards lubrication, etc.

In order to simplify the following calculations no allowances have been made for losses due to friction and other resistances.

THE LEVER

113. The **lever** is one of the simplest and most widely applied of all the simple machines. It consists of a bar, Fig. 17, supported on a block or bearing, A, called a fulcrum about which it can move freely. The applied force f' acts at some point along this bar and the resistance r acts at some other point.

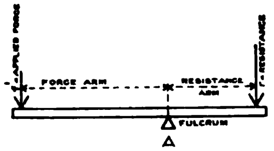

Fig. 17

114. *The perpendicular distance from the fulcrum to the line indicating the direction of the applied force is called the* **force arm** *of a lever and the perpendicular distance from the fulcrum to the line indicating the direction of the resistance is called the* **resistance arm.**

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*The value of k is usually determined by experiment.

115. There are three classes of levers. In the *first class*, Fig. 18. the applied force and resistance are on opposite sides of the fulcrum. In the *second class*, Fig. 19, the applied force and resistance are both on the same side of the fulcrum and the force arm is longer than the resistance arm. The *third class*, Fig. 20, differs from the second class in that the resistance arm is longer than the force arm.

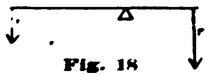

Fig. 18

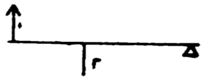

Fig. 19

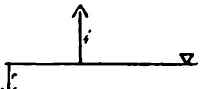

Fig. 20

116. If x = length of force arm and y = length of resistance arm, the distances through which the applied force and resistance act are proportional to x and y respectively. Therefore, the formula,

$$f'x = ry, \qquad (35)$$

may be substituted for the general formula (33).

Either this formula or the general formula may be used in calculating lever problems according as it is easier to measure the lengths of the force and resistance arms or the distances through which the applied force and resistance act.

117. PROBLEM.—If a lever of the first class is just balanced and the applied force is 300 lbs. while the resistance is 50 lbs., what is the length of the force arm if the resistance arm is 2 ft. long?

SOLUTION.—Substituting the values, $f' = 300$ lbs., $r = 50$ lbs. and $y = 2$ ft. in formula (35), we have

$$300\,x = 50 \times 2;$$

$$\text{or, } x = \tfrac{1}{3} \text{ ft. or 4 in., } \textit{Ans.}$$

118. PROBLEM.—If a lever of the first class has a force arm 6 in. long and a resistance arm 10 in. long, how far will the resistance act if the applied force acts through a distance of 4 in.?

SOLUTION.—Since the distances through which the applied force and resistance act are proportional to the force and resistance arms respectively we have the proportion,

$$6 : 10 :: 4 : s.$$

$$\text{Solving, } s = \frac{4 \times 10}{6} = 6\tfrac{2}{3} \text{ in., } \textit{Ans.}$$

119. **Problem.—If an applied force of 100 lbs. acts through a distance of 8 in. on a lever of the second class and just overcomes a resistance of 400 lbs., through what distance is the resistance overcome?**

Solution.—In this case substituting in formula (33) the values, $f' = 100$ lbs., $d' = 8$ in. and $r = 400$ lbs., we have

$$100 \times 8 = 400 \times s;$$

$$s = \frac{100 \times 8}{400} = 2 \text{ in., } \textit{Ans.}$$

120. **Problems.—(1) What resistance will a force of 75 lbs. balance if the force arm of a lever is 8 in. long and the resistance arm 6 in. long?**

(2) What force will be required to balance a resistance of 240 lbs. if the force and resistance arms are 3 ft. and 4 ft. long respectively?

(3) If it is required to support a resistance of 230 lbs. with a force of 10 lbs. and the resistance arm is 2 in. long, how long should the force arm be?

(4) If it is desired to change a movement of 8 in. to 5 in., using a lever with arms 11¾ in. long, how far from the fulcrum will the new hole have to be drilled?

Answers.—(1) 100 lbs. (2) 320 lbs. (3) 46 in. (4) 7.34 in.

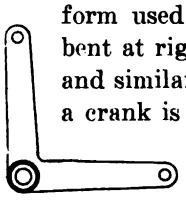

Fig. 21

121. So far all the levers considered have been straight. There are, however, many different shapes. A very common form used in signal work is one in which the arms are bent at right angles to each other, Fig. 21. Levers of this and similar shapes are called **cranks.** The object of using a crank is to change the direction of motion, thus, if one arm moves in a horizontal direction the other may move vertically as in the case of the cranks at the foot of a signal pole, see Fig. 22.

122. Cranks are made which have angles greater or less than 90 deg. between their arms. However, the same formulas should be employed that are used for a straight lever, *remembering that the distances of the applied force and resistance from the fulcrum must be measured along lines perpendicular to the directions of motion of the applied force and resistance respectively.*

123. PROBLEMS ILLUSTRATING THE APPLICATION OF THE LEVER TO A MECHANICAL INTERLOCKING SYSTEM.—(1) A semaphore, Fig. 22, requires 60 lbs. at point A to move it. Point A is connected to a crank at point B, 6 in. from the fulcrum C. How much force is required to move the semaphore, if applied to the crank at a point, D, 9 in. from the fulcrum?

(2) Point A on a semaphore is connected with a crank BCD at point B; point D which is 10 in. from C moves just 8 in. What is the distance from B to C if point A is required to move exactly 4 in.?

(3) Each arm of a crank at the foot of a signal pole is 9 in. long and the end of one arm of the crank moves 6 in. If a new hole is drilled in the other arm 6 in. from the fulcrum what will the movement be at that point?

ANSWERS.—(1) 40 lbs. (2) 5 in. (3) 4 in.

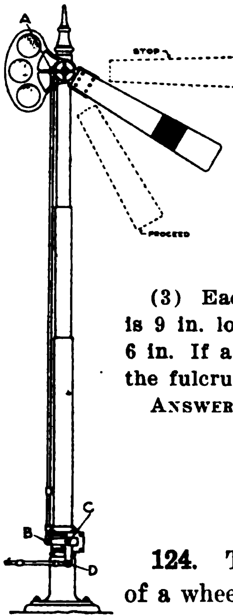

Fig. 22

THE WHEEL AND AXLE

124. The **wheel and axle** is a machine composed of a wheel and an axle fastened rigidly together. The diameter of the wheel is larger than that of the axle and consequently a point on the circumference of the wheel will move further in one revolution than a point on the circumference of the axle. A common form is the ordinary **windlass** in which a crank carrying a handle is used instead of a wheel.

125. The wheel and axle may be considered as a sort of perpetual lever, Fig. 23. The radius of the wheel, AO, corresponds to the long arm of the lever, and the radius of the axle, BO, corresponds to the short arm. By turning the wheel one revolution the rope or cable is wound once around the axle and the resistance acts through a distance equal to the circumference of the axle, while the applied force acts through a distance equal to the circumference of the wheel. Applying the fundamental law of machines, *the applied force multiplied by the*

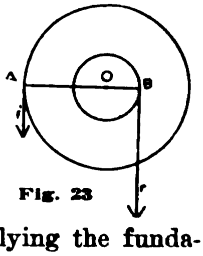

Fig. 23

circumference of the wheel equals the resistance multiplied by the circumference of the axle. The circumferences are directly proportional to the radii which may therefore be put in their places. Then the law for the wheel and axle will read: *The applied force multiplied by the radius of the wheel equals the resistance multiplied by the radius of the axle.*

126. This law may be expressed in a formula as follows: If f' = applied force, x' = radius of wheel, r = resistance and y' = radius of axle; then

$$f'x' = ry'. \qquad (36)$$

THE PULLEY

127. The **pulley** is a wheel or combination of wheels made with edges grooved to carry a rope or chain. There are two kinds of pulleys, fixed and movable.

A **fixed pulley,** Fig. 24, is fastened to some solid support and is used only to change the direction of motion. *Therefore the applied force equals the resistance and acts through the same distance.*

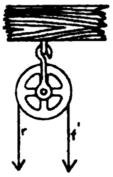

Fig. 24

128. A **movable pulley** may be considered as a perpetual lever of the second class as shown clearly in Fig. 25. The resistance acts against the pulley at its axle, O, the point B is the fulcrum and the applied force acts at point A. Therefore since OB is one half of AB, the applied force is equal to one half the resistance and as the pulley is supported by two parts of the same rope, one half of the resistance acts against each of these parts. Thus in Fig. 26,

$$f' = \tfrac{1}{2}\ r.$$

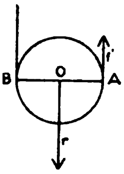

Fig. 25

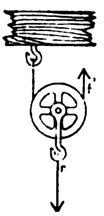

Fig. 26

129. Many different combinations of pulleys are made, but ordinarily a single rope passes around all the sheaves* as in the common block and tackle.

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*Wheels.

In this case the resistance is equal to the applied force multiplied by the number of parts of the rope supporting the movable pulley; or the applied force is equal to the resistance divided by the number of parts supporting the movable pulley.

130. To express this rule in a formula let f' represent the applied force, r the resistance, and N the number of parts of the rope supporting the movable pulley; then

$$r = f'N; \qquad (37)$$

$$\text{or, } f' = \frac{r}{N}. \qquad (38)$$

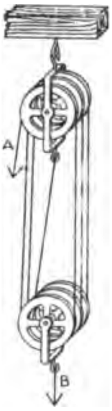

Fig. 27

131. PROBLEM.—**If a force of 10 lbs. acts on the end, A, of the rope in Fig. 27, what is the resistance acting at B, if the force at A is just balanced?**

SOLUTION.—**Substituting in formula (37) the values, $f' = 10$ and N equals 6, we have**

$$r = 10 \times 6 = 60 \text{ lbs., } \textit{Ans.}$$

It should be noted *that if the applied force acts through a certain distance, the resistance is overcome through only $\frac{1}{N}$ times that distance.* Thus, in this problem, if point A is moved 6 ft. point B is moved only 1 ft. (Art. 109).

THE INCLINED PLANE

132. The **inclined plane** AB, Fig. 28, is a smooth inflexible surface inclined so as to make an acute angle H with the horizontal plane. When an object is placed on such a plane the force of gravity, DE, may be resolved into a component, DG, at right angles to the plane and a component, DF, parallel to the plane. DF tends to make the object slide down the plane and motion is produced if it is great enough to overcome the friction between the object and the plane. Thus, when the angle H is large, DF is large and the body moves rapidly, and if the angle is so small that DF is less than the friction, the body remains at rest.

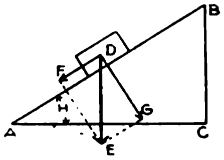

Fig. 28

133. A force tending to push an object up a plane may work at various angles. When the applied force acts in a direction parallel to the plane, Fig. 29, *the applied force multiplied by the length of the plane, AB, equals the resistance of the object multiplied by the height of the plane BC.*

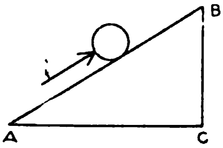

Fig. 29

134. Expressing this rule by a formula, if f' = applied force, L = length of plane, r = resistance and h = height of plane; then

$$f'L = rh. \qquad (39)$$

135. Problem.—What force parallel to the plane is required to push a ball weighing 100 lbs. up an inclined plane, if the length of the plane is 15 ft. and the height at the raised end 3 ft.?

Solution.—Substituting in formula (39) the values, $r = 100$ lbs., $L = 15$ ft. and $h = 3$ ft., we obtain

$$f'\ 15 = 100 \times 3;$$

$$f' = \frac{100 \times 3}{15} = 20 \text{ lbs., } \textit{Ans.}$$

136. When a force tending to push an object up a plane acts in a direction parallel to the horizontal plane, Fig. 30, *the applied force multiplied by the length, AC, of the base equals the resistance of the object multiplied by the height of the plane, BC.*

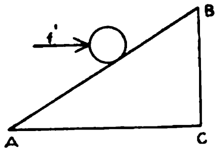

Fig. 30

137. Expressing this rule by a formula, if f' = applied force, L' = length of base, r = resistance and h = height of plane; then

$$f'L' = rh. \qquad (40)$$

THE WEDGE

138. The **wedge** is a piece of hard wood or metal shaped like two inclined planes placed with their bases together,

Fig. 31. It is commonly used in splitting wood, blocking up heavy objects such as buildings, ships, etc. Unlike the other simple machines the usefulness of the wedge depends upon friction. It is usually acted upon by an intermittent force like the blows of a hammer, and after each stroke is held in position by friction. If, because the wedge is improperly shaped or for some other reason, the friction is not sufficient to hold it in the position to which it has been driven it will fly back and its effect will be lost.

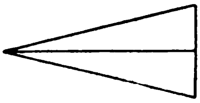

Fig. 31

On account of the difficulty of measuring the applied force and friction, the exact application of the general law of machines is of little use in this case, although the longer a wedge is made in comparison with its width the greater its power.

THE SCREW

139. The **screw** is a cylinder, usually of metal, with a ridge or thread formed in an advancing spiral on its surface. The ridge may be on the outer surface as in a common wood screw or on the inner surface as in a nut. Very often the two kinds are used together as in the ordinary bolt and nut. The screw is virtually an inclined plane.

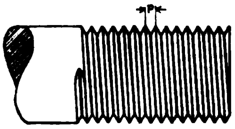

Fig. 32

140. The distance, p, Fig. 32, between any two consecutive turns of the thread measured in a direction parallel with the axis of the cylinder is called the **pitch** of the screw.

141. Force may be applied to a screw by means of a lever, or equivalent device, and acts in the circumference of a circle whose radius is equal to the length of the lever. Thus in one revolution, the force will act through a distance equal to this circumference and the resistance through a distance equal to the pitch of the screw.

142. Applying the fundamental law of machines, *the force multiplied by the circumference of the circle through which it acts is equal to the resistance multiplied by the pitch of the screw.*

143. Expressing this rule by a formula, if $f' =$ applied force, $C =$ circumference of circle or distance in inches through which f' acts, $r =$ resistance and $p =$ pitch of screw or distance through which r acts; then

$$f'C = rp. \qquad (41)$$

144. Problem.—If a force of 50 lbs. acts at the end of a 12 in. wrench turning a screw with four threads per inch of length, what resistance will the screw overcome?

Solution.—Substituting in formula (41) the values, $f' = 50$ lbs., $C = 2\pi \times 12$ in., and $p = \frac{1}{4}$ in., we obtain

$$50 \times 2 \times 3.14 \times 12 = r \times \tfrac{1}{4};$$

$$r = 50 \times 2 \times 3.14 \times 12 \times 4 = 15{,}072 \text{ lbs., } \textit{Ans.}$$

In this case it is necessary to turn the screw 4 revolutions or to apply force through a distance of 301.44 in. in order to overcome the resistance through a distance of 1 in.

COMPOUND MACHINES

145. A combination or series of simple machines acting on each other is called a **compound machine.** The general law of machines may be applied by starting with the first of the series and considering it as a simple machine. Then the resistance acting against this machine will be the force acting on the next simple machine of the series, etc.

EXAMINATION QUESTIONS AND PROBLEMS

(1) Express in words the meaning of formulas (6), (7), (8), (22) and (27). For example, formula (1) may be expressed thus:

The velocity equals the distance divided by the time.

(2) If a train running at uniform speed travels 10 mi. in 13 min., 20 sec., what is its velocity in feet per second?

(3) How long would it take a man walking at a rate of 6 ft. per sec. to travel 8 mi.?

(4) A car running down grade increased its speed uniformly. If it had a velocity of 10 mi. per hr. when passing one signal and 50 mi. per hr. when passing another, 3,300 ft. beyond the first; (*a*) What was its average velocity? (*b*) How long did it require to traverse the distance between signals?

(5) If a train running 75 mi. per hr. is stopped in $3\frac{2}{3}$ min.; (*a*) What is the retardation? (*b*) What distance does it travel in coming to rest?

(6) If a locomotive starting from rest is accelerated at a rate of 10 ft. per sec. per sec.; (*a*) How far does it travel in the third second? (*b*) In what distance would it reach a speed of 45 mi. per hr.?

(7) (*a*) Draw a diagram showing the method of obtaining the value and direction of the resultant of two forces, one of 30 lbs. and the other of 50 lbs., acting at an angle of 45 deg. to each other. (*b*) Draw a diagram showing the value, direction, and position of the resultant of two parallel forces, one of 25 lbs., and the other of 200 lbs., both acting in the same direction with their points of application 9 ft. $4\frac{1}{2}$ in. apart.

(8) If an object was dropped from a window 257.28 ft. high; (*a*) How long did it take to reach the ground? (*b*) How far did it fall in the third second? (*c*) What was its velocity at the moment it struck the ground?

(9) (*a*) What is the mass of a body that weighs 160.8 lbs.? (*b*) What is the weight of a body if its mass is 15?

(10) (*a*) If a force of 10 lbs. acts on a body whose mass is 2, what acceleration is produced? (*b*) What force is required to accelerate a body weighing 1,608 lbs. at a rate of 10 ft. per sec. per sec.?

(11) What force is necessary to restrain a body weighing 16.08 lbs. to a circular path if it makes 600 r. p. m. and the radius is 5 ft.?

(12) (*a*) If a weight of 100 lbs. is raised 30 ft. how much work is done? (*b*) If a telegraph pole is dragged 50 ft. and an average force of 150 lbs. exerted in the operation, what amount of work is done?

(13) What is the difference between work and power?

(14) An average force of 50 lbs. is required to move a hand car at the rate of 15 mi. per hour, what power is required?

(15) An elevator motor is required to raise a cage, whose net weight is 1,000 lbs., through a distance of 110 ft. in 20 sec. What power must it develop?

(16) How much kinetic energy is stored in a body weighing 321.6 lbs. if it moves at a rate of 90 ft. per sec.?

(17) How much potential energy is stored in a body if it weighs 500 lbs. and is placed at an elevation of 25 ft.?

(18) What is the period of vibration of a pendulum 289.44 ft. long?

(19) What is the proper way to measure the force and resistance arms of a lever? Draw a diagram illustrating your answer.

(20) If the force arm of a lever is 13 ft. long and the resistance arm 26 in. long, what resistance acts to just balance an applied force of 10 lbs.?

(21) In Fig. 22 how far from C would hole B have to be drillled if CD is 11¾ in. long and B is required to move 4 in. when D moves 8 in.?

(22) A wheel 3 ft. in diameter is connected to an axle 6 in. in diameter, what resistance at the circumference of the axle will an applied force of 15 lbs. at the circumference of the wheel overcome?

(23) A block and tackle is composed of two pulleys with four sheaves each, connected by a continuous rope. If one end of the rope is fastened to the fixed pulley, what resistance would be overcome by a force of 10 lbs. at the other end? Draw a diagram illustrating your solution.

(24) A plane is 24 ft. long and one end rests on the ground while the other end is 6 ft. high. What force parallel to the plane would be required to roll up a barrel of cement weighing 300 lbs.?

(25) If a screw has two threads per inch and is turned by a crank 18 in. long, what resistance will an applied force of 10 lbs. overcome?

ANSWERS TO EXAMINATION PROBLEMS

(1) ?

(2) 66 ft. per sec.

(3) 1 hr., 57 min., 20 sec.

(4) (*a*) 30 mi. per hr. (*b*) 1 min., 15 sec.

(5) (*a*) 0.5 ft. per sec. per sec. (*b*) 2 mi., 1,540 ft.

(6) (*a*) 25 ft. (*b*) 217.8 ft.

(7) ?

(8) (*a*) 4 sec. (*b*) 80.4 ft. (*c*) 128.64 ft. per sec.

(9) (*a*) 5. (*b*) 482.4 lbs.

(10) (*a*) 5 ft. per sec. per sec. (*b*) 500 lbs.

(11) 9,859.6 lbs.

(12) (*a*) 3,000 ft. lbs. (*b*) 7,500 ft. lbs.

(13) ?

(14) 2 H. P.

(15) 10 H. P.

(16) 40,500 ft. lbs.

(17) 12,500 ft. lbs.

(18) 9.42 sec.

(19) ?

(20) 60 lbs.

(21) 5.87 in.

(22) 90 lbs.

(23) 80 lbs.

(24) 75 lbs.

(25) 2,260.8 lbs.

TABLE OF SYMBOLS*

a = acceleration in feet per second per second.

C = circumference of circle in inches through which applied force acts.

d = distance in feet.

d_1 = distance in feet traversed in any particular unit of time.

d' = distance through which f' acts.

F = centrifugal force in pounds.

f = force in pounds.

f' = applied force.

g = acceleration due to gravity in feet per second per second.

h = height in feet.

$H.P.$ = horse power.

k = a constant.

$K.E.$ = kinetic energy.

L = length of inclined face of inclined plane.

L' = length of base of inclined plane.

l = length of pendulum in feet.

m = mass.

N = number of parts of rope supporting a movable pulley.

n = revolutions per minute.

p = pitch of screw.

$P.E.$ = potential energy.

R = radius of curvature in feet.

r = resistance or resisting force.

s = distance through which r acts.

T = period of vibration of pendulum in seconds.

t = time in seconds.

v = velocity in feet per second.

v_0 = average velocity.

v_1 = initial velocity.

v_2 = final velocity.

W = work in foot-pounds.

w = weight in pounds.

x = length of force arm of lever.

x' = radius of wheel.

y = length of resistance arm of lever.

y' = radius of axle.

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*The symbols and formulas used in the text are collected in these tables for convenient reference and need not be studied.

TABLE OF FORMULAS

The formulas are numbered consecutively throughout the text and are placed and numbered in the same order in the following table. As an aid in referring to the text the page number is given directly after the formula number.

Velocity

		Page No.
$v=\frac{d}{t}$.	(1)	(6)
$d=tv$.	(2)	(6)
$t=\frac{d}{v}$.	(3)	(6)
$v_0=\frac{v_1+v_2}{2}$.	(4)	(8)
$v_0=\frac{v_2}{2}$.	(5)	(8)

Acceleration

$v_2=at$.	(6)	(9)
$v_2=\sqrt{2ad}$.	(7)	(9)
$d=\frac{at^2}{2}$.	(8)	(9)
$d=\frac{v_2t}{2}$.	(9)	(9)
$d_1=\frac{a(2t-1)}{2}$.	(10)	(9)
$t=\sqrt{\frac{2d}{a}}$.	(11)	(9)
$a=\frac{2d}{t^2}$.	(12)	(9)
$a=\frac{v^2_2}{2d}$.	(13)	(9)

Falling Bodies

$v_2=gt$.	(14)	(17)
$v_2=\sqrt{2gd}$.	(15)	(17)
$d=\frac{gt^2}{2}$.	(16)	(17)
$d_1=\frac{g(2t-1)}{2}$.	(17)	(17)
$t=\sqrt{\frac{2d}{g}}$.	(18)	(17)

Mass and Force

		Page No.
$w=mg$.	(19)	(19)
$m=\frac{w}{g}$.	(20)	(19)
$f=ma$.	(21)	(19)
$f=\frac{w}{g}a$.	(22)	(19)
$F=\frac{mv^2}{R}$.	(23)	(20)
$F=\frac{wv^2}{gR}$.	(24)	(20)
$v=\frac{2\pi Rn}{60}$.	(25)	(20)

Work and Energy

$W=fd$.	(26)	(22)
$H.P.=\frac{fd}{550t}$.	(27)	(23)
$K.E.=wh$.	(28)	(25)
$K.E.=\frac{wv^2_1}{2g}$.	(29)	(25)
$P.E.=wh$.	(30)	(25)

Pendulum

$T=\pi\sqrt{\frac{l}{g}}$.	(31)	(28)
$l=g\left(\frac{T}{\pi}\right)^2$.	(32)	(28)

Simple Machines

$f'd'=rs$.	(33)	(30)
$kf'd'=rs$.	(34)	(31)
$f'x=ry$.	(35)	(32)
$f'x'=ry'$.	(36)	(35)
$r=f'N$.	(37)	(36)
$f'=\frac{r}{N}$.	(38)	(36)
$f'L=rh$.	(39)	(37)
$f'L'=rh$.	(40)	(37)
$f'C=rp$.	(41)	(39)

MECHANICS OF LIQUIDS

TRANSMISSION OF PRESSURE

146. Liquids and gases are perfectly elastic; that is, if the volumes are decreased by pressure upon their surfaces, they will, upon the removal of the pressure, resume their original volumes.

Gases are compressible; liquids are practically incompressible, so that regardless of the pressure, the volume of an enclosed liquid is nearly constant. Thus water enclosed in a strong vessel and subjected to a pressure of 14.7 lbs. per sq. in. is decreased in volume only about $\frac{1}{200}$ of 1% of its original volume.

147. The law describing the action of fluids when an external force or pressure is applied to them, known as **Pascal's Law**, may be stated as follows:

Pressure exerted upon a fluid enclosed in a vessel is transmitted undiminished in every direction, regardless of the shape of the vessel.

*_The pressures exerted by a fluid upon equal areas of a retaining vessel are equal; or stated differently, the pressures on two sections of the surface of a retaining vessel are proportional to the areas of the sections._

The pressure of a fluid on a vessel acts in a direction perpendicular to the surface acted upon.

148. The facts stated above are applied in the operation of hydraulic machinery of which the **hydraulic press** is a good example. Its essential parts consist of a small piston X, Fig. 33, and a large piston Y, working in cylinders M and N which are connected by the pipe C. Pipe C and the spaces below the pistons are completely filled with some liquid such as water or oil.

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*Disregarding the weight of the fluid.

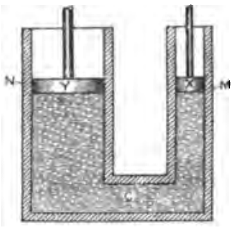

Fig. 33

According to Pascal's Law, the pressures on two unequal surfaces of a retaining vessel are proportional to the areas of those surfaces. If $A =$ the area of X, $A' =$ the area of Y, $f' =$ the pressure on X, or the applied force, and $r =$ the pressure on Y, or the resistance; then

$$f' : r :: A : A'. \qquad (42)$$

On account of the fact that the volume of an enclosed liquid is nearly constant whatever the pressure, if X is forced down decreasing the volume in cylinder M a certain amount, Y will be forced up so as to increase the volume in cylinder N an equal amount.

If $d' =$ the distance X is moved and $z =$ the decrease of volume in cylinder M; then

$$z = Ad'.$$

Likewise if $s =$ the distance Y is moved, then since the increase of volume in cylinder N equals the decrease of volume in cylinder M,

$$z = A's.$$

$$\text{Therefore } Ad' = A's,$$

$$\text{or } A : A' :: s : d'.$$

but by formula (42) $f' : r :: A : A'$.

$$\text{Hence } f' : r :: s : d'. \qquad (43)$$

That is, as in any other machine,* the force is inversely proportional to the distance traveled.

The principle of the hydraulic press has a great number of applications. For example, it is used in a form of jack designed for raising great weights, and in hydraulic punching and riveting machines.

149. Problem.—If the diameter of the smaller cylinder in Fig. 33 is 1 in. and the diameter of the larger cylinder is 1 ft.; (*a*) How

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*Neglecting friction and other losses.

great a weight on Y will a weight of 2 lbs. on X support? (*b*) If Y is to move 3 in. how far will X have to move?

Solution.—(*a*) Substituting the values $A = 0.7854 \times (1)^2$,* $A' = 0.7854 \times (12)^2$ and $f' = 2$ lbs. in formula (42), we get

$$2 : r :: 0.7854 \times 1^2 : 0.7854 \times 12^2.$$

$$\text{Therefore } r = \frac{0.7854 \times 12^2 \times 2}{0.7854 \times 1^2} = 288 \text{ lbs., } \textit{Ans.}$$

(*b*) Substituting the values $s = 3$ in., $r = 288$ lbs., and $f' = 2$ lbs. in formula (43), we have

$$2 : 288 :: 3 : d'.$$

$$\text{Therefore } d' = 432 \text{ in., } \textit{Ans.}$$

150. Problems.—Fig. 34 shows a diagram of a hydraulic press. (1) A force of 20 lbs. is applied at F. The force arm *x* of the lever is ten times as long as the resistance arm *y*. If the diameter of the small piston is ¼ in. and that of the large piston is 8 in.; (*a*) What is upward pressure of R? (*b*) If R is to move 1 in. how far must F move?

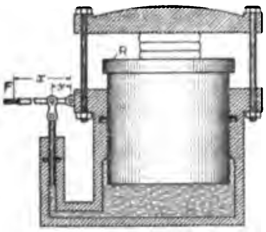

Fig. 34

(2) It is desired to exert an upward pressure at R of 200 tons. What force must be applied at F?

Answers.—(1) (*a*) 204,800 lbs. (*b*) 10,240 in. (2) 39.06 lbs.

PRESSURE DUE TO GRAVITY

151. Each particle in a body of liquid is acted upon by gravity. The pressure due to this action is, according to Pascal's Law, transmitted equally in all directions and there is, therefore, on any particle in a body of liquid equal pressure in all directions.

The pressure per unit area at any point below the surface of a liquid depends on the depth of the point. This may be explained by considering a vertical column of particles extending from the point to the surface. The particle at the surface may be considered to have only the force of gravity acting on it. The next particle below in addition to this force is acted upon by the weight of the particle above it. Likewise the third particle in the column is acted upon by the weight of the two particles above it which it transmits to the particles below in addition to its own weight. Thus the last particle transmits its own weight as well as the weight of all the particles above to a surface placed at the given point and the pressure on the surface is proportional to the number of particles or depth below the surface of the liquid.

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*See Mensuration in the appendix to Drawing, Part III.

152. *The* **downward pressure** *on the base of a vessel depends upon the depth of the liquid, the area of the base, and the weight of a unit volume of the liquid, and is independent of the shape of the vessel and the amount of liquid.* For example, if vessels with equal bases but different volumes, A and B, Fig. 35, were filled with water to the same depth the pressures on the bases would be equal.

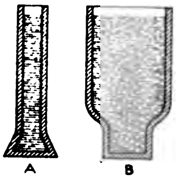

Fig. 35

The downward pressure on any horizontal surface is equal to the area of the surface, multiplied by the depth of this surface below the surface of the liquid multiplied by the weight of a unit volume of the liquid.

If A = the area of a horizontal surface, h' = the depth, W' = the weight of a unit volume of the liquid, and P = the downward pressure; then

$$P = Ah'W'.^* \qquad (44)$$

The quantity W' varies for different liquids. A cubic foot of water at a temperature of 62° F. weighs 62.35 lbs.

153. Problem.—Find the pressure on the bottom of a tank 4 ft. wide and 6 ft. long if it is filled with water to a depth of 4 ft.

Solution.—By the conditions of the problem $A = 4 \times 6 = 24$ sq. ft., $h' = 4$ ft., $W' = 62.35$ lbs. Substituting these values in the formula (44), we get

$$P = 24 \times 4 \times 62.35 = 5{,}985.60 \text{ lbs., } \textit{Ans.}$$

154. *The lateral pressure on a vertical surface is equal to the area of the surface pressed upon, multiplied by the depth of the center of area of this surface below the surface of the liquid, multiplied by the weight of a unit volume of the liquid.*

If A = the area of the surface pressed upon, h'' = the depth of the center of this area, W' = the weight of a unit volume of the liquid, and P' = the pressure on the vertical surface, then

$$P = Ah''W'. \qquad (45)$$

155. Problem.—Find the total pressure on the sides and ends of a tank 2 ft. wide, 2 ft. deep, and 3 ft. long if it is half filled with water.

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*As noted before the symbols throughout the same formula must be expressed in the same units. Thus if W' is expressed in pounds per cubic foot, A must be expressed in square feet and h' in feet.

Solution.—Since the tank is half filled, the water in the tank is 1 ft. deep. Therefore each end will have 2 × 1 sq. ft. of surface pressed upon and each side will have 3 × 1 sq. ft. of surface pressed upon. The center of areas of these surfaces will be ½ ft. below the surface of the water. Hence, $A = 2 + 2 + 3 + 3 = 10$ sq. ft., $h'' = \frac{1}{2}$ ft., $W' = 62.35$ lbs. Substituting these values in the formula (45), we get

$$P' = 10 \times \tfrac{1}{2} \times 62.35 = 311.75 \text{ lbs., } \textit{Ans.}$$

156. *The upward pressure on any horizontal surface is the same as the downward pressure on an equal surface similarly placed with respect to the surface of the liquid,* for the reason that, according to Pascal's Law, the pressure on any particle in a body of liquid is equal in all directions. Formula (44) will give the upward pressure on any horizontal surface if we let h' = the depth of that surface below the surface of the liquid and P = the *upward* pressure.

157. Problems.—(1) What is the pressure on the bottom of a tank 16 ft. square if it is filled with water to a depth of 10 ft.?

(2) (*a*) What is the total pressure on the four sides of the tank in problem (1)? (*b*) What is the total pressure on the sides and bottom of the tank?

(3) If a tub 2½ ft. in diameter is floating on a pond and its bottom is 8 in. below the surface of the water, what is the upward pressure on the tub?

Answers.—(1) 159,616 lbs. (2) (*a*) 199,520 lbs. (*b*) 359,136 lbs. (3) 204.04 lbs.

158. If a body is submerged in a liquid it is acted upon by forces pressing against it in all directions and *therefore will be supported by a force equal to the weight of the volume of liquid which it displaces.* This can be shown by Fig. 36 which represents a cubical body submerged in water. The lateral pressures, F, are equal and opposite in direction and therefore balance each other. The pressure downward = $A \times CE \times W'$, and the pressure upward = $A \times DE \times W'$; (formula 44). The difference between CE and DE = CD. Hence the difference between the pressure upward and the pressure downward is equal to $A \times CD \times W'$. But $A \times CD \times W'$ is the weight of a volume of water whose

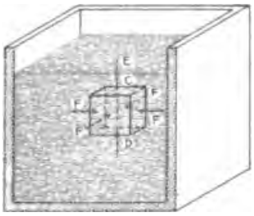

Fig. 36

base is A and whose height is CD; that is, of a volume of water equal to the volume of the cube. Therefore, the upward pressure is greater than the downward by the weight of the water displaced. Hence if a body is placed in a liquid it will sink until it has displaced a volume of the liquid equal in weight to the weight of the body. If the body is not entirely submerged we know that a volume of the liquid equal to the volume of the body weighs more than the body, but if the body is entirely submerged we know that the liquid weighs less than the body.

159. When all the forces acting on a liquid are in equilibrium the liquid will be at rest. If a quantity of liquid is confined in a vessel and the liquid is at rest the surface of the liquid is **horizontal** or, as we usually say, **level.**

DENSITY AND SPECIFIC GRAVITY

160. The **density** of a substance is the quantity of matter contained in a unit volume of the substance.

If Q = the quantity of matter, D_1 = the density, and V = the number of units of volume; then

$$D_1 = \frac{Q}{V}; \text{ or, if } V = 1,\ D_1 = Q.$$

The ratio between the density of a given substance and the density of some substance taken as a standard is called the **specific density.**

The **specific gravity** *of a substance is the ratio between the weight of a certain volume of the substance and the weight of an equal volume of some other substance taken as a standard.*

Since the weight of a substance at a given place varies directly as the quantity of matter in it the specific density of a substance will always be numerically equal to its specific gravity.

The adopted standard for measuring the specific density or the specific gravity of liquids and solids is pure water at 39.1 deg. F., at which temperature its density is greatest. The weight of a cubic foot of water at 39.1 deg. F. is 62.42 lbs. As it is more convenient to make measurements at ordinary temperatures the weight of a cubic foot at 62 deg. F., viz. 62.35 lbs., is commonly used.

161. To obtain the specific gravity of a solid it is necessary to find the weight of the solid and the weight of an equal volume of water and then to take the ratio between the two quantities so obtained. For solids that do not dissolve in water this can be done by immersing the solid in a vessel completely filled with water and then weighing the water that overflows from the vessel. For solids that are soluble a similar method can be employed; namely, the solid instead of being placed in water is placed in some liquid in which it will not dissolve and of which the specific gravity is known.

162. To find the specific gravity of liquids a specific gravity bottle (i.e., a bottle which will hold a known weight of water) is filled with the liquid and weighed. The ratio of the weight of the liquid to the known weight of the same amount of water will be the specific gravity of the liquid.

163. There is another way in which the specific gravity of liquids may be determined. If salt is dissolved in water a certain volume of the solution weighs more than an equal volume of water. Therefore, as any solid which floats in a liquid displaces an amount of the liquid equal to its own weight, a solid floating in water will displace a greater amount than it would if it were floated in the salt solution. If the same solid is floated in different liquids it will sink to different depths. Since its weight remains constant the depths to which it sinks show the respective volumes of equal weights of the liquids displaced. If a solid which has a uniform section, such as a cylinder which floats in a vertical position, sinks to a depth d in water and to a depth d' in some other liquid, the specific gravity of the latter will be $\frac{d}{d'}$.

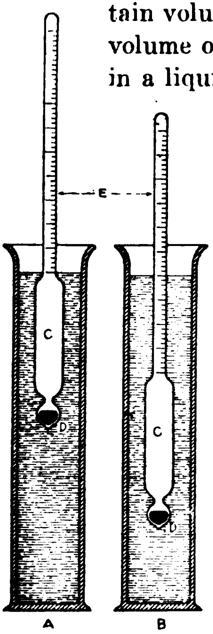

Fig. 37

164. This principle is made use of in the **hydrometer**. Fig. 37 shows an ordinary form of this instrument; view A represents it floating in water; view B, in alcohol. It consists of a glass tube, C, filled with air ending in a bulb, D, which is filled with mercury so that the whole instrument will float in a vertical position. The stem, E, is usually hollow and contains a paper scale, although sometimes the scale is cut on the glass. The scale may be divided into equal divisions of arbitrary length or it may be graduated so that the specific gravity can be read directly.

165. A common form of the hydrometer is Beaumé's. It has an arbitrary scale and requires the use of a table to convert its readings into specific gravities. Another form convenient in many ways is the specific gravity hydrometer which has a scale from which the specific gravity can be read directly.

166. In addition to its use in determining specific gravity the hydrometer is used in ascertaining either the amount of water or of solid matter in solutions, or whether a solution is of the proper strength, i. e., contains the desired amount of substance dissolved.

When using the hydrometer it is necessary, if accurate results are desired, to take into account the temperature of the solution. The higher the temperature, the less the density, or specific gravity, of a given solution and so the greater the depth to which the hydrometer will sink. This variation can be avoided by keeping the temperature constant for any particular solution.

WATER POWER

167. Any body of water elevated above the sea level has potential energy by virtue of its position. The amount of energy depends upon the volume of the water and its height above sea level. The amount of energy that can be put to practical use depends upon the amount of fall that can be obtained at any one place.

168. The gross horsepower of a waterfall may be calculated by the following formula, in which $Q' =$ the number of cubic feet of water flowing over the fall in one minute; $H =$ the distance in feet between the surface of the water at the top of a fall and that at its foot; W' = the weight of a cubic foot of water = 62.35 lbs.; and $G. H. P.$ = gross horsepower of fall.

$$G.\,H.\,P. = \frac{Q'HW'}{33{,}000}. \qquad (46)$$

Only a portion of the gross horsepower of a fall can be turned to practical use, the amount depending upon local conditions.

169. Problem.—If 10,000 cu. ft. of water flows over a certain fall 15 ft. high in 1 min., what is the useful horsepower if 50% of the gross horsepower can be developed?

Solution.—From the conditions of the problem $Q' = 10{,}000$ cu. ft., $H = 15$ ft., and $W' = 62.35$ lbs. Substituting these values in formula (46), we get

$$G.\,H.\,P. = \frac{10{,}000 \times 15 \times 62.35}{33{,}000} = 283.4 \text{ H. P.}$$

Since only 50% of the $G. H. P.$ can be developed, the useful horsepower = 283.4 × .50 = 141.7 H. P., *Ans.*

170. Problems.—(1) If 1,000 cu. ft. of water flows over a fall 20 ft. high in 1 min., what is the gross horsepower of the fall?

(2) What is the useful horsepower that can be derived from a fall 15 ft. high if 8,750 cu. ft. of water pass over in 1 min., and 40% of the gross horsepower is developed?

Answers.—(1) 37.7 + H. P. (2) 99.2 — H. P.

171. A water-wheel is a machine used to transform water power into mechanical power. There are several types of water-wheel, but the impulse wheel and turbine are the only ones extensively used. The impulse wheel, Fig. 38, is designed to be used when a small quantity of water at a high pressure is available. The turbine is designed to be used where a large quantity of water at a comparatively low pressure is available.

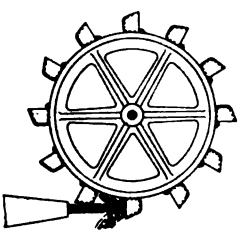

Fig. 38

CAPILLARY ACTION

172. If a glass tube with a small bore is placed in a glass of water the water will rise in and around the tube to a point somewhat higher than its level in the glass. View A, Fig. 39 illustrates this. If the same glass tube is placed in a glass of mercury the mercury will be depressed, view B, Fig. 39. In the first case the liquid *wets the surface* of the tube; in the second case it *does not.*

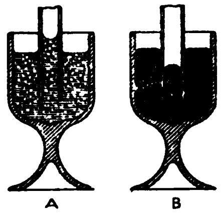

Fig. 39

173. *Whenever a liquid wets the surface of a solid with which it is in contact the liquid will be raised and whenever the liquid does not wet the surface it will be pushed down.* This action is called **capillary attraction** or **capillary action.**

174. Examples of capillary attraction are very common; the action of blotting paper, of sponges and of lamp wicks are all familiar illustrations. A wick is usually made of cotton fibers or loosely twisted threads woven together. As the capillary attraction of a wick depends upon the spaces between the fiber as much as upon the fiber, it is necessary, if the wick is to work properly, to keep it clean.

MECHANICS OF GASES

ATMOSPHERIC PRESSURE

175. The **atmosphere, or air,** is a thick layer of gases, largely nitrogen and oxygen, which surrounds the earth. Since the atmosphere is acted upon by the force of gravity it exerts considerable pressure at the earth's surface and this pressure is, according to Pascal's Law, equal in all directions at any particular point.

176. The **barometer** is an instrument for measuring the pressure of the atmosphere. The principle of the barometer may be explained by Fig. 40. This figure represents a glass tube a yard or more long, with one end closed. The tube is completely filled with mercury and placed with its open end under the surface of mercury in a cup. Since there is nothing to hold the mercury up in the tube, except the pressure of the atmosphere on the surface A, it will fall until the downward pressure at B and the downward pressure of the air, on an area of the surface A equal to the cross section area of the mercury column, are equal. The height BC of the mercury in the tube will be about 30 in. and the pressure or weight of this column of mercury will be equal to the pressure or weight of a column of air extending from the surface of the mercury to the extreme limit of the atmosphere and having a cross section area equal to the cross section area of the mercury column.

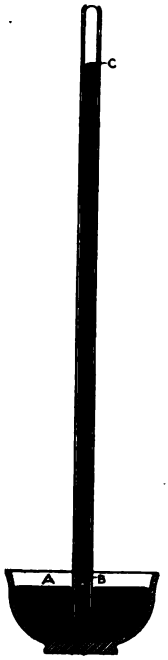

Fig. 40

The **mercurial barometer** is essentially the apparatus just described. To be of practical use, however, it must be arranged so that the height BC of the mercury column can be easily read.

177. Since the pressure of the air on a given area is the same as the pressure exerted on an equal area by a column of mercury that the air pressure will support, the pressure of the air may be found as follows when the height of the mercury column is known, in this case assumed to be 30 in. The inside dimension of the tube does not affect the height of the mercury unless it is very small when capillary action has some appreciable effect; hence it may be assumed, for convenience, that the tube has an inside cross section area of 1 sq. in. There will then be 30×1 or 30 cu. in. of mercury in the tube.

The specific gravity of mercury is 13.6; i. e., a cubic inch of mercury weighs 13.6 times as much as a cubic inch of water. A cubic foot of water weighs 62.35 lbs., therefore a cubic inch weighs $\frac{62.35}{1728}$ or .036 lbs. From which it follows that 30 cu. in. of mercury weighs $.036 \times 13.6 \times 30$, or $14.68 +$ lbs.

Therefore in this case the atmosphere supports 14.68 lbs. of mercury, or in other words, the atmospheric pressure is 14.68 lbs. per sq. in.

178. A form of barometer which is much used because of its convenience in handling is the **aneroid barometer,** Fig. 41. It consists of a metal chamber from which the air is exhausted and of which one part is thin corrugated metal. Variations in the atmospheric pressure cause this corrugated part to move an amount proportional to the pressure. This motion is multiplied by a systems of levers and transmitted to a pointer which thus indicates the changes in pressure. The scale of the aneroid barometer is arranged so that the pointer indicates the height in inches of a mercury column acted upon by the same conditions.

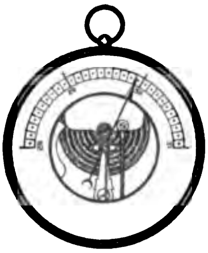

Fig. 41

179. If a barometer is kept in the same place it will be noticed that the atmospheric pressure varies from time to time. When the pressure increases or remains high the weather will probably be clear. When the pressure decreases or remains low the weather will probably be stormy.

Since the height of the air decreases in proportion to the elevation, the atmospheric pressure decreases as the height above sea level increases. At sea level the barometer stands at about 30 in. and its height decreases approximately $\frac{1}{10}$ in. for an increase of 100 ft. in elevation. By observing the height of the barometer it is possible to tell approximately the height above sea level of any particular locality.

EFFECT OF PRESSURE

180. The volume of matter changes with a change in the pressure on its surface. As before noted in solids and liquids this change is very slight and can be disregarded even when the pressure becomes very much greater than usual. With gases, however, a variation in pressure is accompanied by a variation in volume which is too large to be disregarded.

181. Boyle's Law.—*The volume of a given mass of gas at a constant temperature is inversely proportional to the pressure upon it.*

Boyle's Law may be expressed by the following formulas:

(1) If P_1 = the pressure upon a given mass of gas and V_1 = the volume of the same mass, then since the volume is inversely proportional to the pressure the ratio $V_1 : \frac{1}{P_1}$ is constant. If K = the constant we have

$$\frac{V_1}{\frac{1}{P_1}} = K, \text{ or } P_1 V_1 = K. \qquad (47)$$

(2) If the volume V_1 is changed to another volume V_2 then the pressure P_1 will become P_2, and since the product of the volume and the pressure is constant, we get

$$P_1 V_1 = P_2 V_2. \qquad (48)$$

182. PROBLEMS.—(1) The volume of a gas under a pressure of 14 lbs. per sq. in. is 10 cu. ft. Under what pressure will the volume be 4 cu. ft. if the temperature remains constant?

SOLUTION.—From the problem $V_1 = 10$ cu. ft., $P_1 = 14$ lbs. per sq. in., and $V_2 = 4$ cu. ft. Substituting these values in formula (48), we get

$$14 \times 10 = P_2 \times 4.$$

$$\text{Therefore } P_2 = \frac{14 \times 10}{4} = 35 \text{ lbs. per sq. in., } \textit{Ans.}$$

(2) The volume of a gas under a pressure of 10 lbs. per sq. in. is 2 cu. ft. What will be its volume under a pressure of 35 lbs. per sq. in. the temperature remaining constant?

SOLUTION.—From the problem $P_1 = 10$ lbs. per sq. in., $V_1 = 2$ cu. ft., and $P_2 = 35$ lbs. per sq. in. Substituting in formula (48), we get

$$10 \times 2 = 35 \times V_2.$$

$$\text{Therefore } V_2 = \frac{20}{35} = \frac{4}{7} \text{ cu. ft., } \textit{Ans.}$$

183. PROBLEMS.—(1) The volume of a gas under 50 lbs. pressure per sq. in. is 100 cu. ft. What will be the pressure if the volume is changed to 10 cu. ft.?

(2) The volume of air under a pressure of 28 lbs. per sq. in. is 10 cu. ft. What will be the volume under 7 lbs. pressure per sq. in.?

ANSWERS.—(1) 500 lbs. per sq. in. (2) 40 cu. ft.

184. Experiments have shown that Boyle's Law is not strictly true for all gases at ordinary temperatures. Such gases as can be condensed to a liquid form by a comparatively low pressure, for instance carbonic acid gas show considerable variation from the law, while other more permanent gases, as oxygen and nitrogen, do not.

PNEUMATIC MACHINES

185. The diagram Fig. 42 represents a **siphon**; XYZ is a bent tube which has been filled with water, the ends closed and the part XY placed in a vessel partly filled with water. If the ends of the tube are opened the water will begin to flow out through the opening at Z. This will continue until the level, L, of the water in the vessel has reached the same level as Z at which time the water will cease flowing.

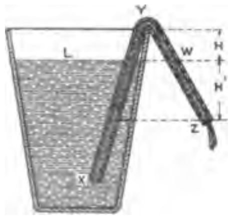

Fig. 42

In order to understand the principle on which the siphon acts it should be noticed that the downward pressure on the surface at L is due to the atmosphere and that this pressure is transmitted equally in all directions so that at point W there is a downward pressure equal to the atmospheric pressure. At the point Z the downward pressure is equal to the atmospheric pressure plus the pressure due to the column of water WZ. The upward pressure at Z is equal to the atmospheric pressure alone. Hence the downward pressure is greater and the water flows out. As the water flows out the height H′ of the column WZ decreases and finally becomes zero; then the downward and upward pressure are equal and the flow stops.

Since the action of the siphon is due to the pressure of the column of water WZ, the greater the height H′, between the levels L and Z the more rapid will be the flow and the greater the volume of water that will flow through the siphon in a given time.

The height H must not be greater than the height of a column of the liquid which the atmospheric pressure will support. The reason for this is clear; when the liquid begins to flow through Z the pressure at Y is lessened and the atmospherie pressure on L forces more liquid up through the part XY of the tube. If the height H is greater than the height of a column of the liquid which the atmospheric pressure will support the liquid will not be forced to the point Y and can not flow into the other part of the tube.

For water the extreme height for H is between 27 ft. and 33 ft., the exact height depending upon the elevation above sea level and the atmospheric pressure. For liquids heavier than water the height is less and for liquids lighter than water the height is greater.

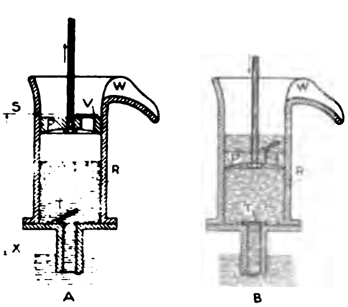

Fig. 43

186. The **suction pump** consists of a cylinder R, Fig 43, in which a piston P, works. There is a valve, T, opening upwards at the upper end of the pipe which connects the pump with the water to be raised and also a valve, V, opening upward in the piston. Suppose now in view A, the piston is moving upward. The space below the piston increases and consequently the pressure on the valve, T, will be lessened so that the greater pressure of the air below will force T open. Then the pressure on the surface of the water in the pipe will also become less and the atmospheric pressure will force the water up through the pipe and through the valve into the cylinder. View B, represents the relative positions of the valves when the piston is moving downward. As soon as the piston begins its downward stroke the pressure on the valve T is increased and the valve will close. As the piston continues descending the pressure on the air or water in the lower part of the cylinder will increase and force open the valve V and so allow the air or water to pass into the upper part of the cylinder. When the piston again begins to move upward the water in the upper part of the cylinder will flow out through W, the pressure on T will become less and more water will be forced up into the lower part of the cylinder.

187. As will be seen from the foregoing description the water is raised through the height XS* solely by the atmospheric pressure on X. Consequently the height XS must not be greater than the height of a column of water which the atmospheric pressure will support; i. e., to raise water at sea level the height XS must not be greater than about 33 ft. and for places above sea level it must be somewhat less.

188. The pump as described in Art. 186 is commonly used to raise water from wells and cisterns. If it is changed by closing the top of the cylinder, attaching a pipe to the spout W, Fig. 44, and, to insure better action, placing a valve V′ at the end of this pipe it can be made to lift water to a height depending only upon the strength of the pump and the force applied to work the piston. Arranged in this way the pump is called a **force pump**. Another type is shown in Fig. 45.

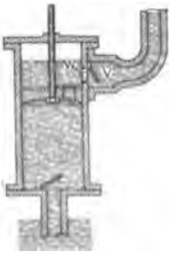

Fig. 44

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*The level S corresponds to the bottom of the piston at the end of the upward stroke.

189. The flow of water from a suction or force pump is under ordinary circumstances discontinuous, the liquid flowing only when the piston is moving in one direction. If an air chamber, A, Fig. 45, is attached to the pipe the action is changed. When the piston is moving down the water is forced into the chamber, A, as well as into the pipe, D, compressing the air in A. When the piston begins to move up the valve V′ closes but the pressure of the air in A continues forcing the water up the pipe D, and the flow of water will be continuous while the pump is working.

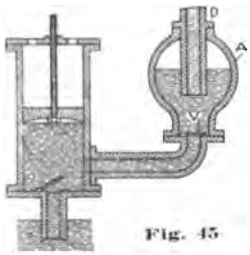

Fig. 45

190. The **vacuum pump** or **air pump** is used to take air out of any closed vessel. Fig. 46 shows the cylinder of a simple form of air pump. The action of this pump is as follows. The vessel from which the air is to be exhausted is connected to the tube A. On the upward stroke, since the volume of the air under the piston is increased its pressure is decreased, and the motion of the piston having opened the valve B, the expansive action of the air in the vessel forces a portion of it into the cylinder. On the downward stroke valve B is closed and the air in the cylinder is compressed until it opens valve C and escapes into the space above the piston. On the next upward stroke this air is forced out of the cylinder through the valve D.

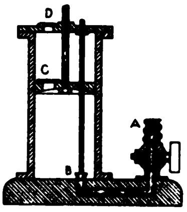

Fig. 46

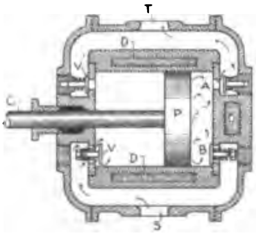

Fig. 47

191. The pump used for compressing air or other gases is practically a force pump whose valves and piston are air tight. Fig. 47 is a diagram of the cylinder of such a pump or **compressor.** The air comes into the cylinder through the pipe S and valve V while the piston P is moving toward the right. When the piston moves in the opposite direction the valve V′ opens and allows the air to pass out through the pipe T to a tank or reservoir. When the piston moves to the right again the valve V′ closes and the atmospheric pressure forces open the valve V so that more air passes into the cylinder. Valves are placed also at A and B so that the piston is forcing air out through T on both strokes. Ordinarily the piston rod C of the pump is connected directly with the piston rod of a steam engine or by means of gears and cranks with an electric motor.

A great deal of heat is evolved during the compression, and to prevent overheating the cylinder it is surrounded by the spaces marked D through which water is continually passing.

EXAMINATION QUESTIONS AND PROBLEMS

(1) State the difference between the compressibility of liquids and of gases.

(2) In a hydraulic jack the diameter of the smaller piston is 0.5 in. and that of the larger piston is 4 in. If the force is applied by means of a lever which has a force arm 1 yd. long and a resistance arm 2 in. long, what force must be applied at the end of the force arm to raise a weight of 5 T. 1,520 lbs.?

(3) If the weight in problem (2) is to be raised 4 in., how far will the force have to move?

(4) What is the pressure on the bottom of a cylindrical tank 12 ft. in diameter if it is filled with water to a depth of 8 ft.?[*]

(5) What is the pressure on the cylindrical surface of a cylindrical tank 16 ft. in diameter if it is filled with water to a depth of 10 ft.?

(6) A cubical object weighing 0.648 lbs. and measuring 3 in. in each direction is placed in water. To what depth below the surface of the water will the lower surface of the object sink providing it floats horizontally?

(7) A cylindrical piece of wood with a lead weight fastened to one end sinks to a depth of 15 inches in a liquid of which the density is 1. To what depth will it sink in a liquid of which the density is 3?

(8) 24,000 cu. ft. of water flows over a fall 22 ft. high in 1 min. What is the horsepower of the fall?

(9) If the atmospheric pressure is 13.22 lbs. per sq. in., how high will the mercury column in a barometer be?

(10) If a hole were drilled in the top of the glass tube illustrated in Fig. 40, what would be the result and why?

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[*] Use $\pi = 3.14$ and 1 cu. ft. of water = 62.35 lbs.

(11) A certain mass of air under a pressure of 15 lbs. per sq. in. occupies 200 cu. ft. of space. What will be the volume of the same mass under a pressure of 75 lbs. per sq. in. provided its temperature does not change?

(12) How must the pressure be changed, provided the temperature remains constant, to reduce the volume of a certain mass of gas to $\frac{1}{10}$ its original volume?

(13) What is the difference between a suction pump and a force pump?

(14) (*a*) What causes the action of a siphon? (*b*) Of a suction pump?

(15) What causes the action of a vacuum pump?

ANSWERS TO EXAMINATION PROBLEMS

(1) ?

(2) 10 lbs.

(3) 384 ft.

(4) 56,384.35 lbs.

(5) 156,623.2 lbs.

(6) 2 in.

(7) 5 in.

(8) 997.6 H. P.

(9) 27 in.

(10) ?

(11) 40 cu. ft.

(12) ?

(13) ?

(14) ?

(15) ?

TABLE OF SYMBOLS*

A and A' = areas of surface.

d' = distance through which f' moves.

f' = applied force.

$G.H.P.$ = gross horsepower.

H = height of fall.

h' = depth of horizontal surface below surface of liquid.

h'' = depth of center of area of vertical surface below surface of liquid.

K = a constant.

P = pressure of liquid on horizontal surface.

P' = pressure of liquid on vertical surface.

P_1 and P_2 = pressures on a mass of gas.

Q' = quantity of water in cu. ft. flowing over a fall in 1 min.

r = resistance.

s = distance through which r moves.

V_1 and V_2 = volumes of a mass of gas corresponding to pressures P_1 and P_2.

W' = weight of a unit volume.

TABLE OF FORMULAS

The formulas are numbered consecutively throughout the text and are placed and numbered in the same order in the following table. As an aid in referring to the text the page number is given directly after the formula number.

		Page No.			Page No.
$f' : r :: A : A'$	(42)	(48)	$G.H.P. = \frac{Q'HW'}{33.000}$	(46)	(55)
$f' : r :: s : d'$	(43)	(48)			
$P = Ah'W'$	(44)	(50)	$P_1V_1 = K$	(47)	(59)
$P' = Ah''W'$	(45)	(50)	$P_1V_1 = P_2V_2$	(48)	(59)

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*The symbols and formulas used in the text are collected in these tables for convenient reference and need not be studied.

HEAT AND LIGHT

HEAT

1. **Heat** is a form of molecular energy consisting of an agitation of the molecules of matter. As this agitation is increased a body becomes warmer, as it is decreased a body becomes cooler. The state of a body with regard to its being warm or cold can be recognized by the sensation of feeling.

2. Heat may be derived from any of the following sources; *the sun, mechanical action, chemical action, electric currents, the interior of the earth, and changes in physical condition.*

The *sun* is the most important source of heat, and directly or indirectly, of nearly all other forms of available energy.

Mechanical action as a source of heat is illustrated by friction in a bearing.

Chemical action as a source of heat is illustrated by the burning of wood; the carbon of the wood unites with the oxygen of the air and a large amount of heat is evolved.

The passage of an *electric current* tends to heat a conductor. If there is a large current and a considerable amount of resistance the heating of the conductor will be very marked, as for example, in an incandescent lamp.

The *interior of the earth* seems to contain a large store of heat, but no extensive practical uses are made of this source of energy.

A change of *physical condition* as a source of heat is well illustrated by the condensation of steam. Thus when steam at 212° F. condenses to water at the same temperature it gives up a large amount of heat.

TEMPERATURE

3. If a warm body, such as a piece of warm iron, is placed in contact with a cold body, the heat of the warm body is decreased as indicated by its becoming cooler and the heat of the cold body is increased as indicated by its becoming warmer. We say the temperature of the warm body decreases and the temperature of the cold body increases. **Temperature,** then, is the condition of a body considered with reference to its ability to give heat to or receive heat from another body. Heat always passes from a body of high to one of low temperature.

In order to measure the temperature of bodies it is necessary to have some unit for comparison. An arbitrary unit that is commonly used is called the degree Fahrenheit and is $\frac{1}{180}$ part of the difference between the temperatures of melting ice and pure water boiling under an atmospheric pressure of 14.7 lbs. per sq. in. To illustrate the way it is expressed a temperature of ten degrees Fahrenheit is written 10 deg. F. or 10° F.

4. It should be noticed that there is a distinction between *temperature* and *quantity of heat.* A pint and gallon of boiling water both have the same temperature, but the quantity of heat in the pint is much less than the quantity in the gallon. The quantity of heat in a body depends not only upon its temperature, but also upon the quantity of matter in it.

TRANSMISSION OF HEAT

5. If one end of a rod of iron is placed in a fire and the other end is held in the hand the temperature of the end held in the hand gradually increases. That is, the heat passes along the rod of iron from one molecule to another until the whole rod is affected. This manner of transmitting heat from a point of high temperature to one of low temperature is called **conduction,** and the power of a substance to conduct heat is called its **thermal conductivity.**

6. Some substances conduct heat much more rapidly than others. Thus metals are the best conductors of heat, liquids are not so good, and gases are very poor conductors. Substances of the same class do not all conduct heat with the same rapidity. Silver, for example, is the best conductor known; if 1,000 represents its conductivity then 811 represents that of copper and 359 that of cast iron.

7. It is seen from Art. 6, that copper is a relatively good heat conductor. It is partly for this reason that it is used in making soldering coppers.* When a soldering copper is heated all parts have nearly the same temperature, but as soon as it is used the point begins to lose its heat more rapidly than the large part of the copper. On account of its good conductivity, however, the heat from the large part passes to the point of the copper and keeps it at a high temperature.

8. Air which has become heated in any manner, above the temperature of the surrounding air, for example by contact with a warm stove, expands, becoming lighter and consequently rising. Therefore it carries the heat away from the stove and coming in contact with other parts of the air and objects which are cooler gives up some of its heat to them. The process of transferring heat in this way is called **convection.** While gases are very poor conductors of heat they can be heated and transfer heat readily by convection.

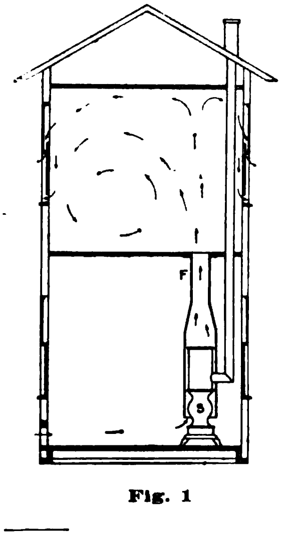

Fig. 1

9. This method of conveying heat is much used in heating buildings. Fig. 1 shows a system that might be used in a small building such as a signal tower. The supply of heat is obtained from an ordinary stove, S, surrounded with a sheet iron jacket arranged with a pipe, F, leading to the room above. The air currents are indicated by arrows.

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*Soldering irons.

10. Although liquids are better conductors than gases they are heated and transfer heat more rapidly by convection than by conduction. This is illustrated by the hot water heating system shown in Fig. 2. As the water in the boiler B is heated by the fire it expands, becomes lighter, and rises to the top of the boiler, displacing the cooler water which settles to the bottom and in turn is heated by the fire. This action sets up a circulation in the boiler by means of which the temperature of the entire body of water is increased.

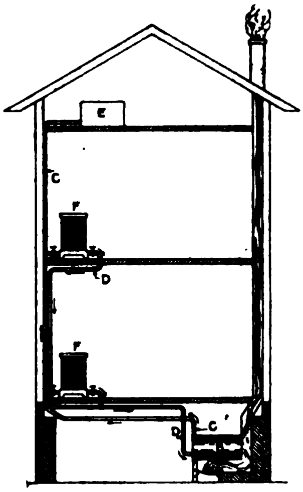

Fig. 2

The hot water rising to the top of the boiler does not stop at that point, however, but continues to rise through pipe C displacing the cold water which flows to the bottom of the boiler, the lowest point in the system, through radiators FF and the pipe D. The action is continuous on account of the fact that as soon as hot water begins to pass through the radiators, it gives up a part of its heat to the metal of the radiator which in turn heats the surrounding air. This results in the water becoming cooler and flowing back to the boiler through D while hot water from C takes its place. E is an open tank placed at the highest point of the system. It acts as a reservoir for the increased volume of the water as it expands when the fire becomes hotter and it keeps the system full as the water contracts when the fire dies down. The system must be kept full of water in order to maintain circulation.

EFFECTS OF HEAT

11. Among the more common effects of heat, besides *changes in temperature* which have already been mentioned, there are *changes of volume*, *changes of state*, and *chemical effects.*

12. If not affected by other conditions, changes in temperature cause changes in the volume of a gas, the volume varying with the temperature. The relation between the temperature and volume is expressed by the following law.

13. **Law of Charles.**—*The volume of a given mass of gas at a constant pressure is proportional to its absolute temperature, or in other words, the volume of a gas increases or decreases* $\frac{1}{491.2}$* *of its volume at 32 deg. F. for every degree F. that the temperature increases or decreases.*

If V_0 = the volume at 32 deg. F. and V_1 = the volume at any other temperature t_1, then

$$V_1 = V_0\left(1 + \frac{t_1 - 32}{491.2}\right). \quad (1)$$

14. PROBLEM.—**The volume of a gas at 32 deg. F. is 10 cu. ft. What will be its volume at 52 deg. F. if the pressure remains constant?**

SOLUTION.—**By the conditions of the problem $V_0 = 10$ cu. ft., and $t_1 = 52$ deg. Substituting these values in formula (1), we have**

$$V_1 = 10\left(1 + \frac{52 - 32}{491.2}\right).$$

Therefore $V_1 = \frac{5112}{491.2} = 10.40$ cu. ft., *Ans.*

15. If, at the same time that the temperature changes from 32 deg. F. to t_1, the pressure changes from P_0 *to* P_1 we have the following formula:

$$V_1 = V_0\frac{P_0}{P_1}\left(1 + \frac{t_1 - 32}{491.2}\right). \quad (2)$$

16. PROBLEM.—**The volume of a gas at 32 deg. F. is 6 cu. ft., if the temperature rises to 50 deg. F. and at the same time the pressure changes from 14 lbs. per sq. in. to 20 lbs. per sq. in., what is the volume of the gas?**

SOLUTION.—**By the conditions of the problem $V_0 = 6$ cu. ft., $t_1 = 50$ deg., $P_0 = 14$ lbs. per sq. in., and $P_1 = 20$ lbs. per sq. in. Substituting these values in formula (2), we have**

$$V_1 = 6 \times \frac{14}{20}\left(1 + \frac{50 - 32}{491.2}\right).$$

Therefore $V_1 = \frac{2138.64}{491.2} = 4.35$ cu. ft., *Ans.*

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*The coefficient $\frac{1}{491.2}$ is not exactly the same for all gases, but is very nearly so.

17. Problems.—(1) The volume of a gas at 32 deg. F. is 5 cu. ft. What will be its volume at 100 deg. F. if the pressure remains constant?

(2) What will be the volume in problem (1) if the pressure changes from 14 lbs. per sq. in. to 18 lbs. per sq. in.?

Answers.—(1) 5.69 cu. ft. (2) 4.42 cu. ft.

18. *If the pressure is maintained constant equal volumes of various gases when heated expand by nearly equal amounts for the same rise in temperature, or if the initial volumes are maintained constant the pressures increase by nearly equal amounts.*

Liquids and solids expand when heated, but not in accordance with the preceding law. Thus if rods of iron and brass are each 1,000 in. long at 0 deg. F., at 100 deg. F. the iron rod will be about 1,000.6 in. long and the brass rod about 1,001.0 in. long, or there will be a difference in expansion of 0.4 in.

19. All substances when heated uniformly tend to expand equally in all directions. The expansion of gases and liquids is measured by the change in volume and is called **cubical expansion.** The expansion of solids is usually measured by the change of some one dimension and is called **linear expansion.** The **coefficient of cubical expansion** is the expansion per unit volume for unit change of temperature. The **coefficient of linear expansion** is the expansion per unit length for unit change of temperature. The coefficient of cubical expansion is equal to three times the coefficient of linear expansion.

20. Problem.—A steel rail is 33 ft. long at 0 deg. F. What will be the amount of expansion in inches if the temperature changes from 0 deg. F. to 80 deg. F. and the coefficient of linear expansion is .00000636?

Solution.—Since steel expands .00000636 of its length for a change of 1 deg. F. in temperature, for a change of 80 deg. it will expand $.00000636 \times 80$ times its length; or the expansion in feet will be $.00000636 \times 80 \times 33$. Therefore the expansion in inches equals

$$.00000636 \times 80 \times 33 \times 12 = .2014848 \text{ in.}, \quad \textit{Ans.}$$

21. Problems.—(1) A copper wire is 1 mi. long at 32 deg. F. What will be its length in feet at 82 deg. F. if the coefficient of linear expansion of copper is .00000887?

(2) A concrete wall is 500 ft. long at — 10 deg. F. What distance in inches must be allowed for expansion if the temperature rises to 100 deg. F. and the coefficient of linear expansion of concrete is .00000795?

ANSWERS.—(1) 5,282.34 ft. (2) 5.247 in.

22. The effect of heat on water is somewhat peculiar. Practically all common substances except water expand on being heated and contract when cooled. If water at 60 deg. F. is cooled it will contract as is expected until it reaches 39.1 deg. F. At that point it reaches its greatest density. If the cooling is continued the water begins to expand and the expansion continues so that ice formed from the water occupies a greater amount of space than the water at 39.1 deg. F. did. It is on this account that water enclosed in pipes or vessels is liable to burst them if allowed to freeze.

23. The fact that gases and some liquids and solids when heated expand at a rate nearly proportional to the increase in temperature is made use of in a class of instruments for measuring temperature. Such instruments are called **thermometers.** A common thermometer, Fig. 3, consists of a glass tube with a fine bore having a bulb at one end. The tube is partly filled with mercury or alcohol, the air taken out of the remaining space, and the tube sealed. When the cube is heated the mercury or alcohol expands and rises; when the tube is cooled the liquid contracts and falls. A scale is either marked on the tube or on a piece of metal or wood to which the tube is securely fastened so that the expansion and contraction of the mercury may be measured.

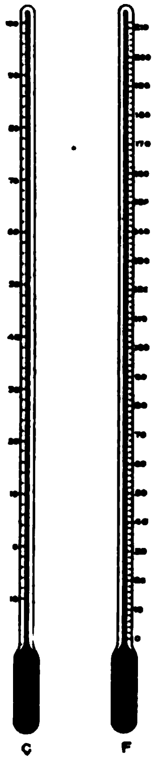

Fig. 3

24. There are two kinds of thermometers that are used commonly, the difference being in the scales. Under ordinary atmospheric pressure water freezes or ice melts at the same temperature, and under a constant pressure of 14.7 lbs. per sq. in. the temperature of steam is always the same. These two temperatures are taken as reference points. On the **Fahrenheit** thermometer the melting point of ice is marked 32 deg. and the temperature of steam or boiling water is marked 212 deg. On the **Centigrade** thermometer the melting point of ice is marked 0 deg. and the temperature of boiling water is marked 100 deg. On the Fahrenheit thermometer then there are 180 divisions or degrees between the freezing and the boiling points and on the centigrade there are 100 divisions or degrees between these points. The Fahrenheit thermometer is used for ordinary purposes, the Centigrade for scientific work.

25. The following formula may be used to reduce a Fahrenheit reading to Centigrade. If $F =$ the Fahrenheit reading and $C =$ the Centigrade reading, then

$$C = \frac{5}{9}(F - 32^\circ). \qquad (3)$$

Similarly a Centigrade reading may be reduced to Fahrenheit by the formula,

$$F = \frac{9}{5}C + 32^\circ. \qquad (4)$$

In applying these formulas it must be remembered to add or subtract 32 deg. algebraically. Temperatures below 0 deg. on either scale are considered negative quantities, those above 0 deg. positive.

26. For temperatures below — 38 deg. F. it is necessary to use alcohol in thermometers since mercury solidifies at about — 39 deg. F. For temperatures up to 675 deg. F. ordinary mercury thermometers may be used. For temperatures higher than that other instruments are used.

27. Whether a body exists in a solid, liquid, or gaseous state depends upon its temperature and the pressure upon it.

Any solid may be turned to a liquid by applying sufficient heat; and a liquid may be changed to a solid by taking heat from it. The temperature at which these changes take place is called the **melting point.**

For any particular substance the melting point is constant if the pressure remains the same. For different substances the melting point varies greatly.

When the process of melting is going on the temperature of the substance remains constant.

28. Any liquid substance can be changed to a vapor if a sufficient quantity of heat is applied; and any vapor can be changed to a liquid if sufficient heat is taken away from it. The temperature at which these changes take place is usually called the **boiling point.**

For any particular substance the boiling point is constant if the pressure is the same. By increasing the pressure the boiling point is raised and by decreasing the pressure it is lowered.

When a liquid begins to boil its temperature remains constant until the boiling is completed if the pressure is not changed.

MEASUREMENT OF HEAT

29. In measuring a quantity of heat the **British thermal unit** (B. T. U.) is the standard commonly used in this country. It is the quantity of heat required to raise the temperature of 1 lb. of pure water 1 deg. F.* In some other countries a standard called the **calorie** is used. It is the quantity of heat required to raise the temperature of 1 kilogramme of pure water 1 deg. C. at or about 4 deg. C. 1 B. T. U. is equal to 0.252 calorie; 1 calorie is equal to 3.968 B. T. U.

30. PROBLEM.—How much heat is required to raise the temperature of 100 lbs. of water from 40 deg. F. to 212 deg. F.?

SOLUTION.—It requires 1 B. T. U. to raise the temperature of 1 lb. of water 1 deg. F. Therefore to heat 100 lbs. 172 deg. it will require

$$100 \times 172 = 17{,}200 \text{ B. T. U., } \textit{Ans.}$$

31. When a solid is changed to a liquid or a liquid to a gas a quantity of heat seems to disappear, for during the process the temperature of the substance does not change. The heat is evidently used up in producing a change in the state of the substance. Heat that disappears in these changes is commonly called **latent heat.** If the change is reversed the same amount of heat which disappears will reappear.

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*Where more than ordinary accuracy is required this should be measured at or about 39.1 deg. F.

When a solid is being melted the quantity of heat necessary to melt a unit weight of the substance is called the **latent heat of fusion.**

When a liquid is changed to a solid a quantity of heat equal to the latent heat of fusion is given out by the substance.

32. PROBLEM.—How much heat will become latent in melting 25 lbs. of cast iron if the latent heat of fusion of cast iron is 59.4 B. T. U. per lb.?

SOLUTION.—59.4 B. T. U. become latent in melting 1 lb. of cast iron; therefore in melting 25 lbs. the amount of heat which will become latent equals

$$25 \times 59.4 = 1{,}485 \text{ B. T. U., } \textit{Ans.}$$

33. PROBLEMS.—(1) How much heat will become latent in melting 500 lbs. of lead if the latent heat of fusion of lead is 9.66 B. T. U. per lb.?

(2) How much heat will be given out if 75 lbs. of zinc which is melted is allowed to harden, if the latent heat of fusion of zinc is 50.03 B. T. U. per lb.?

(3) How much water at 185 deg. F. will be required to melt 15 lbs. of ice at 32 deg. F. if the latent heat of fusion of ice is 144 B. T. U. per lb.?

ANSWERS.—(1) 4,830 B. T. U. (2) 3,752.25 B. T. U. (3) 14.11 lbs.

34. When a solid or liquid is changing to a vapor the quantity of heat necessary to vaporize a unit weight of the substance is called the **latent heat of vaporization.**

When a vapor changes back to a liquid or solid a quantity of heat equal to the latent heat of vaporization reappears.

35. PROBLEM.—How much heat will it require to turn 5 lbs. of alcohol which is at the boiling point into vapor if the latent heat of vaporization of alcohol is 364.3 B. T. U. per lb.?

SOLUTION.—It requires 364.3 B. T. U. to turn 1 lb. of alcohol at the boiling point to vapor; therefore the amount of heat required to turn 5 lbs. to vapor equals

$$5 \times 364.3 = 1{,}821.5 \text{ B. T. U., } \textit{Ans.}$$

36. PROBLEMS.—(1) How much heat becomes latent in changing 50 lbs. of water at 212° F. into steam at atmospheric pressure if the latent heat of vaporization of water is 965.7 B. T. U. per lb.?

(2) If 100 lbs. of water have been turned to steam how much heat will reappear if the steam is allowed to condense to water at 212 deg. F.?

ANSWERS.—(1) 48,285 B. T. U. (2) 96,570 B. T. U.

37. Vaporization may take place in two ways. If the liquid does not seem to be disturbed but the vapor rises slowly from the surface the process is called **evaporation.** If the liquid is disturbed and the vapor forms not only at the surface but also in the substance the process is called **ebullition** or **boiling.**

38. The rate at which vaporization will take place is increased by the following changes in condition: (1) an increase in the temperature of the substance, (2) an increase in the amount of surface, (3) a decrease in the pressure, and (4) a continual change of the air in contact with the surface.

39. The boiling point of liquids is changed by various things. (1) The boiling point is higher if solids are dissolved in the liquid. (2) The boiling point is raised if the pressure on the surface of the liquid is increased and lowered if it is decreased.

40. As an illustration of the chemical effect of heat the common parlor match affords a good example. When the match head is heated sufficiently by friction or other means the materials of which it is composed combine. This combination in turn produces more heat thus raising the temperature of the wood to a point where it will combine with the oxygen of the air.

SPECIFIC HEAT

41. The amount of heat required to raise the temperature of any body of matter one degree is called the **thermal capacity** of the body. In measuring the thermal capacity of bodies, water is usually taken as a standard. The ratio of the quantity of heat required to raise the temperature of a definite weight of any substance one degree, to the quantity of heat required to raise the temperature of an equal weight of water one degree from a temperature of 39.1 deg. F. is called the **specific heat** of the substance.

The specific heat of any substance varies somewhat with its temperature, but may be considered constant where great accuracy is not required.

42. The specific heat of a substance may be determined by the method of mixture. The body whose specific heat is to be determined is heated to a definite known temperature and then immersed in a vessel of water of which the weight and temperature are known, or in a vessel of liquid of which the weight, temperature and specific heat are known. The temperature of both will soon be the same and this is measured. All of the heat lost by the body will be gained by the water. If $s =$ the specific heat of the body, $w =$ its weight, and $t =$ its temperature; $s' =$ the specific heat of the water or liquid, $w' =$ its weight and $t' =$ its temperature; and $T =$ the final temperature of both, then $sw(t - T)$, which represents the quantity of heat lost by the body, is just equal to $s'w'(T - t')$, which represents the quantity of heat gained by the water, providing of course, that none of the heat is lost by radiation or by conduction. That is,

$$sw(t - T) = s'w'(T - t'), \qquad (5)$$

$$\text{or } s = \frac{s'w'(T - t')}{w(t - T)}. \qquad (6)$$

43. Problem.—If 5 lbs. of water at 40 deg. F. are placed in a vessel with 10 lbs. of mercury at 80 deg. F. and the resulting temperature is 42.5 deg. F., what is the specific heat of mercury, the specific heat of water being 1?

Solution.—From the conditions of the problem $w' = 5$ lbs., $t' = 40$ deg., $s' = 1$, $w = 10$ lbs., $t = 80$ deg. and $T = 42.5$ deg. Substituting these values in formula (6) we get

$$s = \frac{5\,(42.5 - 40)}{10\,(80 - 42.5)}.$$

Therefore $s = .0333$, *Ans.*

44. Problem.—A piece of steel weighing 2 lbs. with a temperature of 500 deg. F. is placed in a tub containing 124.8 lbs. of water at 32

deg. F. If the resulting temperature is 32.87 deg. F., what is the specific heat of steel?

ANSWER.—.116.

45. PROBLEM.—A piece of cast iron weighing 10 lbs. at a temperature of 400 deg. F. is placed in a tub containing 100 lbs. of water at 60 deg. F. What is the resulting temperature if the specific heat of iron is 0.1298 and of water 1?

SOLUTION.—By the conditions of the problem $t = 400$ deg., $w = 10$ lbs., $w' = 100$ lbs., and $t' = 60$ deg., and $s' = 1$. Substituting these values in formula (5) we get

$$0.1298 \times 10 \times (400 - T) = 1 \times 100\,(T - 60).$$

Simplifying $1.298\,(400 - T) = 100\,(T - 60)$,

or $519.2 - 1.298\,T = 100\,T - 6000$.

Therefore $T = 64.35$ deg., *Ans.*

46. PROBLEM.—A piece of steel weighing 4 lbs. heated to a temperature of 700 deg. F. is placed in a tub containing 125 lbs. of water at 40 deg. F. What is the resulting temperature if the specific heat of steel is 0.1165?

ANSWER.—42.45 deg. F.

HEAT AND WORK

47. It has been found that a definite amount of mechanical energy can be converted into a definite amount of heat. Thus 778 ft. lbs. are equivalent to 1 B. T. U. This quantity of mechanical energy is known as the **mechanical equivalent of heat.**

48. PROBLEM.—A certain journal uses up 15,000 ft. lbs. of energy in friction. How much heat is generated at the journal assuming that all the mechanical energy used up in friction is turned to heat?

SOLUTION.—By Art. 48, 778 ft. lbs. of mechanical energy are equivalent to 1 B. T. U. Therefore 15,000 ft. lbs. equal

$$\frac{15{,}000}{778} = 19.28 \text{ B. T. U., } \textit{Ans.}$$

49. PROBLEMS.—(1) How many heat units are equivalent to 250,000 ft. lbs. of mechanical energy?

(2) What is the mechanical equivalent of 125 B. T. U.?

ANSWERS.—(1) 321.33 B. T. U. (2) 97,250 ft. lbs.

50. Several mechanical devices are used to convert heat into mechanical energy. Among the most important of these is the steam engine, the essential parts of which are a cylinder in which a piston is forced back and forth by the steam and a valve arranged to admit the steam first on one side of the piston and then on the other. Fig. 4 shows a diagram of a simple horizontal engine. A is the cylinder; B, the piston; C, the piston rod which is joined by the connecting rod D to the crank E; F and F′ are openings, called ports, through which the steam passes from the valve box into the cylinder; and G is the slide valve which controls the passage of the steam in and out of the cylinder.

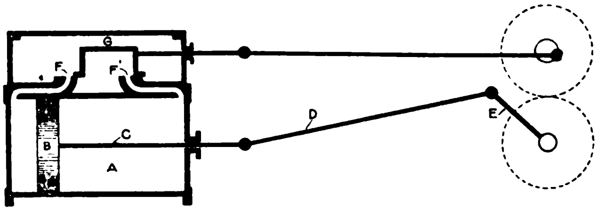

Fig. 4

The action of the steam engine is due to the pressure of the steam. That in turn is due to the action of the molecules of water or steam under the influence of heat. The steam engine, then, may properly be called a heat engine, for it is a contrivance for changing heat energy to mechanical energy.

51. The gas or gasoline engine is another type of heat engine that is used extensively. Its action is illustrated by the diagrams in Figs. 5, 6, 7 and 8. In Fig. 5 the piston is moving down

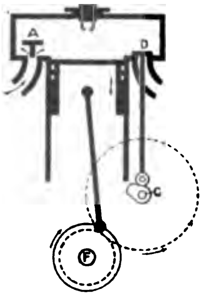

Fig. 5

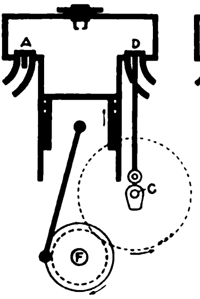

Fig. 6

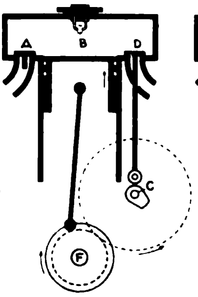

Fig. 7

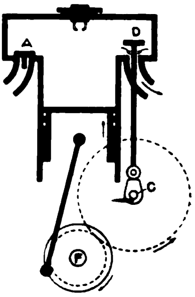

Fig. 8

drawing a charge of air and gas into the cylinder through the inlet pipe and valve A. The air is mixed with gas or gasoline vapor in the proper proportion by some form of mixing valve or carburetor* attached to the inlet pipe.

At the end of this down stroke valve A closes and the piston moving up, Fig. 6, compresses the charge until, toward the end of the stroke, the pressure reaches a value varying from about 45 to 80 lbs., per sq. in., depending on the design of the engine.

Just before the piston reaches the end of this up stroke the mixture is ignited, Fig. 7, usually by an electric spark B between two wires, one or both of which are insulated from the cylinder. The spark is produced by the action of an electric current which is allowed to flow through a spark coil,* at the right instant, by a special form of switch* operated by the cam shaft C.

After ignition the mixture burns very rapidly and the pressure, greatly increased by the action of the heat on the gases, forces the piston down on the third stroke thus changing a portion of the heat energy of the gases into mechanical energy.

At the end of the third stroke the exhaust valve D is opened by the valve gear and the burned gases escape through the exhaust pipe to the atmosphere as the piston moves up on the fourth stroke, Fig. 8.

As heat energy is transformed into mechanical energy only during the third stroke, a fly wheel* must be attached to the crank shaft F in order to store energy to operate the engine during the other three strokes and also to furnish steady power to some other machine or device.

Only a portion of the heat energy is transformed into a mechanical energy. The portion that is not transformed escapes, partly with the exhaust gases and partly by heating the cylinder walls. In order to prevent the cylinder walls from becoming too hot they are usually surrounded by a chamber, called a water jacket, through which cooling water passes. In some cases small engines are cooled by air currents circulating around the cylinder which is provided with ribs or projections to increase the cooling surface.

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*Not shown in the diagrams.

LIGHT

RADIANT ENERGY

52. To explain certain facts about heat and light such, for example, as the transference of heat and light from the sun, it is necessary to assume that there is some medium pervading all space. This medium, which is called **ether,** is capable of receiving energy from the molecular vibrations of a body and transmitting it. This manner of transmitting energy is called **radiation** and the energy transmitted by radiation is known as **radiant energy.** Heat and light are forms of radiant energy.

53. Radiant energy falling upon a body, usually called incident radiation, may be transmitted, as when light falls on a pane of glass; or reflected, as when light falls on a mirror; or absorbed, as when light falls on a dark surface. Any object that absorbs radiant energy is heated at the same time.

54. A number of bodies in an enclosed space exchange radiant energy, or heat, until all are in the same condition, that is, in a condition of thermal equilibrium. Even then radiation does not cease, but each body receives just as much energy as it gives out so that the state of equilibrium is maintained.

55. Radiant energy is propagated along straight lines known as **rays.** A collection of parallel rays is called a **beam,** Fig. 9; and a collection of divergent or convergent rays, a **pencil,** Fig. 10.

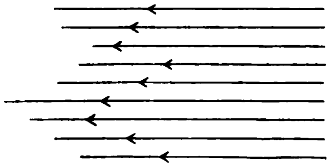

Fig. 9

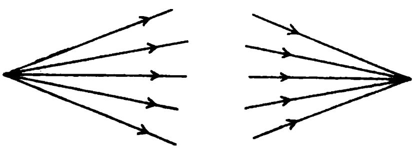

Fig. 10

56. From the preceding paragraphs it will be noted that heat may be transmitted from one body to another by **radiation** as well as by conduction and convection, which have already been mentioned. Thus the heat of the sun reaches the earth by radiation. The radiation of heat follows the laws for the radiation of light.

57. Light is a form of radiant energy capable of affecting the nerves of the eye.

58. Any body of matter when sufficiently heated will produce light. When in such a condition a body is said to be in a state of **incandescence** and is called a **luminous body.** The sun and artificial sources of light, such as an incandescent lamp, are examples of luminous bodies.

Certain peculiar substances, such as sulphide of calcium, have the power of absorbing light when exposed to it without becoming hot and then, if placed in a dark room, emit light for some time. This property is called **phosphorescence.**

59. The velocity of light as determined by experiment is 186,000 mi. per sec., so that transmission for ordinary distances is practically instantaneous. The velocity varies for different mediums and is greater in rare mediums, such as air, than in dense mediums, such as glass.

60. All luminous bodies are visible, but by far the greater part of all bodies on the earth are not luminous, yet they are visible at times because of light which they receive from luminous bodies. A body, such as the moon, which is not luminous, but is made visible by light received from a luminous body, is said to be **illuminated.**

61. When a luminous or highly illuminated body is seen against a dark back-ground it appears larger than it really is. This effect is called **irradiation.** The effect becomes greater as the contrast between the luminous or illuminated body and the dark back-ground is increased.

62. Certain bodies which allow the light falling on them to pass through practically undiminished are called **transparent**

bodies, for example, air and glass; other bodies while allowing the light to pass through to a greater or less extent seem to scatter it and are called **translucent,** for example, fog and ground glass; still other bodies stop or absorb all the light falling on them and are called **opaque bodies,** for example, a piece of wood or of iron.

Visible objects can be seen distinctly through transparent bodies; they can be seen very indistinctly through translucent bodies, but cannot be seen through opaque bodies.

SHADOWS

63. If an orange is held between a lighted candle and a screen the light will be cut off from a part of the screen; we usually describe this by saying that the orange "casts its shadow" on the screen. What actually takes places is that the orange, being opaque, stops the light that falls upon it and consequently the space and part of the screen on the opposite side of the orange from the candle receive no direct light. Fig. 11 illustrates this, showing the light as though coming from just one point of the flame. If the flame is large the light coming from all the points causes the shadow to take the form shown in Fig. 12. All of the light will be cut off from the space A, B, C, D. This part of the shadow is called the **umbra.** Only part of the light will be cut off from the outer section ADF, BCE. This part of the shadow is called the **penumbra.**

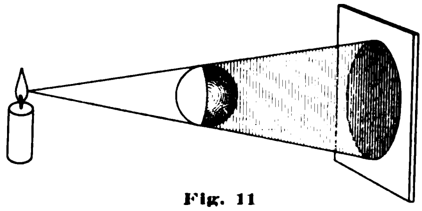

Fig. 11

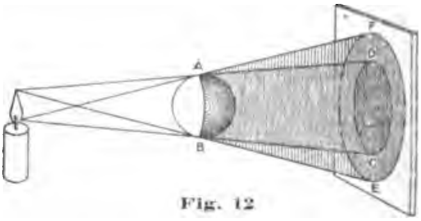

Fig. 12

64. If light from a luminous body, or from a body brightly illuminated, as by light from the sun, is allowed to pass through a small opening and fall upon a screen, a picture, or **image,** of the object from which the light comes will be seen upon the screen, as illlustrated by Fig. 13. As the rays of light all cross at D the image will be inverted.

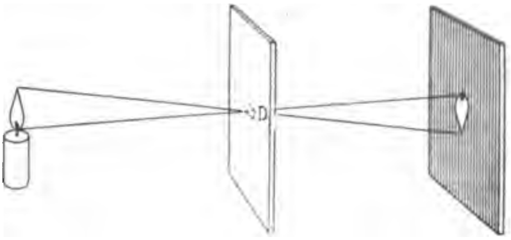

Fig. 13

PHOTOMETRY

65. The quantity of light received by any unit surface is usually called the **intensity of illumination** of that surface.

66. *The intensity of illumination of a surface depends upon the angle at which the light meets the surface.* If the surface is at right angles with the direction of propagation of the light the intensity will be greatest, and the intensity will decrease as the angle between the surface and the light becomes less. Fig. 14 illustrates this clearly. If a beam of light meets a surface at an angle of 90° it will illuminate a section represented by AC. If it meets the surface at a smaller angle it will illuminate a larger section represented by AB.

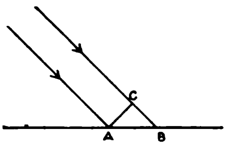

Fig. 14

67. *The intensity of illumination of a surface depends also upon its distance from the source of illumination and varies inversely as the square of the distance of the surface from the source.* Fig. 15 illustrates this. The distance from the light L to A is $\frac{1}{2}$ the distance from L to B. The surface of A is $\frac{1}{4}$ of the surface of B. The total amount of light which falls on A is just equal to the amount which would fall on B if A were removed. But as there is 4 times as much surface in B as in A the intensity of illumination of B is ¼ that of A. That is, as the distance is increased in the ratio of 1 : 2 the intensity of illumination on the same surface is decreased in the ratio of 1 : 4; or in other terms, if I = the intensity of illumination of the surface, B = the brightness of the light and d = the distance of the illuminated surface from the light, then

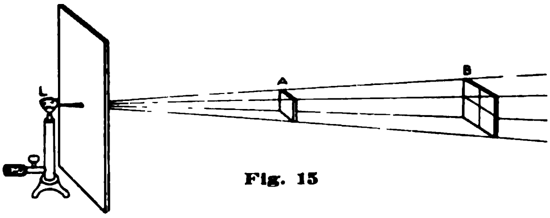

Fig. 15

$$I = \frac{B}{d^2}.$$

68. The standards used in measuring the brightness of lights are artificial lights such as a sperm candle burning 120 grains per hour or a standardized incandescent lamp supplied with a constant current.

69. The instrument used to measure the brightness of lights is called a **photometer.** A good example is the Bunsen photometer, Fig. 16, which consists of a bar having a scale divided into 1,000 parts.[*] A standard lamp is placed at one end and the light to be measured at the other. Between the two lights there is a movable screen C made of unsized paper which is opaque. The central part of the screen is provided with a spot made translucent by the application of paraffin. Mounted with the screen are two oblique mirrors, M, arranged so that a person

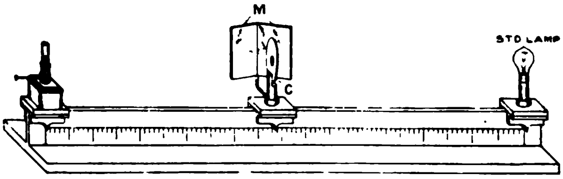

Fig. 16

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[*] On account of the small size of the cut the scale shown is divided into only 100 parts.

looking into them can see both sides of the screen at once. To compare the brightness of the two lights the screen is moved along the bar until the intensity of illumination on both sides is equal, that is until they appear the same. The distances from the lights are then noted.

If B_1 = the brightness of the standard lamp, B_2 = the brightness of the other light, d_1 = the distance of the screen from the standard lamp, and d_2 = the distance of the screen from the other light, then from Art. 67,

$$I = \frac{B_1}{d^2_1} \text{ and } I = \frac{B_2}{d^2_2}.$$

$$\text{Therefore } \frac{B_1}{d^2_1} = \frac{B_2}{d^2_2},$$

$$\text{or } B_1 : B_2 :: d^2_1 : d^2_2. \qquad (7)$$

70. PROBLEM.—What is the candle power of a lamp when it is 900 units from the screen in a Bunsen photometer and the screen is 100 units from a standard lamp of 1 C. P.*?

SOLUTION.—From the conditions of the problem $B_1 = 1$ C. P., $d_1 = 100$, and $d_2 = 900$. Substituting these values in formula (7) we get

$$1 : B_2 :: 10{,}000 : 810{,}000.$$

Therefore $B_2 = 81$ C. P., *Ans.*

71. PROBLEM.—The distance between the screen and a standard lamp of 1 C. P. is 200 units and between the screen and an electric light is 800 units. What is the candle-power of the electric light?

ANSWER.—16 C. P.

72. In using the photometer or other instrument depending upon the equality of two fields of view it is essential, if accurate results are to be obtained, that the light from the two sources should be of the same color. Differences in color make accurate judgments of relative brightness impossible.

REFLECTION

73. A ray of light, CD, Fig. 17, falling upon a polished surface, AB, will be turned back or reflected, in the direction DE.

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*Candle-power.

The ray CD is called the **incident ray**; and the ray DE, the **reflected ray.** The angle S, between the incident ray and the line DF which is perpendicular to AB, is called the **angle of incidence** and the angle R, between DF and DE, is called the **angle of reflection.**

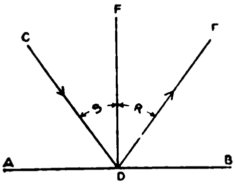

Fig. 17

74. Law of Reflection.—*Whatever the nature of the surface upon which light falls the angle of reflection is always equal to and in the same plane with the angle of incidence.*

75. When a beam of light falls on a polished surface it is reflected in such a manner that an image of the source of the light may be seen. This is termed **regular reflection,** see Fig. 18. When a beam of light falls on an unpolished surface it is reflected in such a manner that no image can be seen. This is called **irregular reflection** or **diffusion,** see Fig. 19.

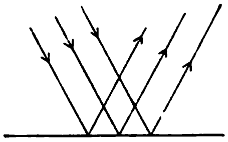

Fig. 18

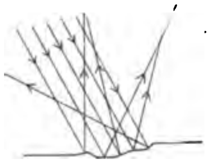

Fig. 19

76. Any polished surface may be used as a mirror; if the surface is plane it is a **plane mirror,** if it is a section of a spherical surface it is a **spherical mirror.** Spherical mirrors are concave or convex according as the inner or outer portion of the spherical surface is used as the reflecting surface. Mirrors are also made of other forms, for example a parabolic mirror.

PLANE MIRRORS

77. If an object is placed before a plane mirror an image of the object will appear in the mirror, apparently at the same distance back of the reflecting surface as the object is in front of it. L, Fig. 20, is one point of an object before a plane mirror MM′. Rays of light pass

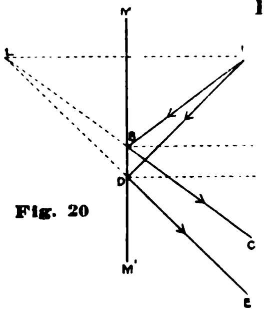

Fig. 20

from the object in different directions. LB and LD represent two of these rays which are reflected in the directions BC and DE respectively, which directions they would have if they came from L′, a point just as far back of the reflecting surface as the point L is in front of it.

Hence if an object represented by the arrow AB, Fig. 21, is placed before a plane mirror MM′ its image will appear behind the mirror in the position A′B′. It should be noted that the image of the object is inverted laterally, the right side of the object appearing on the left side in the image.

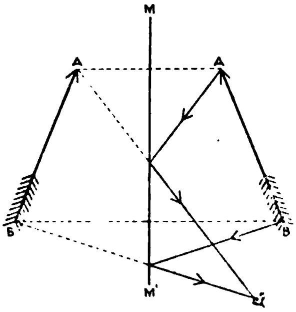

Fig. 21

SPHERICAL MIRRORS

78. AB, Fig. 22, represents a concave mirror of which C is the **center of curvature.** The angle ACB is called the **aperture** of the mirror. D is the **center** of the mirror. The line CD is called the **principal axis.** Any other line through C intersecting the mirror, as CE, is called a **secondary axis.**

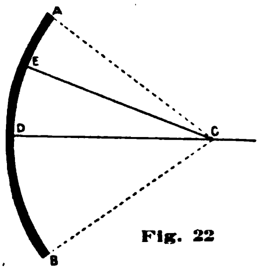

Fig. 22

79. When parallel rays of light fall on a concave mirror they are reflected as shown in Fig. 23 and practically all pass through the point F. This point is called the **principal focus** of the mirror. It is a **real focus,** i. e., all the rays of light actually cross there, and is on the principal axis half way between the center of the mirror, D, and the center of curvature, C. The distance FD is called the **principal focal length** of the mirror.

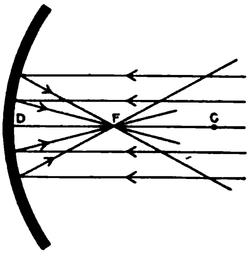

Fig. 23

80. If rays of light diverging from a point G, Fig. 24, on the opposite side of the center of curvature from the mirror, fall upon a concave mirror they will be reflected and practically all pass through a point H between C and F. Or if they diverge from H they will practically all be reflected to G. G and H are called **conjugate foci.**

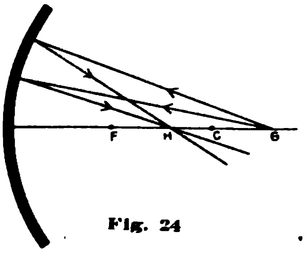

Fig. 24

81. If rays of light diverging from a point M, Fig. 25, between the principal focus F and the center of the mirror D, fall upon a concave mirror they will be reflected and take a direction as if they came from the point N. Since the light appears to come from N, but actually does not, it is called the **virtual focus.**

82. The image formed by reflection of rays of light from a spherical mirror may be constructed geometrically. The construction depends upon two principles:

(1) *A ray of light parallel to the principal axis is reflected through the principal focus if the focus is real or as though it came from the principal focus if the focus is virtual.*

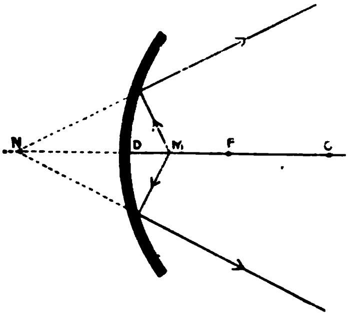

Fig. 25

(2) *A ray along a secondary axis is reflected in the direction from which it came.*

The image of one point of an object is at the place where all the rays from that point intersect. The intersection of two rays from a point determine that place; therefore to determine the image of a point two rays are selected; one parallel to the principal axis, and one along a secondary axis.

83. Figs. 26 and 27 illustrate the method of constructing an image formed by a concave mirror.

X′Y′, Fig. 26, is an image of the object XY. It is smaller than the object, inverted and is called a **real image**; for if a screen is placed at that point it will appear on the screen.

84. The size and position of the image depend largely upon the position of the object. The following cases may arise.

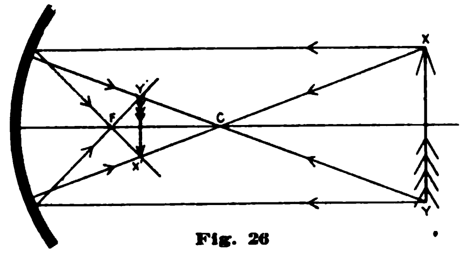

Fig. 26

(1) If the object is so far away that the rays falling on the mirror are all practically parallel the image will appear as a point at the principal focus.

(2) If the object comes nearer the center of curvature the image, which is inverted, also approaches the center of curvature and increases in size.

(3) If the object is at the center of curvature, an inverted image the same size as the object will appear at the center of curvature.

(4) If the object is between the center of curvature and the principal focus the image will be larger than the object, inverted and on the opposite side of the center of curvature. These statements can be verified by constructions similar to the one shown in Fig. 26.

(5) Fig. 27 shows the image formed when the object is between the principal focus and the mirror. It will be noticed that the rays from X diverge after reflection as though they came from the virtual focus X′ and similarly those from Y diverge as though coming from Y′. The image, then, is erect, larger than the object, back of the mirror and is a **virtual image,** i. e., it can be seen in the mirror but cannot be thrown on a screen.

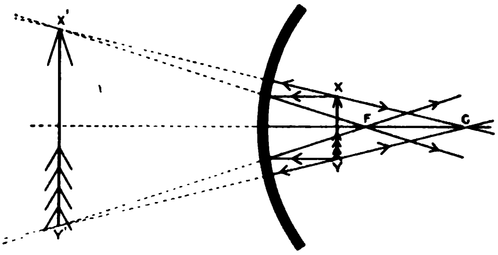

Fig. 27

85. Fig. 28 shows the method of constructing the image formed by a convex mirror. All images formed by convex mirrors are smaller than the object, erect and virtual.

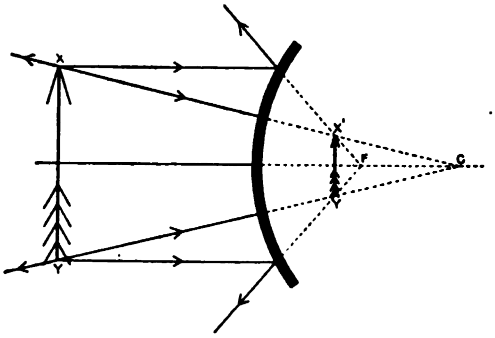

Fig. 28

86. Spherical mirrors should not be made with wide apertures. If the angle ACB, Fig. 29, is greater than about 10° the image will be indistinct because the rays of light falling on the outer portions of the mirror do not, after reflection, pass through the focus. Fig. 29 shows this clearly; the parallel rays falling on the outermost portions of the mirror are reflected according to the law of reflections so that they pass through a point between the principal focus, F, and the center of the mirror, D. This is called **spherical aberration.**

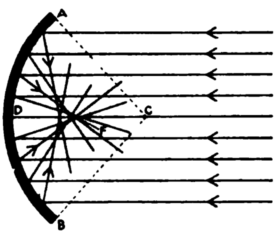

Fig. 29

Spherical aberration can be prevented by using spherical mirrors of small aperture or by the use of parabolic mirrors.

87. Fig. 30 shows a section of a parabolic mirror. It is so shaped that parallel rays falling on any part of the mirror will be reflected so as to pass through the focus, F.

If a light is placed at the focus of a parabolic mirror the reflected rays will be parallel and form a beam of light.

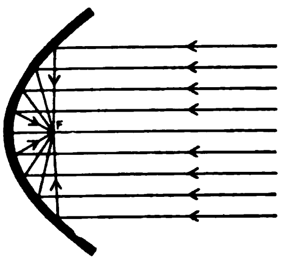

Fig. 30

Parabolic reflectors are used in locomotive headlights and in search lights.

REFRACTION

88. When a ray of light passes obliquely from one medium to another, as from air to glass, its direction is changed. This change of direction is called **refraction.**

The cause of this will be clear when we remember that while the velocity of light is constant for any homogeneous medium, i. e., any medium that has the same structure throughout, it is different for different mediums.

For example, suppose that a beam of light passing through air strikes a pane of glass, AB, Fig. 31, at an oblique angle. Any two rays X and Z, of the beam are parallel. Ray X strikes the glass first and is retarded so that while it goes from K to L, Z goes from M to N. As Z enters the glass it too is retarded and both rays are again parallel. However, as soon as X reaches the point P it passes into the air and at once begins to travel at its former rate so that it goes from P to Q while Z is going from R to S. When Z passes into the air it also resumes its former rate and X and Z are again parallel and travel in the same direction they were going before entering the glass. The beam of light is refracted twice, once on entering and once on leaving the glass.

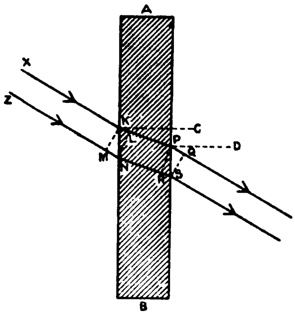

Fig. 31

It should also be noticed that in the first case the ray is bent toward the perpendicular, CK, to the bounding surfaces and in the second case it is bent away from the perpendicular, DP.

89. Suppose ACB, Fig. 32, represents a ray of light passing from air into water, MN the surface of the water, and PQ a perpendicular to the surface

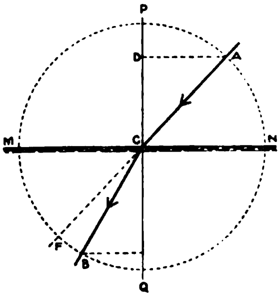

Fig. 32

MN at C. The angle ACP is called the **angle of incidence**; BCQ, the **angle of refraction**; and FCB, the **angle of deviation.** Draw a circle with C as the center and a radius equal to 1 unit. From A and B draw lines AD and BE perpendicular to PQ. Then line AD is called the *sine* of the angle ACP and BE the *sine* of the angle BCQ. The ratio, or quotient, $\frac{AD}{BE}$ is called the **index of refraction.**

90. The following table gives the indices of refraction for some of the more common transparent mediums.

Medium	*Index of Refraction*
Air	1.00
Water	1.33
Alcohol	1.37
Turpentine	1.46
Crown glass	1.50
Flint glass	1.60
Diamond	2.42

91. It is possible, if the indices of refraction of two mediums are known, to construct geometrically the refracted ray. To illustrate this assume that a ray of light passes from air into flint glass and that the angle of incidence is 30 deg.

Let AC, Fig. 33, represent the incident ray meeting the surface MN of the flint glass at C, the angle of incidence ACP being 30 deg. The index of refraction of flint glass is 1.60; expressing this as a fraction we get $\frac{160}{100}$ or $\frac{8}{5}$. With C as a center draw two circles, one with a radius of 8 units, the other with a radius of 5 units, 8 and 5 being the terms of the fraction expressing the index of refraction. From the point D where the smaller circle cuts the ray AC draw a line perpendicular to MCN and extend this line until it cuts the larger circle at E. Draw a line connecting EC and extend it below MCN to F. The line CF is the refracted ray.

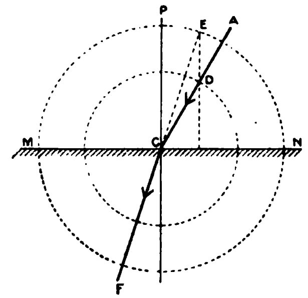

Fig. 33

92. The **laws of refraction** may now be stated as follows:

(1) *When passing into a denser medium a ray of light is refracted toward the perpendicular to the surface; when into a rarer, from the perpendicular.*

(2) *For any two given mediums, the sines of the angles of incidence and of refraction bear a constant ratio to each other, or in other words, the index of refraction for any two mediums is constant.*

(3) *The incident ray and the refracted ray are in the same plane.*

TOTAL REFLECTION

93. Light passing from a denser to a rarer medium is refracted from the perpendicular; consequently the angle of refraction is greater than the angle of incidence. This leads to a peculiar phenomenon under certain conditions. Consider the case of light passing from water to air, Fig. 34. A ray AC, will be refracted to the path, CA′; another ray, BC, having an angle of incidence greater than that of AC will be refracted to the path CB′. If other rays are considered having still greater angles of incidence one will be found the refracted ray of which will pass along the surface of the water; this is represented by DC which is refracted along the surface, CD′. A ray, EC, having an angle of incidence greater than DCP′ is not refracted but is wholly reflected in the path CE′. This phenomenon is called **total reflection**.

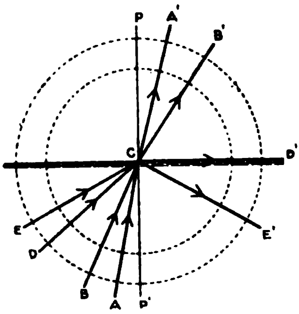

Fig. 34

94. When a ray travels in a medium in such a direction that on emerging it passes along the surface, the angle which it makes with the perpendicular is called the **critical angle** for that medium. Thus in Fig. 34, since the ray DC after passing C travels along the surface CD′, the angle DCP′ which it makes with the perpendicular is the critical angle for water, i. e., about 48½ deg.

The critical angle for crown glass is about 41 deg. and for diamond about 24 deg.

95. Fig. 35 shows a section of a piece of crown glass cut so as to produce total reflection. A ray X entering as shown strikes the surface AB at an angle of 45°, which is greater than the critical angle of crown glass, and is totally reflected.

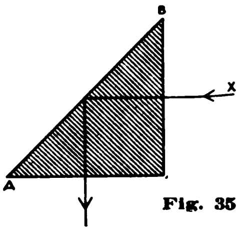

Fig. 35

96. A **plate** is a transparent medium bounded by parallel plane surfaces. When a ray of light passes through a plate at an oblique angle it is reflected twice as shown in Fig. 31, and is said to undergo **lateral aberration.**

97. A transparent medium bounded by two intersecting planes is called a **prism.** Fig. 36 shows the path of a ray of light through a prism. It should be noticed that the ray is refracted away from the thin part of prism toward the thick part. The angle STW formed by the lines of direction of the incident and refracted rays is called the **angle of deviation.**

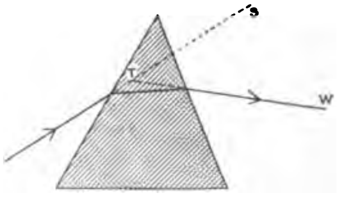

Fig. 36

A prism is called a *reflecting prism* when the light is totally reflected, as in Fig. 35, and a *refracting prism* when the light passes through as in Fig. 36.

98. A transparent medium bounded by two curved surfaces or by one curved and one plane surface is called a **lens.** The boundary surfaces of lenses are usually sections of spherical surfaces.

99. Lenses may be divided into two general classes: (a) **convex lenses** which are thickest in the middle, (1), (2), (3),

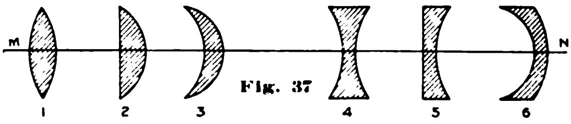

Fig. 37

Fig. 37; (b) **concave lenses** which are thickest at the edge, (4), (5), (6), Fig. 37. There are three kinds in each class named as follows; (1) double-convex, (2) plano-convex, (3) convex-meniscus, (4) double-concave, (5) plano-concave, (6) concave-meniscus. The double-convex and double-concave are types of the respective classes and are used in the following treatment.

100. The line passing through the centers of curvature of the surface bounding a lens is called the **principal axis** of the lens. Thus MN, Fig. 37, is the principal axis of the lenses (1) to (6).

In every lens there is a point on the principal axis such that a ray of light passing through is not changed in direction but suffers slight lateral aberration. This is called the **optical center** of the lens. Fig. 38 shows a ray of light, XYZW, passing through the optical center, C, of a convex lens.

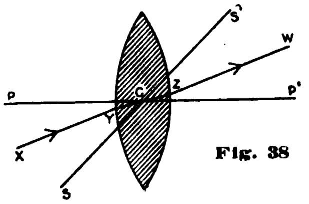

Fig. 38

Any line other than the principal axis passing through the optical center of the lens is called a **secondary axis**. PP′, Fig. 38, is the principal axis and SS′ is a secondary axis.

101. The path of a ray of light through a lens may be traced by the method given in Art. 91. Suppose AB, Fig. 39, is a concave lens of crown glass of which the refractive index is 1.50 or $\frac{3}{2}$. XY is a ray of light falling on the lens at Y. C and C′ are the centers of curvature of the two curved surfaces of AB. The line connecting Y and C is perpendicular to the surface at Y. With Y as a center draw two arcs with radii of 2 and 3 units respectively. From the point where XY intersects

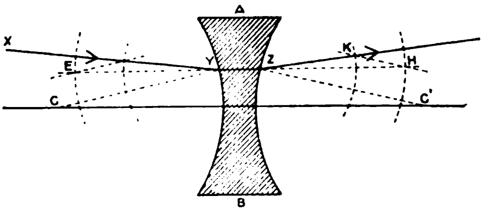

Fig. 39

the smaller arc draw a line parallel to YC intersecting the larger arc at E. Then EY is the direction of the refracted ray and YZ is the path of the ray through the lens. With Z as a center draw two arcs having radii of 2 and 3 units respectively. Extend the line YZ until it intersects the outer arc at H. From H draw a line parallel to ZC′ intersecting the smaller arc at K. Then connect Z and K. ZK is the path of the ray after leaving the lens.

By a similar method the path of a ray through any lens or set of lenses may be traced.

FOCI OF CONVEX LENSES

102. A beam of light passing through a convex lens, AB, Fig. 40, will be refracted in such a way that it will practically all pass through one point F. This point is called the **principal focus.** The distance from the principal focus F to the optical center C is called the **focal length** of the lens.

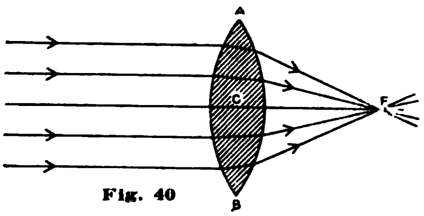

Fig. 40

If rays of light diverging from F pass through the lens they will on emerging form a beam.

103. Rays of light diverging from a point, D, Fig. 41, more than twice the focal length of the lens away from the lens, will

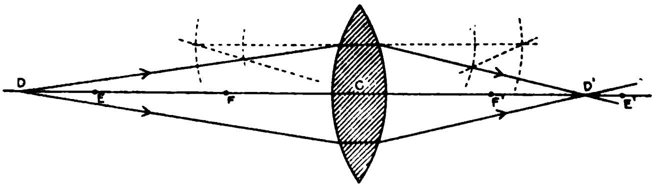

Fig. 41

be brought to a focus at a point D′ between the principal focus, F′, and a point E′ at twice the focal length. Conversely if they diverge from D′ they will be brought to a focus at D. D and D′ are called **conjugate foci.** If the rays diverge from E, Fig. 41, a point at twice the focal length from C, they will come to a focus at E′ and conversely if they diverge from E′ they will come to a focus at E. E and E′ are each twice the focal length from C and are called **secondary foci.**

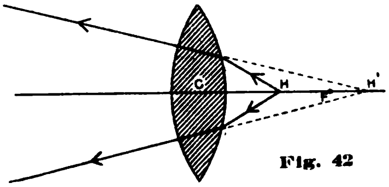

Fig. 42

104. If the rays of light diverge from a point H, Fig. 42, between the principal focus F and the center C, they are still diverging after passing through the lens, as if they came from a point H′; H′ is a **virtual focus.** Conversely, if the incident rays are converging they will be brought to a focus at a point between the principal focus and the center.

FOCI OF CONCAVE LENSES

105. A beam of light passing through a concave lens, AB, Fig. 43, is refracted in such a way that it diverges as though it came from the point F. F is a virtual focus and is called the **principal focus.** The distance from F to the optical center C is the **focal length.**

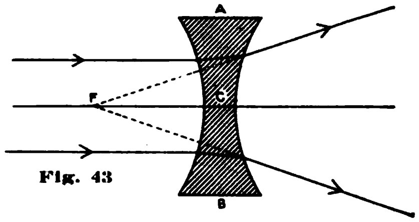

Fig. 43

106. If a diverging pencil of light passes through a concave lens, on emerging it is still more divergent. This is illustrated in Fig. 44. A pencil of light diverging from D after passing through the lens AB is diverging as if it came from D′. D′ is a virtual focus. As the amount of divergence of the incident rays increases the focus D′ moves nearer the lens.

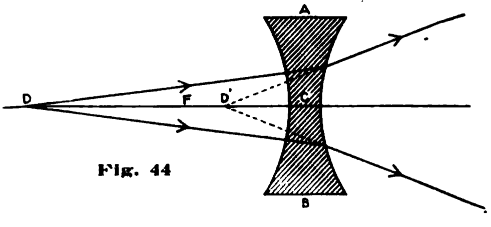

Fig. 44

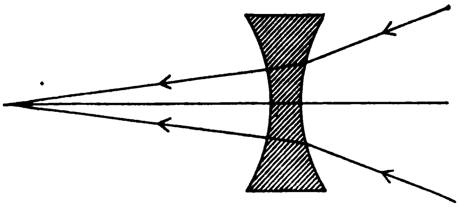

Fig. 45

107. If a converging pencil passes through a concave lens the emerging rays may be converging, parallel, or diverging according to the amount of convergence of the incident rays. Figs. 45, 46 and 47 illustrate the three possible cases.

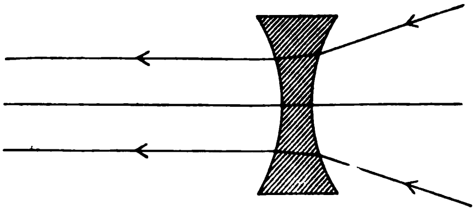

Fig. 46

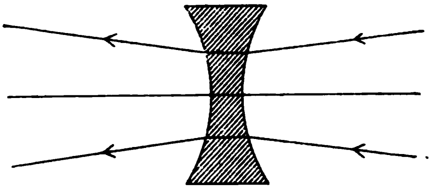

Fig. 47

IMAGES FORMED BY LENSES

108. The image formed by a lens may be found by a geometrical construction similar to that used in constructing the images formed by spherical mirrors. The following principles upon which the construction depends are very similar to those given in Art. 82.

(1) *A ray parallel to the principal axis after refraction passes through the principal focus, if that is real, or takes a direction as if it came from the principal focus, if that is virtual.*

(2) *A ray passing through the optical center does not have its direction changed.*

In applying these principles to construction the result is very nearly exact* and the construction is greatly simplified if the refraction is supposed to take place at one point instead of two. In the following constructions the refraction is thought of as occuring at points on a line perpendicular to the principal axis passing through the optical center. Fig. 48 shows the method of constructing the image formed by a convex lens.

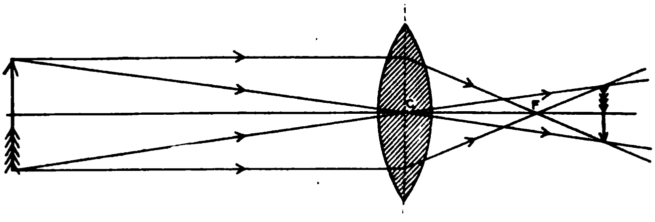

Fig. 48

109. The size and position of the image depend largely upon the position of the object with respect to the lens.

(1) If the object is so far away that the rays are all parallel they will come together at the principal focus and the image will appear as a point.

(2) If the object is nearer than in the first case, but more than twice the focal length from the lens, the image will be real, inverted, smaller than the object and at a distance from the lens greater than the focal length, but less than twice the focal length. This is illustrated in Fig. 48.

(3) If the object is distant twice the focal length, the image will be real, inverted, the same size as the object and at the same distance from the opposite side of the lens.

(4) If the object is at a distance greater than the focal length and less than twice the focal length the image will be real, inverted, larger than the object, and more than twice the focal length away from the lens.

(5) If the object is at the principal focus the emerging rays will be parallel and there will be no image.

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*For thin lenses or small apertures.

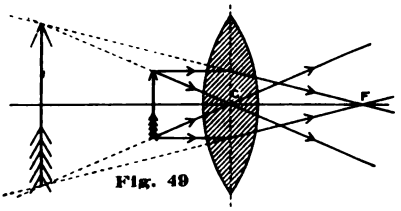

Fig. 49

(6) If the object is at a distance less than the focal length from the lens, the image will be virtual, erect, and larger than the object. This is illustrated in Fig. 49.

110. Fig. 50 shows the same method of construction applied to finding the image formed by a concave lens.

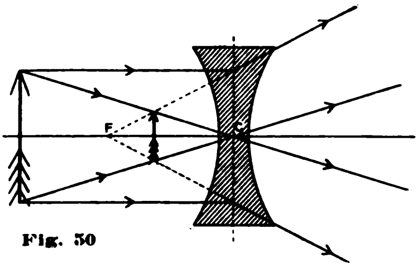

Fig. 50

The images formed by concave lenses are always virtual and erect.

111. Spherical aberration occurs with spherical lenses as well as with spherical mirrors; that is, the rays of light passing through different portions of the lens do not cross the principal axis at the same place. More or less blurring of the image is due to this cause, but can be nearly prevented by using only a small portion of the center of the lens or by using a lens the curvature of which is lessened toward the edge.

COMPOSITION OF LIGHT

112. If a beam of sunlight is allowed to pass through a transparent prism, on emerging, the various rays are no longer parallel

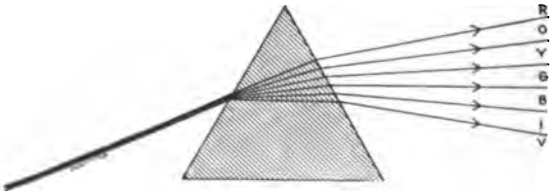

Fig. 51

but diverge as shown in Fig. 51. If the emergent pencil is allowed to fall on a screen it does not appear white, but as a colored band. This colored light is called a **spectrum**; it contains all the colors, having red at one end and violet at the other. The colors are blended so that it is impossible to tell where one ends and the other begins, but the different colors, red, orange, yellow, green, blue, indigo and violet are clearly discernable, the red being refracted the least and the violet the most. The formation of a spectrum by means of a refracting prism is called **dispersion.**

It is evident that light instead of being just one kind of radiant energy is composed of different kinds with different properties. Thus certain kinds of radiant energy are refracted more than other kinds and cause a different sensation. For example, that kind giving a sensation of violet is refracted more than that giving a sensation of red. Light that consist of one particular kind of radiant energy is *homogeneous;* thus, pure red light and pure violet light are homogeneous. Light that consists of two or more kinds of radiant energy is *non-homogeneous;* thus, light from the sun, or an incandescent lamp is non-homogeneous.

113. When light passes through a lens it is dispersed in the same manner as when it passes through a prism. Thus an image of a bright light thrown on a screen by means of a double convex lens is surrounded by colors due to dispersion. This dispersion caused by lenses is called **chromatic aberration.** Chromatic aberration can be corrected by using two lenses made of glass differing in refractive power instead of a single lens. A lens of this type is called **achromatic.** Fig. 52 shows the path of a ray through an achromatic lens.

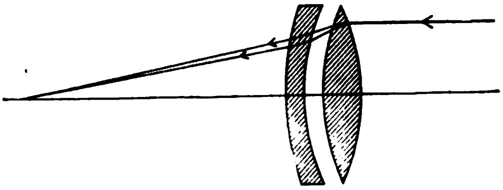

Fig. 52

114. Many substances have the power of stopping or absorbing light of certain kinds while they transmit light of other kinds. For example, if sunlight falls upon a piece of red glass only red light passes through, the rest being absorbed.

This can be well shown by placing a piece of ruby glass between a gas flame and a refracting prism. The spectrum of the flame before the glass is interposed shows all the colors, but when the light passes through the ruby glass all of the colors except the red disappear.

115. The sensations due to light are two, **intensity** or **brightness,** and **color.** Color sensations are due to homogeneous light. Thus, when a beam of homogeneous light falls on the eye a color is seen corresponding to that particular kind of radiant energy.

116. The color of a non-luminous object is due to the fact that the object absorbs all the radiant energy falling on it except that of a particular kind which it reflects. Thus a green leaf in sunlight absorbs the rays of all the colors composing sunlight except the green which are reflected. If a green leaf is placed in a pure red light it appears very dark or black while in a green light it appears even more intensely green than in sunlight.

117. It has been found by experiment that by mixing three colors in proper proportions any color can be matched so far as the sensation of color is concerned. These three colors, called **primary colors,** are red, green and violet. It has also been found that certain colors when mixed produce white. Such colors are called **complementary.** Red and bluish green, orange and blue, and indigo and yellow are complementary colors. As an example of mixing color suppose that a disc is arranged so that it can be rotated rapidly. The eye sees the surface of the disc by reflected light and, if portions of this surface are colored differently, the colors may be distinguished from each other as long as the disc is stationary. However, if the disc is rotated rapidly, although each portion reflects its color just as before, the eye is unable to follow the rapid motion and apparently the different colors blend or mix together and form one. Thus, if orange and blue are mixed in the proper proportion by this method, the resulting color is nearly white.

The preceding statements do not hold true, however, with regard to mixing pigments*. For example, consider the action of two pigments such as yellow and blue. According to Art. 116 each pigment reflects certain colors in the light falling on it, absorbing the others. Thus the yellow pigment reflects yellow together with some orange and green absorbing the other colors. Similarly the blue pigment reflects blue and some green and indigo. Therefore a mixture of these two pigments will appear green since the colors reflected by the blue, with the exception of green, are absorbed by the yellow, and the colors reflected by the yellow, with the exception of green, are absorbed by the blue leaving only green to be reflected.

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*Paints.

118. The most distinct colors are red, yellow, green, and blue. On account of their distinctiveness these colors are best adapted to signaling purposes.

119. By the use of a refracting prism it is possible to determine whether the light transmitted by any particular piece of glass is pure in color or not, i. e., whether it transmits homogeneous or non-homogeneous light. For example, a piece of glass which to the eye transmits only red light may be found to transmit also some yellow or green light.

OPTICAL INSTRUMENTS

120. An **optical instrument** is one necessary to vision, i. e., the eye, or one used to assist vision. The parts of the eye directly employed in the act of seeing are a diaphram moved by muscles so as to regulate the amount of light entering the eye; a lens, the form of which can be changed by muscles so as to accommodate the eye either to seeing objects nearby or at a distance; and a sensitive coating containing the ends of the nerves of sight.

There are certain defects of vision due to imperfect formation of the lens or to imperfect shape of the eye. The most common of these are **near-sight, far-sight** and **astigmatism.** These can be corrected by placing lenses of the proper shape before the eye.

121. A commonly accepted explanation of the color sense is that there are in the eye three different sets of nerves which respond to impressions of the primary colors, red, green, and violet and which may be called red nerves, green nerves, and violet nerves, respectively. Since any color sensation can be obtained by mixing these three, other colors which are seen and white light are only combined sensations of two or three of the primary colors.

122. Color blind persons are those who lack one or the other of these sets of nerves and who consequently cannot distinguish the color which would in an ordinary person affect that set of nerves. Very commonly the red nerves are lacking so that for a person so afflicted all the colors which he sees can be matched by combinations of green and violet light.

The ordinary test for color blindness consists in having a person pick out from a number of various colored skeins of wool those which match three or more test skeins. The test skeins are of at least three different colors, light green, rose and red. A person lacking red nerves will commonly match the light green with pale yellow and red with pure blues or blues containing some purple.

123. The color nerves of a person looking at one color for a long time become fatigued so that the person does not get as clear a sensation of that color as he does of other colors. To illustrate this suppose a person wore a pair of green spectacles for some time, upon taking them off the greens would appear dim to him while the reds and violets would appear brighter than usual.

A somewhat similar effect is seen when two colors are placed side by side. If two complementary colors such as blue and orange are next to each other they cause one another to appear more brilliant.

124. There are several different optical instruments employing lenses or reflectors or both. Among these are the telescope, which is used to enlarge the images of distant objects; the microscope, which is used to give magnified images of very small objects; the camera, which is used to throw images upon a plate sensitive to light; and various sorts of lanterns, some used like the projection lantern to cast enlarged images on a screen and others, such as the search-light and head-light, to project a large amount of light in one direction.

SIGNAL LAMPS

125. The signal lamp is a form of lantern used for the purpose of projecting light in one or more directions. It is constructed essentially as follows. A source of light, such as a kerosene lamp A, Fig. 53, is enclosed in a case B to protect the lamp from the weather and prevent the light from traveling in any except the desired direction. The arrows indicate the currents supplying air to the flame and carrying away the products of combustion. A lens C is arranged to alter the distribution of light and project as much as possible in the desired direction and sometimes a reflector D is added to increase the intensity.

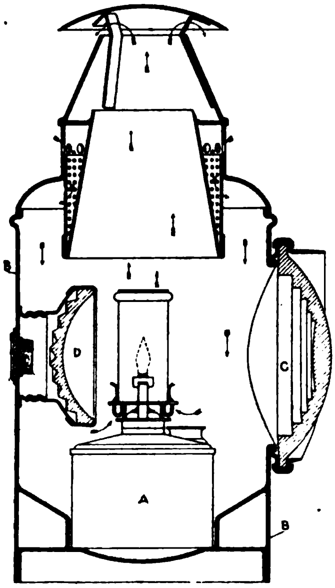
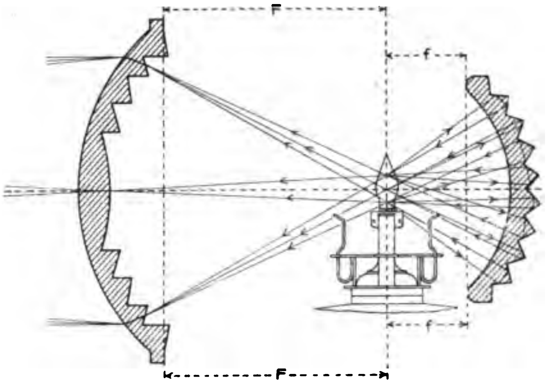


Fig. 53

126. The lenses used in signal lamps are generally of the form shown in Figs. 53 and 54. This form has taken the place of the plano-convex type of lens, which was originally used, for the following reasons. A corrugated lens does not vary as much in thickness as a plano-convex lens, making it possible to secure a more uniform color throughout a colored lens. Spherical aberration, always present in a plano-convex lens, is avoided by properly designing the curvature of the outer corrugations. Although the corrugations may be placed either on the inside or outside surface of the lens, the former position is used so as to give a smooth outer face that is easy to clean and that does not afford as good a lodging place for dirt, snow and ice, as the corrugations would if placed outside.

127. Fig. 54 is a diagram illustrating the correct arrangement of the source of light, lens and reflector. It should be noted that



F = focal length of lens
f = focal length of reflector

Fig. 54

the focal length of a signal lens is measured from the edge, as shown in the figure, instead of from the optical center, a focal length of 3½ in. being generally used.

The diagram shows that a considerable portion of the light radiating from the flame passes through the lens which changes its direction very nearly into a beam of parallel rays. The result is that a surface at 100 ft. from the source of light, instead of receiving only about one quarter of the light that would fall on a similar surface at 50 ft., receives very nearly the same amount. Thus the intensity of the light is decreased with distance only by the amount that the air absorbs and by the fact that the rays must diverge slightly to allow the light to spread over a sufficient area to be seen on curves, etc.

The reflector shown, called a prism reflector, is made of glass and depends for its action on the principle of total reflection described in Arts. 93, 94 and 95. Each ray is reflected twice and sent to the lens through the point in the flame from which it came. Thus the light from the back of the flame is caused to pass through the lens instead of being lost as would occur without the reflector. The distance f, called the focal length of the reflector, should have the proper value to accomplish this result. Reflectors of polished metal should be spherical in form and placed so that the center of curvature coincides with the center of the flame. They possess the disadvantage of tarnishing and losing their polish quickly.

128. It is important that the lens, source of light, and reflector should initially occupy correct positions with regard to each other as shown in Fig. 54, and that these positions should be maintained in service. The result of as small a change in position as $\frac{1}{8}$ in. will decrease the value of a lens or reflector by an appreciable amount and, if the center of the flame is not in the axis of the lens, the projected light will be somewhat deflected from the desired path.

129. Colored light may be obtained from a signal lamp by using a colored lens, in which case the color is not changed. If it is necessary to obtain different colors at different times from the same lamp an uncolored lens is used and different colored glasses, called roundels, are mounted in a movable frame in front of the lens so that the light may be projected through any roundel desired. The colors most used are red, yellow and green since they are very distinct and glass of these colors can be made to transmit enough light to enable the color to be distinguished at a considerable distance.

130. In some cases it is desirable to place two colors, such as red and green, side by side. Fig. 55 shows a section and front view of a lamp arranged for this purpose. The paths of the rays of light are indicated on the plan. It will be noticed that those passing through the uncolored lens A are reflected at an angle of 90 deg. by mirror B before passing through the green roundel C. Therefore the green light from C and red light from lens D have the same direction.

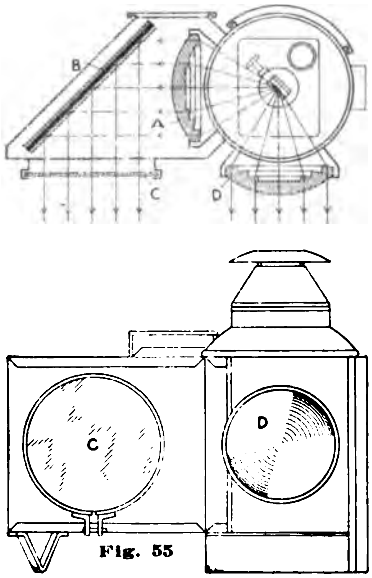


Fig. 55

EXAMINATION QUESTIONS AND PROBLEMS

(1) Explain the action of the thermometer.

(2) (*a*) What three methods are there of transmitting heat? (*b*) Give an example of each, different from that given in the text.

(3) A tank holding 30 cu. ft. is filled with gas at a pressure of 50 lbs. per sq. in. and at a temperature of 32 deg. F. What will be the pressure if the temperature increases to 62 deg. F.?

(4) The coefficient of linear expansion of lead is .00001582. How many inches will a lead pipe 200 ft. long expand if its temperature changes from —10 deg. F. to 90 deg. F.?

(5) Reduce 40 deg. Centigrade to the corresponding Fahrenheit reading.

(6) Reduce —13 deg. Fahrenheit to the corresponding Centigrade reading.

(7) How much heat will be required to change 10 lbs. of ice at 32 deg. F. to steam at 212 deg. F., if the latent heat of fusion of ice is 144 B. T. U., and the latent heat of vaporization of water is 965.7 B. T. U.?

(8) A piece of iron weighing 5 lbs. at a temperature of 1,000 deg. F. is placed in a tub containing 125 lbs. of water at 60 deg. F. What is the resulting temperature if the specific heat of iron is 0.1298 and of water is 1?

(9) What property of a gas is used in transforming heat energy into mechanical energy?

(10) What is the meaning of the term "specific heat"?

(11) Explain why a vessel containing water is sometimes broken when the water freezes.

(12) What is the difference between luminous and illuminated bodies?

(13) Give original examples of transparent, translucent, and opaque bodies.

(14) Two lamps illuminate a screen with equal intensity. If the first is a lamp of 16 C. P., and is 8 ft. from the screen, what is the candle-power of the second if it is 12 ft. from the screen?

(15) State the law of reflection and give an example.

(16) What advantage is there in using a parabolic mirror?

(17) Construct geometrically the path of a ray of light passing from air into water assuming the angle of incidence to be 30 deg.

(18) (*a*) Explain what is meant by the sine of the angle of incidence. (*b*) By the index of refraction.

(19) Explain the phenomenon of total reflection.

(20) What are the primary colors? Explain why they are called primary.

(21) What is spherical aberration and how can it be corrected?

(22) Explain how a red light may be produced from a light such as that furnished by a kerosene lamp.

(23) How may a green glass be tested for the purpose of finding out if it transmits any other color?

(24) State the reasons for using corrugations on a signal lens.

(25) Of what advantage is a reflector?

ANSWERS TO EXAMINATION PROBLEMS

(1) ?

(2) ?

(3) 53.05 lbs. per sq. in.

(4) 3.7968 in.

(5) 104 deg. F.

(6) —25 deg. C.

(7) 12,897 B. T. U.

(8) 64.8 deg. F.

(9) ?

(10) ?

(11) ?

(12) ?

(13) ?

(14) 36 C. P.

(15) to (25) ?

TABLE OF SYMBOLS*

B_1 = brightness of standard lamp.

B_2 = brightness of any light.

C = degrees Centigrade.

d_1 = distance from screen to standard lamp.

d_2 = distance from screen to any light.

F = degrees Fahrenheit.

P_0 and P_1 = pressures on a mass of gas corresponding to volumes V_0 and V_1 respectively.

s = specific heat of a body.

s' = specific heat of water or other liquid.

T = final temperature.

t = temperature of a body.

t' = temperature of water or other liquid.

t_1 = any temperature other than 32° F.

V_0 = volume of a gas at 32° F.

V_1 = volume of a gas at a temperature t_1.

w = weight of a body.

w' = weight of water or other liquid.

TABLE OF FORMULAS

The formulas are numbered consecutively throughout the text and are placed and numbered in the same order in the following table. As an aid in referring to the text the page number is given directly after the formula number.

		Page No.
$V_1 = V_0\left(1 + \frac{t_1 - 32}{491.2}\right)$	(1)	(5)
$V_1 = V_0\frac{P_0}{P_1}\left(1 + \frac{t_1 - 32}{491.2}\right)$	(2)	(5)
$C = \frac{5}{9}(F - 32°)$	(3)	(8)
$F = \frac{9}{5}C + 32°$	(4)	(8)
$sw(t - T) = s'w'(T - t')$	(5)	(12)
$s = \frac{s'w'(T - t')}{w(t - T)}$	(6)	(12)
$B_1 : B_2 :: d_1^2 : d_2^2$	(7)	(21)

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*The symbols and formulas used in the text are collected in these tables for convenient reference and need not be studied.

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